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RANDIC TYPE SDI INDEX OF CERTAIN GRAPHS

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Abstract. In this paper, we calculate the Randic type SDI index of double graphs, subdivision graphs and complements of some standard graphs.

Keywords: Randic type SDI index, double graphs, subdivision graphs, k-complement, k(i)-complement.

AMS Subject Classification: 05C50

1. Introduction

Let G be a graph with n vertices $\{v_1,\ v_2,\ \cdots,\ v_n\}$ and m edges. In this paper we consider only simple, undirected, loop free and signless graphs.

A topological index is a numerical quantity of a molecule that is evaluated from the structural graph of a molecule. In chemistry, topological indices are used for modelling physical, pharmacological, biological and other properties of chemical compounds. One of these topological indices is the Randic type Squire Degree Involved index (Randic type SDI index) which is defined in terms of the vertex degrees as follows:

$$R_{SDI}(G) = \sum_{uv \in E(G)} (d_u)^2 (d_v)^2$$

The complement of a graph G is a graph $\overline{G}$ having the same vertex set such that two distinct vertices of $\overline{G}$ are adjacent if and only if they are not adjacent in G.

Let G be a graph and $P_k = \{V_1,\ V_2,\ \cdots,\ V_k\}$ be a partition of its vertex set V. Similarly to the complement of a graph, the k-complement of G is defined as follows: For all V_i and V_j in P_k for $i \neq j$, remove the edges between V_i and V_j and add the edges between the vertices of V_i and V_j which are not in G and is denoted by $\overline{G_k}$.

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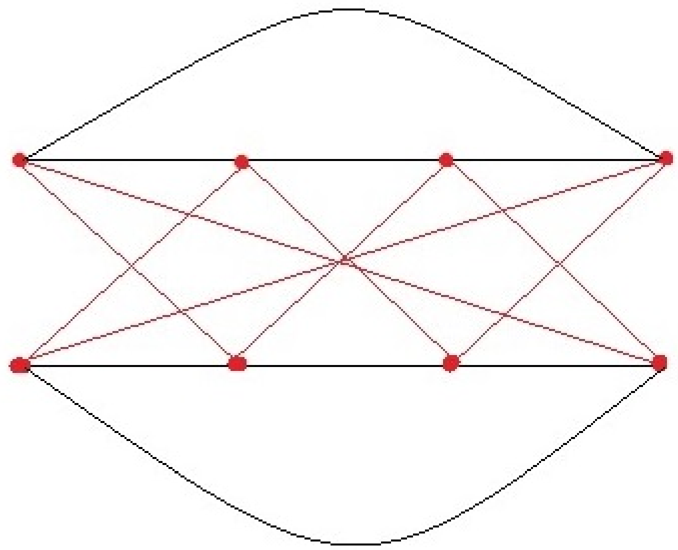
Let G be a graph and $P_k = \{V_1,\ V_2,\ \cdots,\ V_k\}$ be a partition of its vertex set V. Then the $k(i)$-complement of G, yet another type of the complement of a graph, is obtained as follows: For each partition set V_r in P_k, remove the edges of G joining the vertices in V_r and add the edges of $\overline{G}$ (complement of G) joining the vertices of V_r, and is denoted by $\overline{G_{k(i)}}$.

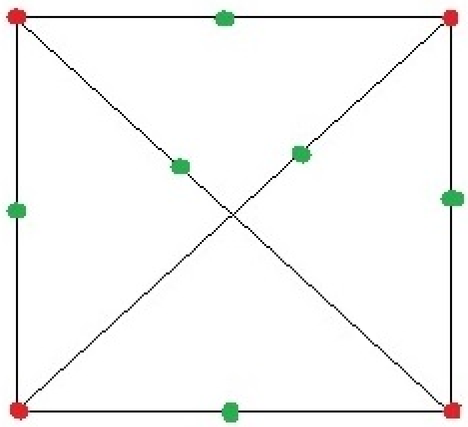
The subgraph or subdivision graph $S(G)$ of a simple graph G is defined as the new graph obtained by adding an extra vertex into each edge of G. The subgraphs have been studied in literature, [5], [10], [6], and [4].

For a graph G with vertex set $V(G) = \{v_1,\ v_2,\ \cdots,\ v_n\}$, we take another copy of G with vertices labelled by $\{v_1,\ v_2,\ \cdots,\ v_n\}$, this time, where v_i corresponds to v_i for each i. If we connect v_i to the neighbours of v_i for each i, we obtain a new graph called the double graph of G. It is denoted by $D(G)$.



Double graph of the Cycle

The double graph of the subdivision graph of a graph G was studied in [6].



Subdivision Graph of the Complete graph

2. Randic type SDI index of some standard graphs

Many graph theorists computed the different topological indices for these standard graphs. In future, it may be helpful to generalize the relationship between different topological indices. Hence we have computed the Randic type SDI index for these graphs.

