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LACEABILITY PROPERTIES IN EDGE TOLERANT CORONA PRODUCT GRAPHS

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Abstract. A connected graph G is termed Hamiltonian-t-laceable if there exists in it a Hamiltonian path between every pair of vertices u and v with the property $d(u, v) = t$, $1 \leq t \leq diam(G)$, where t is a positive integer. The corona product of G and H, denoted by GoH is obtained by taking one copy of G called the center graph, $|V(G)|$ copies of H called the outer graph and taking the i^{th} vertex of G adjacent to every vertex of the i^{th} copy of H where $1 \leq i \leq |V(G)|$. In this paper, we establish laceability properties in the edge tolerant corona product graph $K_n o P_m$.

Keywords: Hamiltonian graph, Hamiltonian laceable graph, Hamiltonian-t-laceable graph, Corona graph.

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1. Introduction

Let G be a finite, simple, connected and undirected graph. Let u and v be two vertices in G. The distance between u and v denoted by $d(u, v)$ is the length of a shortest path in G. G is Hamiltonian laceable if there exists in it a Hamiltonian path between every pair of vertices at an odd distance. G is Hamiltonian-t-laceable if there exists in G a Hamiltonian path between every pair of vertices u and v with the property $d(u, v) = t$, $1 \leq t \leq diam(G)$, where t is a positive integer. Throughout this paper, P_m and K_n will denote the path graph and complete graph with m and n vertices respectively.

Laceability in the brick products of even cycles was explored by Alspach et.al. in [1]. A characterization for a 1-connected graph to be Hamiltonian-t-laceable for $t = 1, 2$ and 3 is given in [3] and this was extended to $t = 4$ and 5 by Thimmaraju and Murali [4]. Leena Shenoy [5] studied Hamiltonian laceability properties in product graphs involving cycles and paths. More results in the laceability properties of product graphs can be found in [6], [7], [8], and [9]. In this paper, we establish laceability properties in the edge tolerant corona product $K_n o P_m$.

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Definition 1.1. *Let G and H be two graphs. The corona product GoH is obtained by taking one copy of G called the center graph, $|V(G)|$ copies of H called the outer graph and taking i^{th} vertex of G adjacent to every vertex of the i^{th} copy of H where $1 \leq i \leq |V(G)|$.*

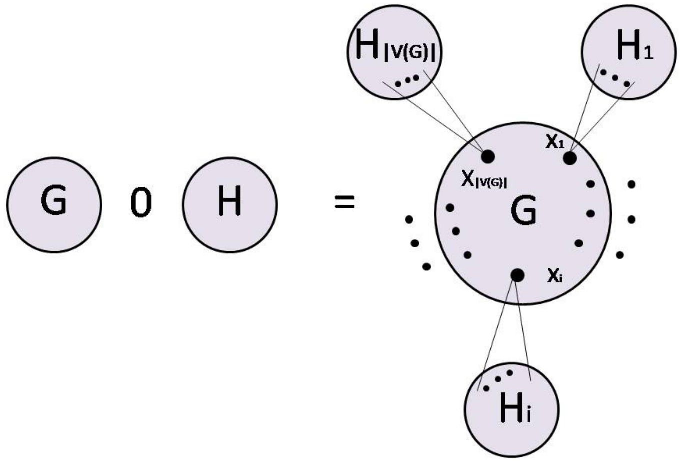

FIGURE 1. The corona product GoH

Definition 1.2. *A graph G^* is k-edge fault tolerant with respect to a graph G if the graph obtained by removing any k edges from G^* contains G, where k is a positive integer.*

Definition 1.3. *Let P be a path between the vertices v_i and v_j in a graph G and let P' be a path between the vertices v_j and v_k. Then, the path $P \cup P'$ is the path obtained by extending the path P between v_i and v_j to v_k through the common vertex v_j (i.e. if $P : v_i....v_j$ and $P' : v_j....v_k$ then $P \cup P' : v_i....v_j.....v_k$).*

2. RESULTS

Theorem 2.1. *For $n \geq 5$ and $m \geq 3$, the $n-2$ edge fault tolerant graph $K_n o P_m$ is Hamiltonian-1-laceable.*

Proof. Let $G = K_n o P_m$, $V(G) = (v_{i,0}; 1 \leq i \leq n) \cup (v_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m)$ where $v_{i,0}$ are the vertices of the complete graph K_n and $v_{i,j}$ are the vertices of the $|V(K_n)|$ copies of P_m, $1 \leq i \leq n$, $1 \leq j \leq m$. Thus, G has $(m+1)n$ vertices, $nC_2 + 2m + 1$ edges and $diam(G) = 3$.

