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EFFECTS OF SINUSOIDAL AND NON-SINUSOIDAL TEMPERATURE MODULATION IN A TRIPLE DIFFUSIVE CONVECTION

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ABSTRACT. The Triple Diffusive convection with time-dependent sinusoidal (cosine) and non-sinusoidal (square and triangular) temperature modulation is studied using linear and non-linear analysis. The expression for Rayleigh number and correction Rayleigh number is obtained by using perturbation method which gives the prospect to control the convection. Effects of various parameters of the problem are individually studied for two cases of temperature modulation namely, (i) in-phase and (ii)out-of-phase. Ginzburg-Landau equation using multi-scale method is derived to study the effects of temperature modulation on heat and mass transfer. It is observed that both solutal Rayleigh numbers stabilizes or destabilizes the system depending on the values of the frequency of modulation.

Keywords: Triple diffusive convection, Temperature modulation, Sinusoidal and non-sinusoidal wave form, Correction Rayleigh number.

AMS Subject Classification: 76E06

1. Introduction

The single component convection was studied first by Bénard [1] and this was further explained by Rayleigh [2] in terms of buoyancy and by Pearson [3] in terms of surface tension. Diffusion process in single component is straightforward but in multicomponent, the system is not that easy compared to single component. The curiosity of oceanography led to the study of double diffusive convection by Stern [4], Turner [5] and many more authors. Furthermore, this inquisitiveness led to another important convective phenomena called triple diffusive convection in which the density depends on three independently diffusing stratifying agencies. Triple diffusive convection can be found in solidification of molten alloys, geothermal lakes, sea water, etc. The onset of motion in a triple diffusive convection is fundamentally different from that of single and double diffusive cases. The presence of third diffusive component in the system leads to a important and remarkable results which are not observed in single and two component convection. Griffiths [6] studied the limiting condition for the formation of salt fingers at an initially sharp density step

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in a three-component system. Poulikakos [7] examined that the presence of third diffusing component with small diffusivity which can seriously alter the nature of the convective instabilities in the system. Later Sameena and Pranesh [8, 9] investigated the heat and mass transfer in a triple diffusive convection by deriving the Ginzburg-Landau equation in a couple stress fluid with and without rotation respectively. Sameena and Pranesh [10] studied the effect of temperature modulation on heat and mass transfer by considering only sinusoidal wave modulation in a couple stress fluid using non-linear analysis. Recently, Raghunatha et. al. [11] studied the effect of cross diffusion in triple diffusive convection in a Oldroyd-B fluid using linear and nonlinear analyses. They found that the cross diffusion either stabilises or destabilises the system depending on the solute concentration values.

One of the functional ways of monitoring convection is by maintaining the non-uniform thermal gradient which solely depends on space. Nonetheless, in various applied circumstances non-uniform thermal gradient finds the source in transient cooling or heating at the boundaries, therefore the basic thermal profiles depend explicitly on time as well as position and these kinds of problems are considered as temperature modulation. Venezian [12] examined the effect of modulation on the onset of thermal convection using perturbation method and concluded that the shift in the critical Rayleigh number is calculated as a function of frequency, and he observed that it is feasible to progress or postpone the onset of convection by time modulation of the wall temperatures. Recently, Siddheshwar et al. [13] studied the effects of temperature modulation and variable viscosity on the onset of Rayleigh-Bénard convection in Newtonian dielectric fluid using linear theory by considering only sinusoidal type. Though many authors have studied the effect of modulation on the onset of thermal convection, to the best of our knowledge, none of them have considered the effect of third component on the onset of convection in a sinusoidal and non-sinusoidal wave temperature modulation. Therefore, the main objective of this paper is to study the effect of third component on the onset of convection and heat mass transfer with sinusoidal (cosine) and non-sinusoidal (square and triangular) wave type of temperature modulation.

