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RESULTS ON SOFT HILBERT SPACES

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Abstract. Molodtsov [6] introduced the notion of soft set which can be considered as a new mathematical approach for vagueness. Das and Samanta [8] first defined the soft vector space and soft norm. Yazar and et al. [9] defined the soft vector space by using the concept of soft point given in [4, 5] and introduced the soft normed spaces in a new point of view. In the present paper, We give some properties of soft inner product spaces and present some examples for soft inner product spaces. Soft Hilbert space is introduced and some related properties are investigated. Finally, soft $\tilde{\ell}_2$ space is given as an example for soft Hilbert spaces.

Keywords: soft sets, soft inner product spaces, soft Hilbert spaces.

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1. Introduction

Molodtsov [6] introduced the notion of soft set to overcome uncertainties which cannot be dealt with by classical methods in many areas such as environmental science, economics, engineering and etc. This theory is applicable where there is no clearly defined mathematical model. There exist several different approaches for introducing a soft point in a soft set. Das and Samanta [8] introduced the concept of soft element by using a function. Then by using soft element Das and Samanta introduced soft real number in [3]. In the studies [4, 5] the soft point is defined by setting some conditions on parameters. Also, Xie [13] introduced the concept of soft point in a different approach. Das and et al. [8] defined a soft vector space by using the concept of soft element. After then they studied on soft normed spaces, soft linear operators, soft inner product spaces and their basic properties [2, 1, 7]. Das and Samanta [11] intro-duced self-adjoint operator and completely continuous operator on soft inner product space. Further, Das and Samanta [12] extended the study of operators on soft inner product spaces. Jun and park [10]

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presented applications of soft sets in Hilbert algebras. Yazar and et al. [9] define the soft vector space by using the concept of soft point given in [4, 5] and introduced the soft normed spaces in a new point of view. In this study, we progress on the study [9] by introducing the soft inner product on soft vector spaces and give some properties of soft inner product spaces. Soft Hilbert space is introduced and some related properties are investigated. Finally, we introduce soft space as an example for soft Hilbert space.

2. Preliminaries

In this section we will introduce necessary definitions and theorems for soft sets. Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denotes the power set of X and $A, B \subseteq E$.

Definition 2.1. [6] *A pair (F, E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$, where $P(X)$ denotes the power set of X. $SS(X)_E$ denotes the family of all soft sets over X with a fixed set of parameters E.*

Definition 2.2. [4, 5] *Let (F, E) be a soft set over X. The soft set (F, E) is called a soft point, denoted by (x_e, E) , if for the element $e \in E$, $F(e) = \{x\}$ and $F(e') = \phi$ for all $e' \in E - \{e\}$ (briefly denoted by $\tilde{x}_e$.)*

Definition 2.3. [4] *Two soft points $(\tilde{x}_e, E)$ and $(\tilde{y}_{e'}, E)$ over a common universe X, we say that the soft points are different if $x \neq y$ or $e \neq e'$.*

Let $SP(\tilde{X})$ be the collection of all soft points of $\tilde{X}$ and $\mathbb{R}(E)^*$ denote the set of all non-negative soft real numbers.

Definition 2.4. [3] *A mapping*

$$\tilde{d} : SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$$

is said to be a soft metric on the soft set $\tilde{X}$ if $\tilde{d}$ satisfies the soft metric conditions.

Let X be a vector space over a field K $(K = \mathbb{R})$ and the parameter set E be the real number set $\mathbb{R}$.

Definition 2.5. [9]*Let (F, E) be a soft set over X. The soft set (F, E) is said to be a soft vector and denoted by $\tilde{x}_e$ if there is exactly one $e \in E$, such that $F(e) = \{x\}$ for $x \in X$ and $F(e') = \phi$, $\forall e' \in E/\{e\}$.*

The set of all soft vectors over $\tilde{X}$ will be denoted by $SV(\tilde{X})$.

Proposition 2.1. [9]*The set $SV(\tilde{X})$ is a vector space according to the following operations;*

(1) $\tilde{x}_e + \tilde{y}_{e'} = (\widetilde{x+y})_{(e+e')}$ *for every* $\tilde{x}_e, \tilde{y}_{e'} \in SV(\tilde{X})$;
(2) $\tilde{r}.\tilde{x}_e = (\widetilde{rx})_{(re)}$ *for every* $\tilde{x}_e \in SV(\tilde{X})$ *and for every soft real number* $\tilde{r}$.