Since $d(v_{i,j}, v_{i,(j+1)}) = d(v_{i,0}, v_{i,j}) = d(v_{i,0}, v_{(i+1),0}) = d(v_{i,0}, v_{(i+j),0}) = 1$ in G for all $1 \leq i \leq n, 1 \leq j \leq m$, it is enough to prove that there exists a hamiltonian path in G between these pairs of vertices.

claim 1. The vertices $v_{i,j}$ and $v_{i,(j+1)}$ are attainable for $i, j \neq 0$

In G, $d(v_{i,j}, v_{i,(j+1)}) = 1$ where $1 \leq j \leq (m-1)$ and the path

$$P : \bigcup_{k=1}^{j} v_{i,j-k+1} \bigcup v_{i,0} \bigcup_{k=0}^{m} v_{(i+2),k} \bigcup v_{(i+3),(m-k)} \bigcup v_{(i+1),k} \left(\bigcup_{t=i+4}^{n+i-1} \bigcup_{k=0}^{m} v_{t,k} \right) \bigcup_{k=0}^{m-j-1} v_{i,(m-k)}$$

in the $n-2$ edge fault tolerant graph G^* has a hamiltonian path between the vertices $v_{i,j}$ and $v_{i,(j+1)}$.

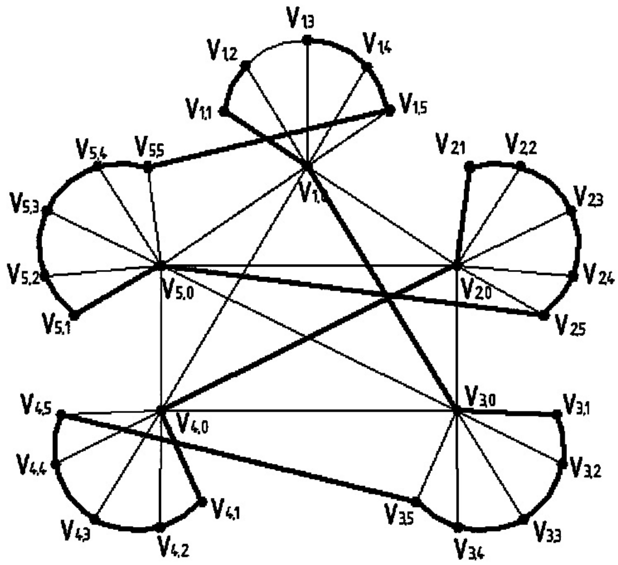

FIGURE 2. Corona product $K_5 o P_5$ with $d(v_{1,2}, v_{1,3}) = 1$

claim 2. The vertices $v_{i,0}$ and $v_{i,j}$ are attainable

In G, $d(v_{i,0}, v_{i,j}) = 1$ where $1 \leq j \leq m$ and the path is $P : v_{i,0} \bigcup_{k=0}^{m} \left[v_{(i+2),k} \bigcup v_{(i+3),(m-k)} \bigcup v_{(i+1),k}\right] \left[\bigcup_{t=i+4}^{n+i-1} \bigcup_{k=0}^{m} v_{t,k}\right] \bigcup_{k=0}^{m-j-1} v_{i,(m-k)} \bigcup_{k=1}^{j} v_{i,k}$ in the $n-2$ edge fault tolerant graph G^* has a hamiltonian path between the vertices $v_{i,0}$ and $v_{i,(j+1)}$.

In G, $d(v_{i,0}, v_{i,1}) = 1$. In this case G^* is a $n-2$ edge fault tolerant graph with the Hamiltonian-1-laceable path as above.

claim 3. The vertices $v_{i,0}$ and $v_{(i+1),0}$ are attainable.

In G, $d(v_{i,0}, v_{(i+1),0}) = 1$ where $i = 1, 2, 3..., n$ and the path is $P : v_{i,0} \bigcup_{k=0}^{m} \left[v_{(i+2),k} \cup v_{(i+3),(m-k)}\right] \bigcup_{t=i+4}^{n+i-1} \bigcup_{k=0}^{m} v_{t,k} \bigcup_{k=0}^{m} v_{i,(k+1)} \bigcup_{k=0}^{m} v_{(i+1),(m-k)}$ in the $n-2$ edge fault tolerant graph G^* has a hamiltonian path between the vertices $v_{i,0}$ and $v_{(i+1),0}$.

claim 4. The vertices $v_{i,0}$ and $v_{(i+j),0}$ are attainable.

