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ON APPROXIMATE METHODS FOR FRACTAL VEHICULAR TRAFFIC FLOW

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ABSTRACT. In this paper, we find the approximate solutions for partial differential equations arising in fractal vehicular traffic flow by using the local fractional Laplace decomposition method (LFLDM) and local fractional series expansion method (LFSEM). These methods provide us with a convenient way to find the approximate solution with less computation as compared with local fractional variational iteration method. The results obtained by the proposed methods (LFLDM) and (LFSEM) are compared with the results obtained by (LFLVIM). Some examples are presented to illustrate the efficiency and accuracy of the proposed methods.

Keywords: Lighthill-Whitham-Richards model, local fractional Laplace decomposition method, local fractional series expansion method, fractal vehicular traffic flow.

AMS Subject Classification: 26A33, 35R11, 74H10.

1. Introduction

Lighthill and Whitham (1955), and Richards (1956) independently presented a macroscopic model of traffic flow to describe the dynamic characteristics of traffic on a homogeneous and unidirectional highway, which is now known as the LWR model in the literature of traffic flow theory. The LWR model is still widely used for the modeling of traffic flow, because of its simplicity and good explanatory power to understand the qualitative behavior of road traffic [1].

The Lighthill-Whitham-Richards model was studied by Li [2] and Wang [3] on a finite length highway and reads as follows

$$\frac{\partial^{\vartheta} M(x,t)}{\partial t^{\vartheta}} + \mu \frac{\partial^{\vartheta} M(x,t)}{\partial x^{\vartheta}} = 0, \ 0 < \vartheta \leq 1 \tag{1}$$

where the initial and boundary conditions are presented as follows

$$\begin{aligned} M(x,0) &= \varphi(x); \\ M(0,t) &= \psi(t). \end{aligned} \tag{2}$$

There are some developed technologies to solve partial differential equations with local fractional operator such as the local fractional variational iteration method [4, 5], local fractional series expansion method [6, 7], local fractional decomposition method [7, 8], local fractional function decomposition method [9, 10], local fractional Laplace variational iteration method [2, 11, 12], and local fractional reduce differential transform method [13].

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In this work, two methods; namely the local fractional Laplace decomposition method and local fractional series expansion method with local fractional operators, are proposed to solve the linear partial differential equations arising in fractal vehicular traffic flow.

2. Local Fractional Laplace Decomposition Method

Let us consider the following, the local fractional differential operator;

$$L_{\vartheta}M(x,t)+R_{\vartheta}M(x,t)=h(x,t) \tag{3}$$

where $L_{\vartheta}=\dfrac{\partial^{\vartheta}}{\partial t^{\vartheta}}$ denotes the linear local fractional differential operator, R_{ϑ} is the remaining linear operator, and $h(x,t)$ is a source term.

Taking local fractional Laplace transform on (3), we obtain

$$\widetilde{L}_{\vartheta}\left\{L_{\vartheta}M(x,t)\right\}+\widetilde{L}_{\vartheta}\left\{R_{\vartheta}M(x,t)\right\}=\widetilde{L}_{\vartheta}\left\{h(x,t)\right\}. \tag{4}$$

By applying the local fractional Laplace transform differentiation property, we have

$$s^{\vartheta}\widetilde{L}_{\vartheta}\left\{M(x,t)\right\}-M(x,0)+\widetilde{L}_{\vartheta}\{R_{\alpha}M(x,t)\}=\widetilde{L}_{\vartheta}\{h(x,t)\}. \tag{5}$$

or equivalently

$$\widetilde{L}_{\vartheta}\left\{M(x,t)\right\}=\frac{1}{s^{\vartheta}}M(x,0)+\frac{1}{s^{\vartheta}}\widetilde{L}_{\vartheta}\left\{h(x,t)\right\}-\frac{1}{s^{\vartheta}}\widetilde{L}_{\vartheta}\left\{R_{\vartheta}M(x,t)\right\}. \tag{6}$$