2. Mathematical Formulation

The horizontal layer of Newtonian fluid is taken between two parallel plates situated at $z = 0$ and $z = d$. The fluid layer is heated and solutes are added from below. Let ΔT, ΔS_1 and ΔS_2 are temperature and solutes 1, 2 difference maintained between the plates. Cartesian coordinates with origin in lower plate and z-axis vertically upwards are taken. Gravity is acting vertically downwards and in addition to fixed temperature difference between the plates an additional perturbation is applied to the wall temperatures varying sinusoidally or non-sinusoidally.

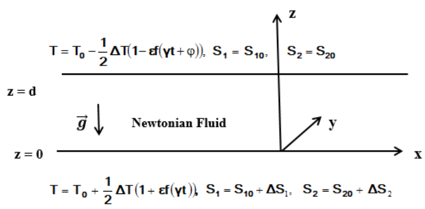

Figure 1. Physical Configuration

Under the Boussinesq approximation the continuity equation, conservation of linear momentum, conservation of energy, conservation of solute 1, conservation of solute 2 and equation of state for the problem are given by:

$$\nabla \cdot \vec{q} = 0, \tag{1}$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} \right] = -\nabla p - \rho g \hat{k} + \mu \nabla^2 \vec{q}, \tag{2}$$

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \chi \nabla^2 T, \tag{3}$$

$$\frac{\partial S_i}{\partial t} + +(\vec{q}.\nabla)S_i = \chi_{S_i} \nabla^2 S_i, \ (i = 1, 2), \tag{4}$$

$$\rho = \rho_0 \left[1 - \alpha_t (T - T_0) + \alpha_{S_i} \big(S_i - S_{i0} \big) \right] \ (i = 1, 2). \tag{5}$$

where, $\vec{q}$ is the velocity vector, ρ is the density, t is time, p is the pressure, μ is the coefficient of viscosity, g is gravity, T is Temperature, S_i is concentration of solute i, ρ_0 is the refence density, α_t is the coefficient of thermal expansion, α_{s_i} is concentration analog of thermal expansion of solute i, χ is thermal diffusivity, χ_{S_i} is solutal diffusivity of solute$i$. The lower and upper plates are maintained with temperature and solutes;

$$T(0,t) = T_0 + \frac{\Delta T}{2}[1 + \epsilon f(\gamma t)], \quad T(d,t) = T_0 - \frac{\Delta T}{2}[1 - \epsilon f(\gamma t + \varphi)], \tag{6}$$

$$S_i = S_{i0} + \Delta S_i \text{ at } z = 0, \qquad S_i = S_{i0} \text{ at } z = d \ (i = 1, 2). \tag{7}$$

where, ϵ is the amplitude of modulation, γ is frequency of modulation and φ is phase angle. The temperature modulation comes into picture in the present problem through the boundary condition (6).

We consider two types of temperature modulation namely:
Case (i): In-phase (Symmetric, $\varphi = 0$) Case (ii): Out-of-phase (Asymmetric, $\varphi = \pi$),

Using the boundary conditions (6) and (7), the solutions of motionless state can be written as.

$$\vec{q_b} = (0,0,0), \qquad S_{ib} = S_{i0} + \Delta S_i \left(1 - \frac{z}{d}\right), \ (i = 1, 2),$$

$$T_b(z,t) = T_0 + \left(1 - \frac{2z}{d}\right) \Delta T + \epsilon \ Re \left[H(m) e^{\frac{mz}{d}} + H(-m) e^{\frac{-mz}{d}} \right] e^{-i\gamma t}, \tag{8}$$

where, $m = (1-i) \left(\frac{\gamma d^2}{2\chi} \right)^{(1/2)}$, $\quad H(m) = \frac{\Delta T}{2} \left[\frac{e^{-i\varphi} - e^{-m}}{e^m - e^{-m}} \right]$,