In G, $d(v_{i,0}, v_{(i+j),0}) = 1$ where $1 < j < (n-2)$ and the path is $P : \bigcup_{k=0}^{m} \left[v_{i,k} \bigcup v_{(i+1),(m-k)}\right] \left[\bigcup_{t=i+2}^{j+i-1} \bigcup_{k=0}^{m} v_{t,k}\right]^{p} \bigcup_{t=i+j+1}^{n+i-1} \bigcup_{k=0}^{m} v_{t,k} \bigcup_{k=0}^{m} v_{(i+j),(m-k)}$ in the $n-2$ edge fault tolerant graph G^*

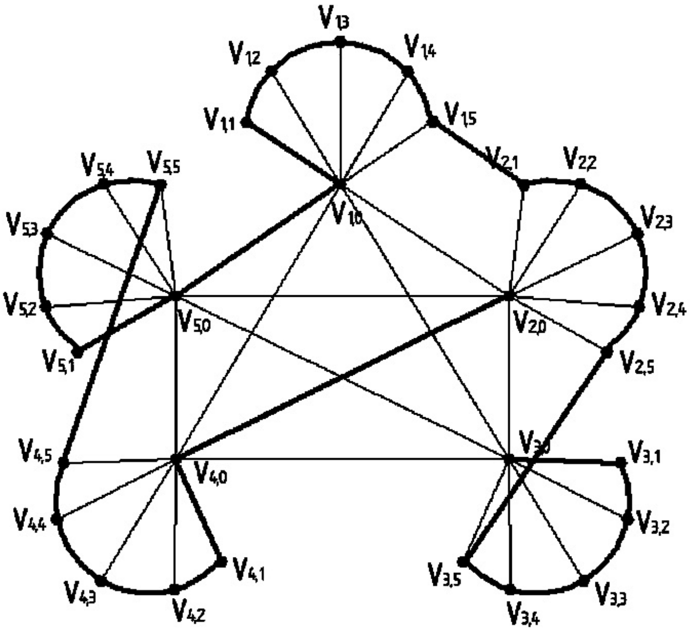

FIGURE 3. Corona product K_5oP_5 with $d(v_{2,0}, v_{3,0}) = 1$

has a hamiltonian path between the vertices $v_{i,0}$ and $v_{(i+j),0}$, where $p = \left\{ \begin{array}{ll} 1 & j \geq 3 \\ 0 & j < 3 \end{array} \right\}$
Hence the proof.

□

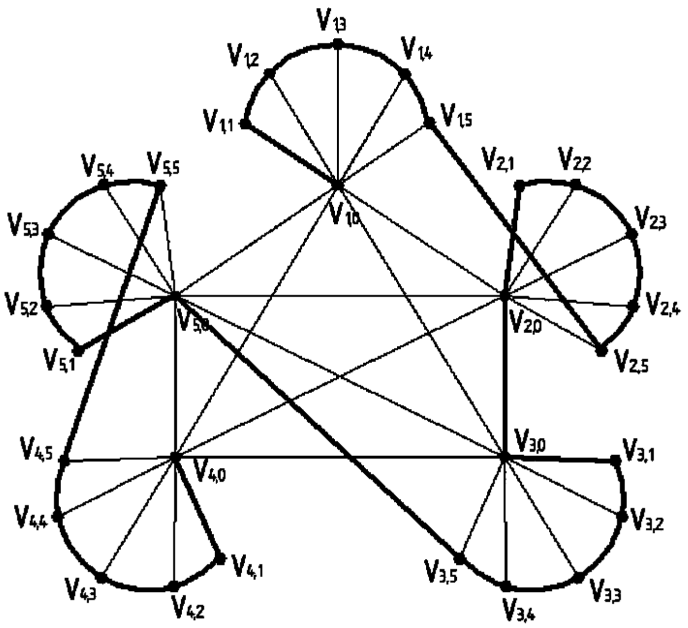

FIGURE 4. Corona product K_5oP_5 with $d(v_{1,0}, v_{4,0}) = 1$

Theorem 2.2. *For $n \geq 5$ and $m \geq 3$, the $n-2$ edge fault tolerant graph $K_n o P_m$ is Hamiltonian-2-laceable.*

Proof. The order of the graph G is same as the Theorem 2.1.