Definition 2.6. *([9]) Let $SV(\tilde{X})$ be a soft vector space. Then a mapping*

$$\|.\| : SV(\tilde{X}) \to \mathbb{R}(E)^*,$$

is said to be a soft norm on $SV(\tilde{X})$, if $\|.\|$satisfies the norm conditions.

The soft vector space $SV(\tilde{X})$ with a soft norm $\|.\|$ on $\tilde{X}$ is said to be a soft normed linear space and is denoted by $(\tilde{X}, \|.\|)$.

3. Soft Hilbert Spaces

Definition 3.1. *Let* $SV(\tilde{X})$ *be a soft vector space. The mapping*

$$< . >: SV(\tilde{X}) \to SV(\tilde{X}) \to \mathbb{R}(E)^*$$

is called a soft inner product on $SV(\tilde{X})$ *iff it satisfies the following conditions, for every* $\tilde{x}_e, \tilde{y}_{e'}, \tilde{z}_{e''} \tilde{\in} SV(\tilde{X})$ *and for every soft reel number* $\tilde{\alpha}$*;*

I1. $< \tilde{x}_e, \tilde{x}_e > \tilde{\geq} \tilde{0}$ *and* $< \tilde{x}_e, \tilde{x}_e >= \tilde{0} \Leftrightarrow \tilde{x}_e = \tilde{\theta}_0$,
I2. $< \tilde{x}_e, \tilde{y}_{e'} >= < \tilde{y}_{e'}, \tilde{x}_e >$,
I3. $< \tilde{\alpha}\tilde{x}_e, \tilde{y}_{e'} >=< \tilde{x}_e, \tilde{\alpha}\tilde{y}_{e'} >= \tilde{\alpha} < \tilde{x}_e, \tilde{y}_{e'} >$,
I4. $< \tilde{x}_e + \tilde{y}_{e'}, \tilde{z}_{e''} >=< \tilde{x}_e, \tilde{z}_{e''} > + < \tilde{y}_{e'}, \tilde{z}_{e''} >$.

The triple $(SV(\tilde{X}), < . >, E)$ *is called soft inner product space.*

Proposition 3.1. *(Parallelogram Law) Let* $(SV(\tilde{X}), < . >, E)$ *be a soft inner product space. For every* $\tilde{x}_e, \tilde{y}_{e'} \tilde{\in} SV(\tilde{X})$

$$\|\tilde{x}_e + \tilde{y}_{e'}\|^2 + \|\tilde{x}_e - \tilde{y}_{e'}\|^2 = 2\left(\|\tilde{x}_e\|^2 + \|\tilde{y}_{e'}\|^2\right)$$

is satisfied.

Theorem 3.1. *Let* $(SV(\tilde{X}), < . >, E)$ *be a soft inner product space.*
For every $\tilde{x}_e, \tilde{y}_{e'} \tilde{\in} SV(\tilde{X})$

$$|< \tilde{x}_e, \tilde{y}_{e'} >| \tilde{\leq} \|\tilde{x}_e\| \, \|\tilde{y}_{e'}\| \tag{1}$$

is hold.

Proof. Let $\tilde{\alpha}$ be a soft scalar. In this case, for every $\tilde{x}_e, \tilde{y}_{e'} \tilde{\in} SV(\tilde{X})$,
we have $< \tilde{x}_e - \tilde{\alpha}\tilde{y}_{e'}, \tilde{x}_e - \tilde{\alpha}\tilde{y}_{e'} > \tilde{\geq} \tilde{0}$.
In other words,

$$< \tilde{x}_e, \tilde{x}_e > -\tilde{\alpha} < \tilde{x}_e, \tilde{y}_{e'} > -\tilde{\alpha}\,[< \tilde{y}_{e'}, \tilde{x}_e > -\tilde{\alpha} < \tilde{y}_{e'}, \tilde{y}_{e'} >] \tilde{\geq} \tilde{0} \tag{2}$$