Taking the inverse of local fractional Laplace transform on Eq. (6), we have

$$M(x,t)=M(x,0)+\widetilde{L}_{\vartheta}^{-1}\left[(\frac{1}{s^{\vartheta}}\widetilde{L}_{\vartheta}\{h(x,t)\}\right]-\widetilde{L}_{\vartheta}^{-1}\left[\frac{1}{s^{\vartheta}}\widetilde{L}_{\vartheta}\left\{R_{\vartheta}M(x,t)\right\}\right]. \tag{7}$$

We are going to represent the solution in an infinite series given below:

$$M(x,t)=\sum_{n=0}^{\infty}M_n(x,t). \tag{8}$$

Substituting (8) into (7), which give us this result:

$$\sum_{n=0}^{\infty}M(x,t)=M(x,0)+\widetilde{L}^{-1}\left[\frac{1}{s^{\vartheta}}L_{\vartheta}\{h(x,t)\}\right]-\widetilde{L}_{\vartheta}^{-1}\left[\frac{1}{s^{\vartheta}}\widetilde{L}\left\{R_{\vartheta}\sum_{n=0}^{\infty}M_n(x,t)\right\}\right]. \tag{9}$$

When we compare the left and right hand sides of (9) we obtain

$$\begin{aligned}
M_0(x,t) &= M(x,0)+\widetilde{L}^{-1}\left[\frac{1}{s^{\vartheta}}\widetilde{L}_{\vartheta}\{h(x,t)\}\right],\\
M_1(x,t) &= -\widetilde{L}_{\vartheta}^{-1}\left[\frac{1}{s^{\vartheta}}\widetilde{L}_{\vartheta}\left\{R_{\vartheta}M_0(x,t)\right\}\right],\\
M_2(x,t) &= -\widetilde{L}_{\vartheta}^{-1}\left[\frac{1}{s^{\vartheta}}\widetilde{L}_{\vartheta}\left\{R_{\vartheta}M_1(x,t)\right\}\right],\\
M_3(x,t) &= -\widetilde{L}_{\vartheta}^{-1}\left[\frac{1}{s^{\vartheta}}\widetilde{L}_{\vartheta}\left\{R_{\vartheta}M_2(x,t)\right\}\right],\\
&\vdots
\end{aligned} \tag{10}$$

The recursive relation, in general form is

$$M_0(x,t) = M(x,0) + \widetilde{L}^{-1}\left[\frac{1}{s^{\vartheta}}\widetilde{L}_{\vartheta}\{h(x,t)\}\right],$$
$$M_{n+1}(x,t) = -\widetilde{L}_{\vartheta}^{-1}\left[\frac{1}{s^{\vartheta}}\widetilde{L}_{\vartheta}\left\{R_{\vartheta}M_n(x,t)\right\}\right]. \tag{11}$$

3. Local Fractional Series Expansion Method

We can written the equation (1) in the form

$$\frac{\partial^{\vartheta}M(x,t)}{\partial t^{\vartheta}} = R_{\vartheta}M(x,t), \tag{12}$$

where $R_{\vartheta}M(x,t) = -\mu L_x^{(\vartheta)}M(x,t)$ is linear local fractional derivative operator of order ϑ with respect to x.

In accordance with the results in [6, 7], there are multiterm separated functions of independent variables x and t reads as

$$M(x,t) = \sum_{i=0}^{\infty} T_i(t)\Phi_i(x), \tag{13}$$

where $T_i(t)$ and $\Phi_i(x)$ are local fractional continuous functions.

In view of (13), we consider

$$T_i(t) = \frac{t^{i\vartheta}}{\Gamma(1+i\vartheta)}, \tag{14}$$

so that

$$M(x,t) = \sum_{i=0}^{\infty} \frac{t^{i\vartheta}}{\Gamma(1+i\vartheta)}\Phi_i(x). \tag{15}$$