Since $d(v_{i,j}, v_{i,(j+s)}) = d(v_{i,j}, v_{(i+s),0}) = 2$ in G for all $1 \leq i \leq n, 1 \leq j \leq m$, it is enough to prove that there exists a hamiltonian path in G between these pairs of vertices.

claim 1. The vertices $V_{i,j}$ and $V_{i,(j+s)}$ are attainable.

In G, $d(v_{i,j}, v_{i,(j+s)}) = 2$ where $1 \leq j \leq (m-2)$, $2 \leq s \leq (m-j)$ and the path is $P : \bigcup_{k=1}^{j} v_{i,(j-k+1)} \cup v_{i,0} \bigcup_{k=j+1}^{j+s-1} v_{i,k} \bigcup_{t=i+1}^{n-1} \bigcup_{k=0}^{m} \left[v_{t,(m-k)} \cup v_{(t+1),k}\right] \bigcup_{k=0}^{m-j-s} v_{i,(m-k)}$ in the $n-2$ edge fault tolerant graph G^* is a hamiltonian path between the vertices $v_{i,j}$ and $v_{i,(j+s)}$.

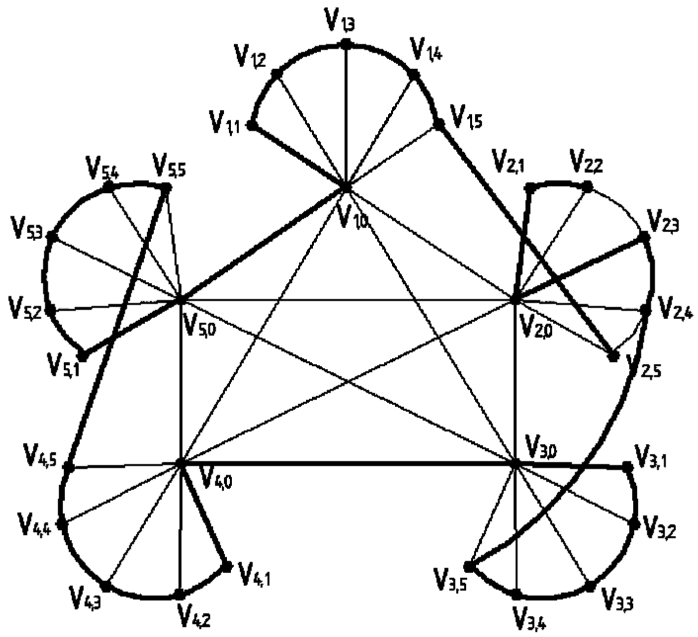

FIGURE 5. Corona product $K_5 o P_5$ with $d(v_{2,2}, v_{2,5}) = 2$

claim 2. The vertices $v_{i,j}$ and $v_{(i+s),0}$ are attainable.

In G, $d(v_{i,j}, v_{(i+s),0}) = 2$ where $1 \leq j \leq m$, $1 \leq s \leq n-i$ and the path is $P : \bigcup_{k=1}^{j} v_{i,(j-k+1)} \bigcup v_{i,0} \bigcup_{k=j+1}^{m} v_{i,k} \left[\bigcup_{t=i+1}^{s+i-1} \bigcup_{k=0}^{m} v_{t,k}\right]^{l} \bigcup_{t=i+s+1}^{n+i-1} \bigcup_{k=0}^{m} v_{t,k} \bigcup_{k=0}^{m} v_{(i+s),(m-k)}$ in the $n-1$ edge fault tolerant graph G^* is a hamiltonian path between the vertices $v_{i,j}$ and $v_{(i+s),0}$ where $l = \left\{ \begin{array}{cc} 1 & s > 1 \\ 0 & s \leq 1 \end{array} \right\}$