The three different wave types of temperature modulation are considerd in the paper, namely:
(i) Cosine : $f(\gamma, t) = Cos(\gamma t)$,
(ii) Square: $f(\gamma, t) = \sum_{n=1,3,5,\cdots}^{\infty} \frac{4}{n\pi} sin(n\gamma t)$,
(iii) Triangular: $f(\gamma, t) = \sum_{n=1,3,5,\cdots}^{\infty} \frac{8}{n^2\pi^2} (-1)^{\frac{n-1}{2}} sin(n\gamma t)$,

3. Stability Analysis

The stability of the system is analysed by introducing the following perturbation on the motionless state and is given by,

$$\vec{q} = \vec{q_b} + \vec{q}\,', p = p_b + p', \ \rho = \rho_b + \rho', T = T_b + T', S_i = S_{ib} + S_i' \ (i = 1, 2), \tag{9}$$

where, the prime indicates that the quantities are infinitesimal perturbations.
Substituting equation (9) in equations (1)-(5), using motionless state solution (8), rendereding resultant equations dimensionless using $(x^*, y^*, z^*) = \left(\frac{x'}{d}, \frac{y'}{d}, \frac{z'}{d}\right)$, $t^* = \frac{t'}{d^2/\chi}$, $q^* = \frac{\vec{q'}}{\chi/d}$, $p^* = \frac{d^2 p'}{\mu\chi}$, $\theta^* = \frac{T'}{\Delta T}$, $\phi^*_{S_i} = \frac{S'}{\Delta S_i}$ and introducing the stream function ψ by $u = \frac{\partial\psi}{\partial z}$, $w = -\frac{\partial\psi}{\partial x}$, we get the following dimensionless equations after eliminating the pressure term.

$$\begin{bmatrix} -\nabla^4 & Ra\frac{\partial}{\partial x} & -R_{S_1}\frac{\partial}{\partial x} & -R_{S_2}\frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 & 0 \\ \frac{\partial}{\partial x} & 0 & -\tau_1\nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 & -\tau_2\nabla^2 \end{bmatrix} \begin{bmatrix} \psi \\ \theta \\ \phi_{S_1} \\ \phi_{S_2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{Pr}\frac{\partial}{\partial t}(\nabla^2\psi) + \frac{1}{Pr}\frac{\partial(\psi,\nabla^2\psi)}{\partial(x,z)} \\ -\frac{\partial\theta}{\partial t} + \delta^2\epsilon_2 f_1\frac{\partial\psi}{\partial x} + \frac{\partial(\psi,\theta)}{\partial(x,z)} \\ -\frac{\partial\phi_{S_1}}{\partial t} + \frac{\partial(\psi,\phi_{S_1})}{\partial(x,z)} \\ -\frac{\partial\phi_{S_2}}{\partial t} + \frac{\partial(\psi,\phi_{S_2})}{\partial(x,z)} \end{bmatrix} \tag{10}$$

In equation (10) the dimensionless groups are $Ra = \frac{\rho_0\alpha_t g\Delta T d^3}{\mu\chi}$ the thermal Rayleigh number, $R_{S_i} = \frac{\rho_0\alpha_{S_i} g\Delta S_i d^3}{\mu\chi}$ the solutal Rayleigh number of solute i, $Pr = \frac{\mu}{\rho_0\chi}$ the Prandtl number, $\tau_i = \frac{\chi_{s_i}}{\chi}$ the ratio of diffusivity of solute i and the heat diffusivity and $\omega = \frac{\gamma d^2}{\chi}$ is the frequency. In the equation(10) we observe that the motionless state solution influences the stability problem through the factor $\frac{\partial T_b}{\partial z}$, which is substituted by

$$\frac{\partial T_b}{\partial z} = -1 + \delta^2\epsilon_2 f_1 \tag{11}$$

where, $f_1 = Re\left[A_1(\lambda)e^{\lambda z} + A_1(-\lambda)e^{-\lambda z}\right] f$, $\lambda = (1-i)\left(\frac{\omega}{2}\right)^{\frac{1}{2}}$, $A_1(\lambda) = \frac{\lambda}{2}\left[\frac{e^{-i\varphi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}\right]$,