In view of (15), we obtain

$$\frac{\partial^{\vartheta}M(x,t)}{\partial t^{\vartheta}} = \sum_{i=0}^{\infty} \frac{t^{i\vartheta}}{\Gamma(1+i\vartheta)}\Phi_{i+1}(x), \tag{16}$$

and

$$R_{\vartheta}M(x,t) = R_{\vartheta}\left(\sum_{i=0}^{\infty} \frac{t^{i\vartheta}}{\Gamma(1+i\vartheta)}\Phi_i(x)\right) = \sum_{i=0}^{\infty} \frac{t^{i\vartheta}}{\Gamma(1+i\vartheta)}\left(R_{\vartheta}\Phi_i\right)(x), \tag{17}$$

Making use of (17), we get

$$\sum_{i=0}^{\infty} \frac{t^{i\vartheta}}{\Gamma(1+i\vartheta)}\Phi_{i+1}(x) = \sum_{i=0}^{\infty} \frac{t^{i\vartheta}}{\Gamma(1+i\vartheta)}\left(R_{\vartheta}\Phi_i\right)(x). \tag{18}$$

Hence, from (18), the recursion reads as follows:

$$\Phi_{i+1}(x) = \left(R_{\vartheta}\Phi_i\right)(x). \tag{19}$$

By using (19), we arrive at the following result:

$$M(x,t) = \sum_{i=0}^{\infty} \frac{t^{i\vartheta}}{\Gamma(1+i\vartheta)}\Phi_i(x). \tag{20}$$

4. Illustrative Examples

In this section, we present the initial and boundary value problems for linear partial differential equations arising in fractal vehicular traffic flow.

Example 4.1. *Let us consider the Lighthill-Whitham-Richards model on a finite length highway as*

$$\frac{\partial^\vartheta M(x,t)}{\partial t^\vartheta} + \mu \frac{\partial^\vartheta M(x,t)}{\partial x^\vartheta} = 0,\ \ 0 < \vartheta \leq 1 \tag{21}$$

where the initial and boundary conditions are presented as follows:

$$\begin{aligned} M(x,0) &= E_\vartheta(x^\vartheta); \\ M(0,t) &= \cosh_\vartheta(\mu t^\vartheta) - \sinh_\vartheta(\mu t^\vartheta). \end{aligned} \tag{22}$$

I. By using LFLDM

In view of (11) and (21) the local fractional iteration algorithm can be written as follows:

$$\begin{aligned} &M_0(x,t) = E_\vartheta(x^\vartheta); \\ &M_{n+1}(x,t) = -\widetilde{L}_\vartheta^{-1}\left[\frac{\mu}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^\vartheta M_n(x,t)}{\partial x^\vartheta}\right\}\right], n \geq 0. \end{aligned} \tag{23}$$

Therefore, from (23) we give the components as follows:

$$M_0(x,t) = E_\vartheta(x^\vartheta); \tag{24}$$

$$\begin{aligned} M_1(x,t) &= -\widetilde{L}_\vartheta^{-1}\left[\frac{\mu}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^\vartheta M_0(x,t)}{\partial x^\vartheta}\right\}\right] \\ &= -\widetilde{L}_\vartheta^{-1}\left[\frac{\mu}{s^{2\vartheta}}E_\vartheta(x^\vartheta)\right] \\ &= -\frac{\mu t^\vartheta}{\Gamma(1+\vartheta)}E_\vartheta(x^\vartheta); \end{aligned} \tag{25}$$

$$\begin{aligned} M_2(x,t) &= -\widetilde{L}_\vartheta^{-1}\left[\frac{\mu}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^\vartheta M_1(x,t)}{\partial x^\vartheta}\right\}\right] \\ &= \widetilde{L}_\vartheta^{-1}\left[\frac{\mu^2}{s^{3\vartheta}}E_\vartheta(x^\vartheta)\right] \\ &= \frac{\mu^2 t^{2\vartheta}}{\Gamma(1+2\vartheta)}E_\vartheta(x^\vartheta); \end{aligned} \tag{26}$$