If $< \tilde{y}_{e'}, \tilde{y}_{e'} >= \tilde{0}$ then $\tilde{y}_{e'} = \tilde{\theta}_0$ and in this case we have $< \tilde{x}_e, \tilde{y}_{e'} >=< \tilde{x}_e, \tilde{\theta}_0 >= \tilde{0}$ and thus the inequality [1] is hold. If $< \tilde{y}_{e'}, \tilde{y}_{e'} > \tilde{>} \tilde{0}$ then by taking $\tilde{\alpha} = \frac{<\tilde{x}_e, \tilde{y}_{e'}>}{<\tilde{y}_{e'}, \tilde{y}_{e'}>}$ from the inequality [2] we have

$$\|\tilde{x}_e\|^2 - \frac{|< \tilde{x}_e, \tilde{y}_{e'} >|^2}{\|\tilde{y}_{e'}\|^2} - \frac{< \tilde{x}_e, \tilde{y}_{e'} >}{\|\tilde{y}_{e'}\|^2}\left[< \tilde{x}_e, \tilde{y}_{e'} > - \frac{|< \tilde{x}_e, \tilde{y}_{e'} >|^2 \, \|\tilde{y}_{e'}\|^2}{\|\tilde{y}_{e'}\|^2}\right] \tilde{\geq} \tilde{0}$$

and consequently we have

$$|< \tilde{x}_e, \tilde{y}_{e'} >| \tilde{\leq} \|\tilde{x}_e\| \, \|\tilde{y}_{e'}\| \, .$$

□

Proposition 3.2. *A soft inner product function is continuous in a soft inner product space. In other words, If* $\{\tilde{x}^n_{e_n}\} \longrightarrow \tilde{x}_e$ *and* $\left\{\tilde{y}^n_{e'_n}\right\} \longrightarrow \tilde{y}_{e'}$*, then* $< \tilde{x}^n_{e_n}, \tilde{y}^n_{e'_n} > \longrightarrow < \tilde{x}_e, \tilde{y}_{e'} >$.

Proposition 3.3. *Let* $(SV(\tilde{X}), < . >, E)$ *be a soft inner product space and* $\{\tilde{x}^n_{e_n}\}, \left\{\tilde{y}^n_{e'_n}\right\}$ *be soft Cauchy sequences in this space. In this case,* $< \tilde{x}^n_{e_n}, \tilde{y}^n_{e'_n} >$ *is also a soft Cauchy sequence.*

Definition 3.2. *Let $SV(\tilde{X})$ be a soft vector space and $E = \mathbb{R}$ be the parameter set. If* $\left\|\tilde{x}^{(i)}_{e^{(i)}}\right\| = \left[\sum\limits_{i=1}^{\infty}\left(x^{(i)}\right)^2 + \sum\limits_{i=1}^{\infty}\left(e^{(i)}\right)^2\right]^{1/2} < \infty$, *for a soft sequence* $\tilde{x}_e = \left\{\tilde{x}^{(i)}_{e^{(i)}}\right\} = \left\{\tilde{x}^{1(i)}_{e^{(i)}_1}, \tilde{x}^{2(i)}_{e^{(i)}_2}, ...\right\} \tilde{\subset} SV(\tilde{X})$ *then the space of these soft sequences is called as soft $\tilde{\ell}_2$ space.*

Proposition 3.4. *If* $\tilde{x}_e = \left\{\tilde{x}^{(i)}_{e^{(i)}}\right\}$, $\tilde{y}_e = \left\{\tilde{y}^{(i)}_{e'(i)}\right\} \tilde{\in} \tilde{\ell}_2$, *then the following inequality is satisfied*

$$\sum_{i=1}^{\infty}\left|\tilde{x}^{(i)}_{e^{(i)}}.\tilde{y}^{(i)}_{e'(i)}\right| \tilde{\leq} \|\tilde{x}_e\| \|\tilde{y}_{e'}\| . \tag{3}$$

Theorem 3.2. *If* $\tilde{x}_e = \left\{\tilde{x}^{(i)}_{e^{(i)}}\right\}$, $\tilde{y}_{e'} = \left\{\tilde{y}^{(i)}_{e'(i)}\right\} \tilde{\in} \tilde{\ell}_2$, *then* $\tilde{x}_e + \tilde{y}_{e'} \tilde{\in} \tilde{\ell}_2$

Proof. Since $\tilde{x}_e = \left\{\tilde{x}^{(i)}_{e^{(i)}}\right\}$, $\tilde{y}_e = \left\{\tilde{y}^{(i)}_{e'(i)}\right\} \tilde{\in} \tilde{\ell}_2$ we have

$$\left\|\tilde{x}^{(i)}_{e^{(i)}}\right\| = \left[\sum_{i=1}^{\infty}\left(x^{(i)}\right)^2 + \sum_{i=1}^{\infty}\left(e^{(i)}\right)^2\right]^{1/2} < \infty$$

and

$$\left\|\tilde{y}^{(i)}_{e'(i)}\right\| = \left[\sum_{i=1}^{\infty}\left(y^{(i)}\right)^2 + \sum_{i=1}^{\infty}\left(e'^{(i)}\right)^2\right]^{1/2} < \infty.$$