4. Linear Stability Analysis

In this section, we obtain the condition for the onset of convection immense useful in the local non-linear analysis to be discussed in the next section. To make this study we neglect the Jacobians in the system (10). Eliminating θ, ϕ_{S_1} and ϕ_{S_2} from the resulting equations (10)and replacing $\epsilon = \delta^2\epsilon_2$ we get equation of ψ in the form:

$$\left[X_1X_2X_3X_4\nabla^2 + [Ra(-1+\epsilon f_1)X_4X_3 + R_{S_1}X_2X_3 + R_{S_2}X_3X_2]\frac{\partial^2}{\partial x^2}\right]\psi = 0, \tag{12}$$

where,

$$X_1 = \left[\frac{1}{Pr}\frac{\partial}{\partial t} - \nabla^2\right], \quad X_2 = \left[\frac{\partial}{\partial t} - \nabla^2\right], \quad X_3 = \left[\frac{\partial}{\partial t} - \tau_1\nabla^2\right], \quad X_4 = \left[\frac{\partial}{\partial t} - \tau_2\nabla^2\right],$$

The boundary conditions for solving equation(10) are obtained in the form:

$$\psi = \frac{\partial^2\psi}{\partial z^2} = \frac{\partial^4\psi}{\partial z^4} = \frac{\partial^6\psi}{\partial z^6} = \frac{\partial^8\psi}{\partial z^8} = 0 \text{ at } z = 0, 1. \tag{13}$$

The eigen value Ra of the equation (12) is obtained by expanding Ra and ψ as a function of small amplitude of modulation ϵ in the form:

$$(Ra, \psi) = (Ra_0, \psi_0) + \epsilon(Ra_1, \psi_1) + \epsilon^2(Ra_2, \psi_2) + \cdots \tag{14}$$

Here, Ra_0 is the solution for the unmodulated system ($\epsilon = 0$), ψ_i and Ra_i ($i \geq 1$) are the correction due to temperature modulation. Using equation (14) in equation (12) and equating the coefficents of corresponding powers of ϵ, we obtain:

$$L\psi_0 = 0, \tag{15}$$

$$L\psi_1 = -Ra_0X_4X_3f_1\frac{\partial^2\psi_0}{\partial x^2} + Ra_1X_3X_4\frac{\partial^2\psi_0}{\partial x^2}, \tag{16}$$

$$L\psi_2 = -Ra_0X_4X_3f_1\frac{\partial^2\psi_1}{\partial x^2} + Ra_1X_3X_4\Big(\frac{\partial^2\psi_1}{\partial x^2} - f_1\frac{\partial^2\psi_0}{\partial x^2}\Big) + Ra_2X_3X_4\frac{\partial^2\psi_0}{\partial x^2}, \tag{17}$$

where,

$$L = \Big[X_1X_2X_3X_4\nabla^2 + (-Ra_0X_4X_3 + R_{S_1}X_2X_3 + R_{S_2}X_3X_2)\frac{\partial^2}{\partial x^2}\Big].$$

The solution of equation (15) is taken in the form $\psi_0 = sin(\pi\alpha x)sin(\pi z)$ and the corresponding eigen value Ra_0 for unmodulated case is given by,

$$Ra_0^{st} = \frac{Rs_1}{\tau_1} + \frac{Rs_2}{\tau_2} + \frac{k^6}{\pi^2a^2}, \tag{18}$$

where, $k^2 = \pi^2(1+\alpha^2)$.
When $R_{S_1} = R_{S_2} = 0$, equation (18) reduces to the expression for the classical Rayleigh number.
Following the analysis of Siddheshwar and Pranesh [14], we get:

$$Ra_1 = 0 \tag{19}$$

$$Ra_{2_c} = \frac{Ra_0^2\pi^2\alpha^2}{2}\sum_{n=1}^{\infty}|B_n(\lambda)|^2|Z_3|^2\left[\frac{L_1(\omega,n)+L_1^*(\omega,n)}{2|L_1|^2}\right], \tag{20}$$