Theorem 2.1.

$$R_{SDI}(G) = \begin{cases} 16(n-2) & \text{if } G = P_n \\ n(n-1)^5 & \text{if } G = S_n^0 \\ 32n(n-1)^5 & \text{if } G = K_{n\times 2} \\ 16n & \text{if } G = C_n, \\ (3n-6)(n-1)^4 & \text{if } G = K_n \\ m^3n^3 & \text{if } G = K_{m,n}, n \\ (n-1)^3 & \text{if } G = K_{1,n-1} \\ 16n(2n^2+1) & \text{if } G = F_n^3 \\ 32n(n^2+1) & \text{if } G = D_n^3 \\ 9(n-1)[n^2-2n+10] & \text{if } G = W_n \\ 16(p+q+3) & \text{if } G = T_{(p,q)} \end{cases}$$

Proof. *We prove the theorem for Friendship graphs. Similar methods can be used for others. Let G be a Friendship graph F_n^3. Here we have $2n$ vertices with degree 2 and $2n$ in the centers of Friendship graph. We have n edges between the vertices of degree 2. There are $2n$ edges betwen the central vertex and remaining vertices of degree 2.*
According to the definition of Randic type SDI index,

$$\begin{aligned} R_{SDI}(F_n^3) &= n(2^2 2^2) + 2n(2^2(2n)^2) \\ &= 16n + 2n(16n^2) \\ &= 16n(2n^2+1). \end{aligned}$$

3. Randic type SDI index of double graphs

Theorem 3.1.

$$R_{SDI}(D(G)) = \begin{cases} 1024n - 2560 & \textit{if } G = P_n, n \geq 3, \\ 64 & \textit{if } G = P_n, n = 2, \\ 1024n & \textit{if } G = C_n, \\ 32n(n-1)^5 & \textit{if } G = K_n, \\ 64n(n-1)^5 & \textit{if } G = S_n^0, \\ 2048n(n-1)^5 & \textit{if } G = K_{n\times 2}, \\ 1024n(2n^2+1) & \textit{if } G = F_n^3, \\ 1792n + 2304n^3 & \textit{if } G = D_n^3, \\ 64(n-1)^3 & \textit{if } G = K_{1,n-1}, \end{cases}$$

Proof. *We prove the theorem for Double graph of Cycle. Similar methods can be used for others. Let G be a Double graph of Cycle $D(C_n)$. Here all the vertices are having the degree 4. Here we have $4n$ edges.*
By the definition of Randic type SDI index,

$$\begin{aligned} R_{SDI}D(C_n) &= 4n(4^2 4^2) \\ &= 1024n \end{aligned}$$

4. Randic type SDI index of complements

Theorem 4.1.

$$R_{SDI}(G) = \begin{cases} \frac{1}{2}(n-1)(n-2)^5 & \textit{if } G = \overline{K_{1,n-1}}, \\ n & \textit{if } G = \overline{K_{n\times 2}}, \\ \frac{n}{2}(n-3)^5 & \textit{if } G = \overline{C_n}, \\ n^6 & \textit{if } G = S_n^0, \\ 32n(n-1)^5 & \textit{if } G = F_n^3, \end{cases}$$

Proof. *We prove the theorem for complement of Star graphs. Similar methods can be used for others. Let G be a complement of Star graph $\overline{K_{(1,n-1)}}$. Here we have $n-1$ vertices with degree $n-2$ and one isolated vertex. We have $\frac{(n-1)(n-2)}{2}$ edges.*
According to the definition of Randic type SDI index,

$$\begin{aligned} R_{SDI}(\overline{K_{(1,n-1)}}) &= \frac{(n-1)(n-2)}{2}[(n-2)^2(n-2)^2] \\ &= \frac{(n-1)(n-2)^5}{2}. \end{aligned}$$

5. Randic type SDI index of subdivision graphs

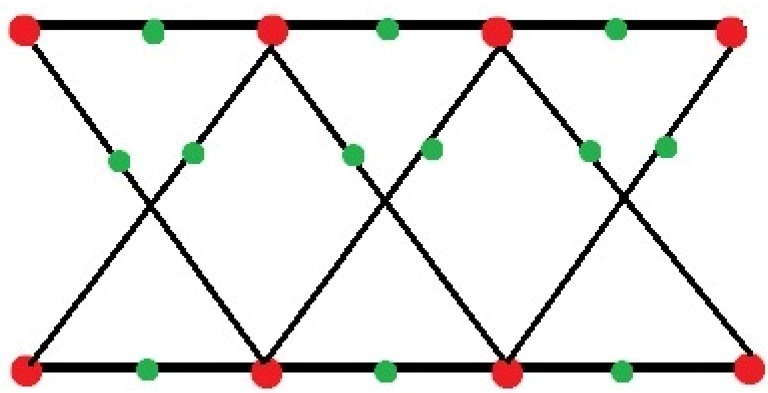
Theorem 5.1.