From here,

$$\begin{aligned}
\|\tilde{x}_e + \tilde{y}_{e'}\|^2 &= \left\|\widetilde{(x+y)}_{e+e'}\right\|^2 = \\
&= \sum_{i=1}^{\infty}\left((x+y)^{(i)}\right)^2 + \sum_{i=1}^{\infty}\left((e+e')^{(i)}\right)^2 \\
&= \sum_{i=1}^{\infty}\left(x^{(i)} + y^{(i)}\right)^2 + \sum_{i=1}^{\infty}\left(e^{(i)} + e'^{(i)}\right)^2 \\
&= \sum_{i=1}^{\infty} x^{2(i)} + y^{2(i)} + x^{(i)}.y^{(i)} + \sum_{i=1}^{\infty} e^{2(i)} + e'^{2(i)} + \sum_{i=1}^{\infty} e^{(i)}.e'^{(i)}
\end{aligned}$$

$$\begin{aligned}
&= \left(\begin{array}{c}\left[\sum\limits_{i=1}^{\infty}\left(x^{(i)}\right)^2 + \sum\limits_{i=1}^{\infty}\left(e^{(i)}\right)^2\right] + \left[\sum\limits_{i=1}^{\infty}\left(y^{(i)}\right)^2 + \sum\limits_{i=1}^{\infty}\left(e'^{(i)}\right)^2\right] \\ + \left[\sum\limits_{i=1}^{\infty} x^{(i)}.y^{(i)} + \sum\limits_{i=1}^{\infty} e^{(i)}.e'^{(i)}\right]\end{array}\right) \\
&= \|\tilde{x}_e\|^2 + \|\tilde{y}_{e'}\|^2 + \sum_{i=1}^{\infty}\left|\tilde{x}^{(i)}_{e^{(i)}}.\tilde{y}^{(i)}_{e'(i)}\right| \\
&\leq \|\tilde{x}_e\|^2 + \|\tilde{y}_{e'}\|^2 + \|\tilde{x}_e\| . \|\tilde{y}_{e'}\| \\
&= \left(\|\tilde{x}_e\| + \|\tilde{y}_{e'}\|\right)^2 < \infty
\end{aligned}$$

and consequently we have $\tilde{x}_e + \tilde{y}_{e'} \tilde{\in} \tilde{\ell}_2$.

□

Proposition 3.5. *Let* $\tilde{x}_e = \left\{\tilde{x}^{(i)}_{e^{(i)}}\right\}$, $\tilde{y}_{e'} = \left\{\tilde{y}^{(i)}_{e'^{(i)}}\right\} \tilde{\in} \tilde{\ell}_2$ *and the function*

$$< . >: \tilde{\ell}_2 \times \tilde{\ell}_2 \longrightarrow \mathbb{R}(E)$$

is defined as follows

$$< \tilde{x}_e, \tilde{y}_{e'} >= \sum_{i=1}^{\infty} x^{(i)}.y^{(i)} + \sum_{i=1}^{\infty} e^{(i)}.e'^{(i)},$$

where $E = \mathbb{R}$. *In this case, the function* $< . >: \tilde{\ell}_2 \times \tilde{\ell}_2 \longrightarrow \mathbb{R}(E)$ *is an inner product on soft vector space* $\tilde{\ell}_2$ *and* $(\tilde{\ell}_2, < . >, E)$ *is a soft inner product space.*

Definition 3.3. *Let* $(SV(\tilde{X}), < . >, E)$ *be a soft inner product space. If this space is complete according to the induced norm by the soft inner product, then* $(SV(\tilde{X}), < . >, E)$ *is said to be a soft Hilbert space.*

Proposition 3.6. *Let* $(\tilde{\ell}_2, < . >, E)$ *be a soft inner product space and* $\tilde{x}_e = \left\{\tilde{x}^{(i)}_{e^{(i)}}\right\}$, $\tilde{y}_{e'} = \left\{\tilde{y}^{(i)}_{e'^{(i)}}\right\} \tilde{\in} \tilde{\ell}_2$, *the function* $\tilde{d} : \tilde{\ell}_2 \times \tilde{\ell}_2 \longrightarrow \mathbb{R}(E)^*$ *defined as follows*

$$\tilde{d}(\tilde{x}_e, \tilde{y}_{e'}) = \left[\sum_{i=1}^{\infty} \left|x^{(i)} - y^{(i)}\right|^2 + \sum_{i=1}^{\infty} \left|e^{(i)} - e'^{(i)}\right|^2\right]^{1/2}$$

is a soft metric.