$$\begin{aligned} M_3(x,t) &= -\widetilde{L}_\vartheta^{-1}\left[\frac{\mu}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^\vartheta M_2(x,t)}{\partial x^\vartheta}\right\}\right] \\ &= -\widetilde{L}_\vartheta^{-1}\left[\frac{\mu^3}{s^{4\vartheta}}E_\vartheta(x^\vartheta)\right] \\ &= -\frac{\mu^3 t^{3\vartheta}}{\Gamma(1+3\vartheta)}E_\vartheta(x^\vartheta); \end{aligned} \tag{27}$$

$$\begin{aligned} M_4(x,t) &= -\widetilde{L}_\vartheta^{-1}\left[\frac{\mu}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^\vartheta M_3(x,t)}{\partial x^\vartheta}\right\}\right] \\ &= \widetilde{L}_\vartheta^{-1}\left[\frac{\mu^4}{s^{5\vartheta}}E_\vartheta(x^\vartheta)\right] \\ &= \frac{\mu^4 t^{4\vartheta}}{\Gamma(1+4\vartheta)}E_\vartheta(x^\vartheta). \end{aligned} \tag{28}$$

Consequently, we obtain

$$\begin{aligned} M(x,t) &= E_\vartheta(x^\vartheta)\left[1+\frac{\mu^2 t^{2\vartheta}}{\Gamma(1+2\vartheta)}+\frac{\mu^4 t^{4\vartheta}}{\Gamma(1+4\vartheta)}+\cdots\right] \\ &\quad -E_\vartheta(x^\vartheta)\left[\frac{\mu t^{\vartheta}}{\Gamma(1+\vartheta)}+\frac{\mu^3 t^{3\vartheta}}{\Gamma(1+3\vartheta)}+\cdots\right] \\ &= E_\vartheta(x^\vartheta)\cosh_\vartheta(\mu t^\vartheta)-E_\vartheta(x^\vartheta)\sinh_\vartheta(\mu t^\vartheta) \end{aligned} \tag{29}$$

II. By using LFSEM

From (19), we obtain the following iterative formula;

$$\Phi_{i+1}(x) = -\mu L_x^{(\vartheta)}\Phi_i(x), \tag{30}$$

where

$$\Phi_0(x) = E_\vartheta(x^\vartheta). \tag{31}$$

In view of (30) and (31), we obtain the approximations given by

$$\begin{aligned} &\Phi_1(x) = -\mu L_x^{(\vartheta)}\Phi_0(x) = -\mu E_\vartheta(x^\vartheta); \\ &\Phi_2(x) = -\mu L_x^{(\vartheta)}\Phi_1(x) = \mu^2 E_\vartheta(x^\vartheta); \\ &\Phi_3(x) = -\mu L_x^{(\vartheta)}\Phi_2(x) = -\mu^3 E_\vartheta(x^\vartheta); \\ &\Phi_4(x) = -\mu L_x^{(\vartheta)}\Phi_3(x) = \mu^4 E_\vartheta(x^\vartheta); \\ &\Phi_5(x) = -\mu L_x^{(\vartheta)}\Phi_4(x) = -\mu^5 E_\vartheta(x^\vartheta); \\ &\vdots \end{aligned} \tag{32}$$

Therefore, by using (20) and (33) we get the solution

$$\begin{aligned} M(x,t) &= \sum_{i=0}^{\infty}\frac{t^{i\vartheta}}{\Gamma(1+i\vartheta)}\Phi_i(x) \\ &= E_\vartheta(x^\vartheta)\left[1-\frac{\mu t^{\vartheta}}{\Gamma(1+\vartheta)}+\frac{\mu^2 t^{2\vartheta}}{\Gamma(1+2\vartheta)}-\frac{\mu^3 t^{3\vartheta}}{\Gamma(1+3\vartheta)}+\frac{\mu^4 t^{4\vartheta}}{\Gamma(1+4\vartheta)}-\cdots\right] \\ &= E_\vartheta(x^\vartheta)\left[1+\frac{\mu^2 t^{2\vartheta}}{\Gamma(1+2\vartheta)}+\frac{\mu^4 t^{4\vartheta}}{\Gamma(1+4\vartheta)}+\cdots\right] \\ &\quad -E_\vartheta(x^\vartheta)\left[\frac{\mu t^{\vartheta}}{\Gamma(1+\vartheta)}+\frac{\mu^3 t^{3\vartheta}}{\Gamma(1+3\vartheta)}+\cdots\right] \\ &= E_\vartheta(x^\vartheta)\cosh_\vartheta(\mu t^\vartheta)-E_\vartheta(x^\vartheta)\sinh_\vartheta(\mu t^\vartheta). \end{aligned} \tag{33}$$