$$R_{SDI}[(S(G))] = \begin{cases} 32(n-\frac{7}{4}) & \text{if } G = P_n \\ 8n(n-1)^3 & \text{if } G = S_n^0, n, \\ 64n(n-1)^3 & \text{if } G = K_{n\times 2}, n, \\ 32n & \text{if } G = C_n, \\ 4n(n-1)^3 & \text{if } G = K_n, \\ m^3n^3 & \text{if } G = K_{m,n}, n, \\ 4(n-1)(n^2-2n+2) & \text{if } G = K_{1,n-1}, \\ 32n(n^2+2) & \text{if } G = F_n^3, \\ 32n(n^2+3) & \text{if } G = D_n^3, n, \\ 4(n-1)[n^2-2n+28] & \text{if } G = W_n, n, \\ 16[2(p+q)+3] & \text{if } G = T_{(p,q)}, \end{cases}$$

Proof. *We prove the theorem for subdivision graph of Tadpole graph. Similar methods can be used for others. Let G be a subdivision graph of Tadpole graph $S(T_{p,q})$. Here we have 2(p+q-1) vertices with degree 2, one pendant vertex and one vertex with degree 3. We have $2(p+q-2)$ edges between the vertices of degree 2. There are three edges stemmed out from the vertex of degree 3 to the three vertices of degree 2. One edge between pendant vertex to the vertex of degree 2.*
By the definition of Randic type SDI index,

$$\begin{aligned} R_{SDI}S(T_{p,q}) &= 3[(2)^2(3)^2] + 2(p+q-2)[(2)^2(2)^2] + (2)^2(1)^2 \\ &= 108 + 32(p+q-2) + 4 \\ &= 16[2(p+q)+3]. \end{aligned}$$

6. Randic type SDI index of subdivision of double graphs



Subdivision of Double graph for Path

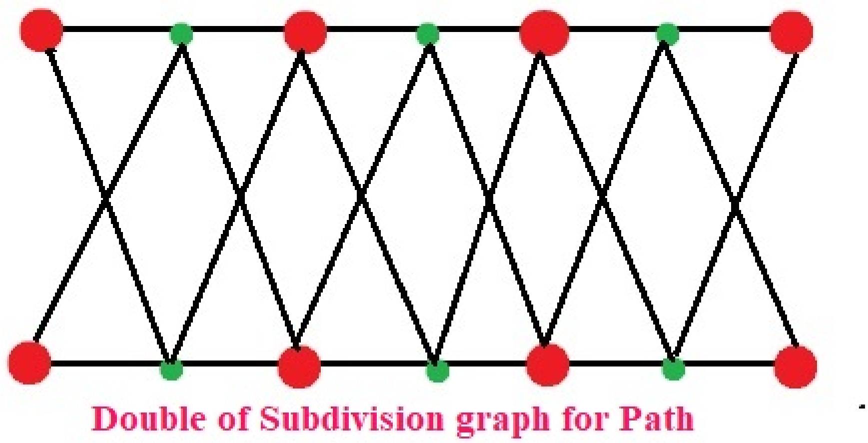
Theorem 6.1.

$$R_{SDI}[S(D(G))] = \begin{cases} 512(n-\frac{7}{4}) & \text{if } G = P_n \\ 64n(n-1)^3 & \text{if } G = K_n \end{cases}$$

Proof. *We prove the theorem for subdivision graph of double graph of Path. Similar methods can be used for others. Let G be a subdivision graph of double graph of Path $S(D(P_n))$. Here we have $4n$ vertices with degree 2, $2n-4$ vertices with degree 4. We have 8 edges between the vertices of degree 2. There are $8n-16$ edges between the vertices of degree 4 and 2.*
By the definition of Randic type SDI index,

$$\begin{aligned} R_{SDI}S(D(P_n)) &= 8[(2)^2(2)^2] + (8n-16)[(2)^2(4)^2] \\ &= 128 + 64(8n-16) \\ &= 512(n - \frac{7}{4}) \end{aligned}$$

7. Randic type SDI index of double graph of subdivision graph



Theorem 7.1.

$$R_{SDI}[S(D(G))] = \begin{cases} 2048n - 3584 & \textit{if } G = P_n, \\ 128n(n-1)^2 & \textit{if } G = K_n, \\ 2048n & \textit{if } G = C_n, \end{cases}$$

Proof. *We prove the theorem for double graph of subdivision graph of Path. Similar methods can be used for others. Let G be a double graph of subdivision graph of Path $S(D(P_n))$. Here we have 4 vertices with degree 2, $4n-6$ vertices with degree 4. We have 8 edges between the vertices of degree 2 and 4. There are $8n-16$ edges between the vertices of degree 4.*
By the definition of Randic type SDI index,

$$\begin{aligned} R_{SDI}S(D(P_n)) &= 8[(2)^2(4)^2] + (8n-16)[(4)^2(4)^2] \\ &= 512 + 256(8n-16) \\ &= 2048n - 3584 \end{aligned}$$

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