where, $L(\omega, n) = Y_1 + iY_2, \quad L_1(\omega, n) = Z_3^*L(\omega, n),$

$$Y_1 = \frac{1}{Pr}\left[\pi^2\alpha^2Pr[-k^4(Rs_2\tau_1 + (Rs_1 - Ra_0\tau_1)\tau_2) + (-Ra_0 + Rs_1 + Rs_2)\omega^2]\right.$$
$$\left. + k^2[-k^8Pr\tau_1\tau_2 + k^4(Z_2 + PrZ_1)\omega^2] - k^2\omega^4\right],$$

$$Y_2 = \frac{\omega}{Pr}\Big[\pi^2\alpha^2Prk^2[Rs_1(1+\tau_2) + Rs_2(1+\tau_1) - Ra_0(\tau_1+\tau_2)] + k^2[k^6(\tau_1\tau_2 + PrZ_2) - k^2(Pr + Z_1)\omega^2]\Big]$$

$$Z_1 = 1 + \tau_1 + \tau_2, \quad Z_2 = \tau_1 + \tau_2 + \tau_1\tau_2, \quad Z_3 = (-i\omega + \tau_1k^2)(-i\omega + \tau_2k^2),$$

$$B_n(\lambda) = \frac{-2n\pi^2\lambda^2\left[e^{\lambda} - e^{-\lambda} + (-1)^n(e^{-\lambda-i\varphi} - e^{\lambda-i\varphi})\right]}{e^{\lambda} - e^{-\lambda}\left[\lambda^2 + (n+1)^2\pi^2\right]},$$

5. Local Nonlinear Stability Analysis

We assume the form of the stream function, temperature and solute concentrations as follows:

$$\psi = A(\tau)sin(\alpha x)sin(\pi z), \tag{21}$$

$$\theta = B(\tau)cos(\alpha x)sin(\pi z) + C(\tau)sin(2\pi z), \tag{22}$$

$$\phi_{S_i} = E_i(\tau)cos(\alpha x)sin(\pi z) + F_i(\tau)sin(2\pi z), \ (i = 1, 2), \tag{23}$$

where, the time dependent amplitudes A, B, C, E_i and F_i, are to be determined from the dynamics of the system.
Substituting equation (21)-(23) in equation (10) and taking the orthogonality condition with the eigen functions associated with the considered minimal modes, we get

$$\frac{1}{Pr}\frac{\partial A}{\partial \tau} = \frac{\pi\alpha}{k^2}\left[-RaB + R_{S_1}E_1 + +R_{S_2}E_2\right] - k^2 A, \tag{24}$$

$$\frac{\partial B}{\partial \tau} = -\pi^2\alpha AC + (-1+\epsilon f)\pi\alpha A - k^2 B, \tag{25}$$

$$\frac{\partial C}{\partial \tau} = \frac{\pi^2\alpha AB}{2} - 4\pi^2 C, \tag{26}$$

$$\frac{\partial E_i}{\partial \tau} = -\pi^2\alpha AF_i - \pi\alpha A - k^2\tau_i E_i,\ (i=1,2), \tag{27}$$

$$\frac{\partial F_i}{\partial \tau} = \frac{\pi^2\alpha AE_i}{2} - 4\pi^2\tau_i F_i,\ (i=1,2), \tag{28}$$

We now use the following regular perturbation expansion in equations (24)- (28):

$$A = \epsilon A_1 + \epsilon^2 A_2 + \epsilon^3 A_3\ ,\ B = \epsilon B_1 + \epsilon^2 B_2 + \epsilon^3 B_3\ ,\ C = \epsilon C_1 + \epsilon^2 B_2 + \epsilon^3 C_3\ ,\ E_i = \epsilon E_{i_1} + \epsilon^2 E_{i_2} + \epsilon^3 E_{i_3}\ ,\quad F_i = \epsilon F_{i_1} + \epsilon^2 F_{i_2} + \epsilon^3 F_{i_3}\ ,\ Ra = Ra_0 + \epsilon Ra_1 + \epsilon^2 Ra_2 + \epsilon^3 Ra_3, \tag{29}$$