From Eqs. (29) and (33), the approximate solution of the given problem (21), by using local fractional Laplace decomposition method and local fractional series expansion method, is the same results as that obtained by the local fractional Laplace variational iteration method.

Example 4.2. *Let us consider the Lighthill-Whitham-Richards model on a finite length highway as:*

$$\frac{\partial^\vartheta M(x,t)}{\partial t^\vartheta}+\frac{\partial^\vartheta M(x,t)}{\partial x^\vartheta}=0,\ \ 0<\vartheta\le 1 \tag{34}$$

where the initial and boundary conditions are presented as follows:

$$\begin{aligned} M(x,0) &= \sinh_\vartheta(x^\vartheta); \\ M(0,t) &= 0. \end{aligned} \tag{35}$$

I. By using LFLDM

In view of (11) and (34) the local fractional iteration algorithm can be written as follows:

$$\begin{aligned} M_0(x,t) &= \sinh_\vartheta(x^\vartheta); \\ M_{n+1}(x,t) &= -\widetilde{L}_\vartheta^{-1}\left[\frac{1}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^\vartheta M_n(x,t)}{\partial x^\vartheta}\right\}\right], n \geq 0. \end{aligned} \tag{36}$$

Therefore, from (36) we give the components as follows:

$$M_0(x,t) = \sinh_\vartheta(x^\vartheta); \tag{37}$$

$$\begin{aligned} M_1(x,t) &= -\widetilde{L}_\vartheta^{-1}\left[\frac{1}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^\vartheta M_0(x,t)}{\partial x^\vartheta}\right\}\right] \\ &= -\widetilde{L}_\vartheta^{-1}\left[\frac{1}{s^{2\vartheta}}\sinh_\vartheta(x^\vartheta)\right] \\ &= -\frac{t^\vartheta}{\Gamma(1+\vartheta)}\sinh_\vartheta(x^\vartheta); \end{aligned} \tag{38}$$

$$\begin{aligned} M_2(x,t) &= -\widetilde{L}_\vartheta^{-1}\left[\frac{1}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^\vartheta M_1(x,t)}{\partial x^\vartheta}\right\}\right] \\ &= \widetilde{L}_\vartheta^{-1}\left[\frac{1}{s^{3\vartheta}}\sinh_\vartheta(x^\vartheta)\right] \\ &= \frac{t^{2\vartheta}}{\Gamma(1+2\vartheta)}\sinh_\vartheta(x^\vartheta); \end{aligned} \tag{39}$$

$$\begin{aligned} M_3(x,t) &= -\widetilde{L}_\vartheta^{-1}\left[\frac{1}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^\vartheta M_2(x,t)}{\partial x^\vartheta}\right\}\right] \\ &= -\widetilde{L}_\vartheta^{-1}\left[\frac{1}{s^{4\vartheta}}\sinh_\vartheta(x^\vartheta)\right] \\ &= -\frac{t^{3\vartheta}}{\Gamma(1+3\vartheta)}\sinh_\vartheta(x^\vartheta); \end{aligned} \tag{40}$$

$$\begin{aligned} M_4(x,t) &= -\widetilde{L}_\vartheta^{-1}\left[\frac{\mu}{s^\vartheta}\widetilde{L}_\vartheta\left\{\frac{\partial^\vartheta M_3(x,t)}{\partial x^\vartheta}\right\}\right] \\ &= \widetilde{L}_\vartheta^{-1}\left[\frac{1}{s^{5\vartheta}}\sinh_\vartheta(x^\vartheta)\right] \\ &= \frac{t^{4\vartheta}}{\Gamma(1+4\vartheta)}\sinh_\vartheta(x^\vartheta). \end{aligned} \tag{41}$$