Hence the proof. □

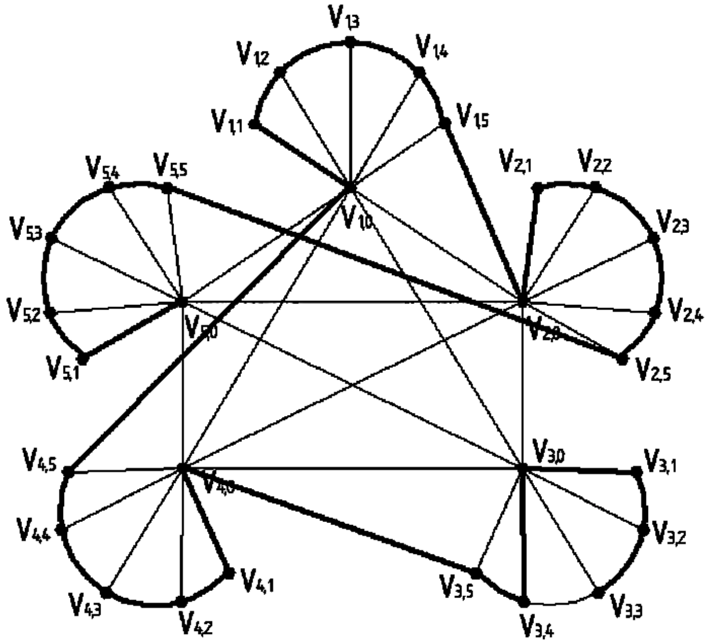

FIGURE 6. Corona product K_5oP_5 with $d(v_{3,3}, v_{5,0}) = 2$

Theorem 2.3. *For $n \geq 5$ and $m \geq 3$, the $n-1$ edge fault tolerant graph $K_n o P_m$ is Hamiltonian-3-laceable.*

Proof. The order of the graph G is same as the Theorem 2.1

Since $d(v_{i,j}, v_{(i+s),q}) = 3$ in G for all $1 \leq i \leq n, 1 \leq j \leq m$, it is enough to prove that there exists a hamiltonian path in G between these pair of vertices.

claim. The vertices $v_{i,j}$ and $v_{(i+s),q}$ are attainable.

In G, $d(v_{i,j}, v_{(i+s),q}) = 3$ where $1 \leq s \leq (n-i)$, $1 \leq j, q \leq m$ and the path is

$$P : \bigcup_{k=1}^{j} v_{i,(j-k+1)} \bigcup v_{i,0} \bigcup_{k=j+1}^{m} v_{i,k} \left[\bigcup_{t=i+1}^{s+i-1} \bigcup_{k=0}^{m} v_{t,k}\right]^{l} \bigcup_{t=i+s+1}^{n+i-1} \bigcup_{k=0}^{m} v_{t,k} \bigcup_{k=1}^{m-q} v_{(i+s),(m-k+1)} \bigcup v_{(i+s),0}$$

$\bigcup_{k=1}^{q} v_{(i+s),k}$ in the $n-1$ edge fault tolerant graph G^* has a hamiltonian path between the vertices $v_{i,j}$ and $v_{(i+s),q}$. Where $l = \left\{ \begin{array}{ll} 1 & s > 1 \\ 0 & s \leq 1 \end{array} \right\}$

Hence the proof. □

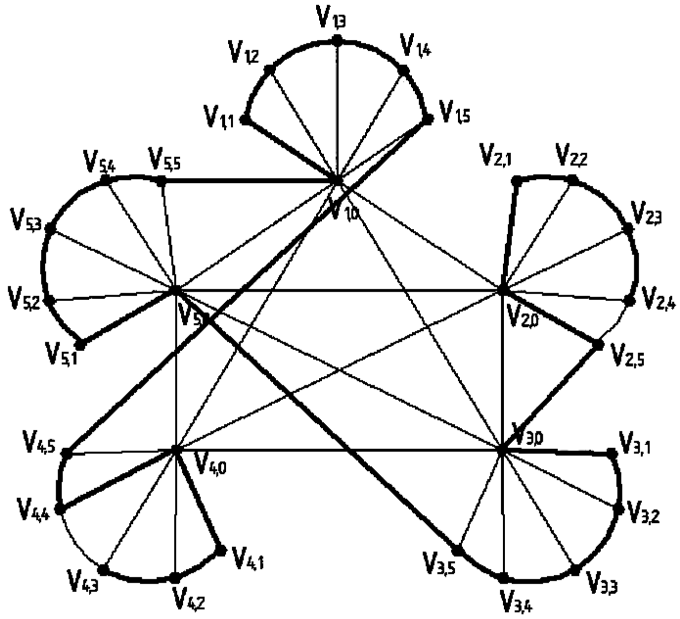

FIGURE 7. Corona product $K_5 o P_5$ with $d(v_{2,4}, v_{4,3}) = 3$

3. Conclusions

Laceability properties of the Corona product of complete graph and path graph has been explored. It is shown that this graph is Hamiltonian-t-laceable with edge fault tolerance $n-2$ or $n-1$. Work on other classes of graphs is presently in progress.

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