Example 3.1. *The soft inner product space* $(\tilde{\ell}_2, < . >, E)$ *is a soft Hilbert space where the parameter set* $E = \mathbb{R}$.

For the soft sequences $\tilde{x}_e = \left\{\tilde{x}^{(i)}_{e^{(i)}}\right\}$, $\tilde{y}_{e'} = \left\{\tilde{y}^{(i)}_{e'^{(i)}}\right\} \tilde{\in} \tilde{\ell}_2$ *the soft inner product is defined as follows,*

$$< \tilde{x}_e, \tilde{y}_{e'} >= \sum_{i=1}^{\infty} x^{(i)}.y^{(i)} + \sum_{i=1}^{\infty} e^{(i)}.e'^{(i)}$$

Let us take a soft Cauchy sequence $\{\tilde{x}^n_{e^n}\}_n$ *in the soft metric space* $(SV(\tilde{X}), \tilde{d})$. *In this case, for every soft reel number* $\tilde{\varepsilon} \tilde{>} \tilde{0}$ *there exists a* $n_0 \in \mathbb{N}$ *such that for* $\forall m, n \geq n_0$

$$\tilde{d}(\tilde{x}^n_e, \tilde{x}^m_e) \tilde{\leq} \sum_{i=1}^{\infty} \left|x^{i,n} - x^{i,m}\right|^2 + \sum_{i=1}^{\infty} \left|e^{i,n} - e^{i,m}\right|^2 \tilde{\leq} \tilde{\varepsilon}^2.$$

Thus for $\forall m, n \geq n_0$ *and* $i = \overline{1, \infty}$ *we have* $|x^n - x^m|^2 + |e^n - e^m|^2 \tilde{\leq} \tilde{\varepsilon}^2$. *Consequently, for every* i , $x^n_e = \left\{\tilde{x}^{i,n}_{e^{i,n}}\right\}$ *is a soft Cauchy sequence. Since* $|x^n - x^m| \tilde{\leq} \tilde{\varepsilon}$, $\{x^n\}$ *is a Cauchy sequence in the space* ℓ_2. *Since* ℓ_2 *is a complete space there exists a sequence* $x \in \ell_2$ *such that* $\{x^n\} \longrightarrow x$. *On the other hand, since* $|e^n - e^m| \tilde{\leq} \tilde{\varepsilon}$, $\{e^n\}$ *is a Cauchy sequence in the space* ℓ_2. *Since* ℓ_2 *is a complete space there exists a sequence* $e \in \ell_2$ *such that* $\{e^n\} \longrightarrow e$.

If we take the sequence $e \in \ell_2$ *as a parameter of the sequence* $x \in \ell_2$ *then we have the soft sequence* $\tilde{x}_e$ *and we obtain* $\left\{\tilde{x}^{(n)}_{e^{(n)}}\right\} \longrightarrow \tilde{x}_e$. *Finally, since* $x \in \ell_2$ *and* $e \in \ell_2$ *we have* $\|x\| = \left[\sum_{i=1}^{\infty} \left|x^{(i)}\right|^2\right]^{1/2} \tilde{<} \frac{\tilde{\varepsilon}}{2}$ *and* $\|e\| = \left[\sum_{i=1}^{\infty} \left|e^{(i)}\right|^2\right]^{1/2} \tilde{<} \frac{\tilde{\varepsilon}}{2}$ *,respectively. Therefore, we have* $\|\tilde{x}_e\| = \left[\sum_{i=1}^{\infty} \left|x^{(i)}\right|^2 + \sum_{i=1}^{\infty} \left|e^{(i)}\right|^2\right]^{1/2} \tilde{<} \tilde{\varepsilon}$ *which means that* $\tilde{x}_e \tilde{\in} \tilde{\ell}_2$. *Since the soft Cauchy sequence* $\left\{\tilde{x}^{(n)}_{e^{(n)}}\right\}$ *is arbitary the soft space* $\tilde{\ell}_2$ *is a soft Hibert space.*

4. Conclusion

We have introduced soft inner product spaces and soft Hilbert spaces in a new point of view. We investigate some properties of soft inner product space and defined the soft $\tilde{\ell}_2$ spaces. Finally we show that the soft $\tilde{\ell}_2$ space is a soft Hilbert sapce.

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Cigdem Gunduz Aras for the photography and short autobiography, see p.63.

Sadi Bayramov for the photography and short autobiography, see p.63.