Consequently, we obtain

$$\begin{aligned} M(x,t) &= \sinh_\vartheta(x^\vartheta)\left[1+\frac{t^{2\vartheta}}{\Gamma(1+2\vartheta)}+\frac{t^{4\vartheta}}{\Gamma(1+4\vartheta)}+\cdots\right] \\ &\quad -\sinh_\vartheta(x^\vartheta)\left[\frac{t^\vartheta}{\Gamma(1+\vartheta)}+\frac{t^{3\vartheta}}{\Gamma(1+3\vartheta)}+\cdots\right] \\ &= \sinh_\vartheta(x^\vartheta)\cosh_\vartheta(t^\vartheta)-\sinh_\vartheta(x^\vartheta)\sinh_\vartheta(t^\vartheta). \end{aligned} \tag{42}$$

II. By using LFSEM

From (19), we obtain the following iterative formula:

$$\Phi_{i+1}(x) = -L_x^{(\vartheta)}\Phi_i(x), \tag{43}$$

where

$$\Phi_0(x) = \sinh_\vartheta(x^\vartheta). \tag{44}$$

In view of (43) and (44), we obtain the approximations given by

$$\begin{aligned}
\Phi_1(x) &= -L_x^{(\vartheta)}\Phi_0(x) = -\sinh_\vartheta(x^\vartheta);\\
\Phi_2(x) &= -L_x^{(\vartheta)}\Phi_1(x) = \sinh_\vartheta(x^\vartheta);\\
\Phi_3(x) &= -L_x^{(\vartheta)}\Phi_2(x) = -\sinh_\vartheta(x^\vartheta);\\
\Phi_4(x) &= -L_x^{(\vartheta)}\Phi_3(x) = \sinh_\vartheta(x^\vartheta);\\
\Phi_5(x) &= -L_x^{(\vartheta)}\Phi_4(x) = -\sinh_\vartheta(x^\vartheta).\\
&\vdots
\end{aligned} \tag{45}$$

Therefore, by using (20) and (46) we get the solution

$$\begin{aligned}
M(x,t) &= \sum_{i=0}^{\infty} \frac{t^{i\vartheta}}{\Gamma(1+i\vartheta)}\Phi_i(x)\\
&= \sinh_\vartheta(x^\vartheta)\left[1 - \frac{t^\vartheta}{\Gamma(1+\vartheta)} + \frac{t^{2\vartheta}}{\Gamma(1+2\vartheta)} - \frac{t^{3\vartheta}}{\Gamma(1+3\vartheta)} + \frac{t^{4\vartheta}}{\Gamma(1+4\vartheta)} - \cdots\right]\\
&= \sinh_\vartheta(x^\vartheta)\left[1 + \frac{t^{2\vartheta}}{\Gamma(1+2\vartheta)} + \frac{t^{4\vartheta}}{\Gamma(1+4\vartheta)} + \cdots\right]\\
&\quad -\sinh_\vartheta(x^\vartheta)\left[\frac{t^{\vartheta}}{\Gamma(1+\vartheta)} + \frac{t^{3\vartheta}}{\Gamma(1+3\vartheta)} + \cdots\right]\\
&= \sinh_\vartheta(x^\vartheta)\cosh_\vartheta(t^\vartheta) - \sinh_\vartheta(x^\vartheta)\sinh_\vartheta(t^\vartheta).
\end{aligned} \tag{46}$$

From Eqs.(42) and (46), the approximate solution of the given problem (34), by using local fractional Laplace decomposition method and local fractional series expansion method, is the same results as that obtained by the local fractional Laplace variational iteration method.

5. Conclusions

In this work, we have successfully provided new applications of the local fractional Adomian decomposition method and the local fractional series expansion method for solving linear partial differential equations arising in fractal vehicular traffic flow. The approximate solutions are obtained in series form that rapidly converges in a closed exact formula with simply computable terms. These techniques can be extended to solve various linear and nonlinear fractional problems in applied science.

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