where, $\tau = \epsilon^2 t$,
Substituting equation (29) in equations (24)-(28) and on comparing the corresponding powers of ϵ in the resulting equations, we get:

$$\mathcal{L}V_1 = 0, \tag{30}$$

$$\mathcal{L}V_2 = \begin{bmatrix} 0 & 0 & \frac{\pi^3\alpha^2}{2k^2}A_1^2 & 0 & \frac{\pi^3\alpha^2}{2k^2\tau_1}A_1^2 & 0 & \frac{\pi^3\alpha^2}{2k^2\tau_2}A_1^2 \end{bmatrix}^{Tr}, \tag{31}$$

$$\begin{aligned}\mathcal{L}V_3 = \Bigg[&\frac{dA_1}{d\tau} + Ra_2\left(-\frac{\pi\alpha}{k^2}A_1\right)\ \left(-\frac{\pi\alpha}{k^2}\frac{dA_1}{d\tau}\right) - f_1\pi\alpha A_1 - \frac{\pi^5\alpha^3}{8\pi^2k^2}A_1^3\ \frac{\pi^3\alpha^2}{k^2}A_1A_2 \\ &-\frac{\pi\alpha}{\tau_1k^2}\frac{dA_1}{d\tau} - \frac{\pi^4\alpha^3A_1^3}{8\tau_1\pi^2k^2}\ \frac{\pi^3\alpha^2}{\tau_1k^2}A_1A_2\ - \frac{\pi\alpha}{\tau_ik^2}\frac{dA_1}{d\tau} - \frac{\pi^4\alpha^3A_1^3}{8\tau_2\pi^2k^2}\ \frac{\pi^3\alpha^2}{\tau_2k^2}A_1A_2\Bigg]^{Tr},\end{aligned} \tag{32}$$

where $V_i = \begin{bmatrix} A_j & B_j & C_j & E_{1_j} & F_{1_j} & E_{2_j} & F_{2_j}\end{bmatrix},\ (j=1,2,3)$,

$$\mathcal{L} = \begin{bmatrix} 0 & -\frac{Ra_0Pr\pi\alpha}{k^2} & 0 & \frac{R_{S_1}Pr\pi\alpha}{k^2} & 0 & \frac{R_{S_2}Pr\pi\alpha}{k^2} & 0 \\ -\pi\alpha & -k^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4\pi^2 & 0 & 0 & 0 & 0 \\ -\pi\alpha & 0 & 0 & -\tau_1k^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4\pi^2\tau_1 & 0 & 0 \\ -\pi\alpha & 0 & 0 & 0 & 0 & -\tau_2k^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -4\pi^2\tau_2 \end{bmatrix},$$

The solution of equation (30) and (31) subject to initial conditions is given by :

$$V_1^T = \begin{bmatrix} A_{10} & -\frac{\pi\alpha}{k^2}A_{10} & 0 & -\frac{\pi\alpha}{\tau_1k^2}A_{10} & 0 & -\frac{\pi\alpha}{\tau_2k^2}A_{10} & 0\end{bmatrix}^{Tr}, \tag{33}$$

$$V_2^T = \begin{bmatrix} A_{20} & -\frac{\pi\alpha}{k^2}A_{20} & \frac{-\pi^3\alpha^2}{8\pi^2k^2}A_{10}^2 & \frac{-\pi\alpha}{\tau_1k^2}A_{20} & \frac{-\pi^3\alpha^2}{8\pi^2\tau_1k^2}A_{10}^2 & \frac{-\pi\alpha}{\tau_2k^2}A_{20} & \frac{-\pi^3\alpha^2}{8\pi^2\tau_2k^2}A_{10}^2\end{bmatrix}^{Tr}, \tag{34}$$

where, A_{10} and A_{20} are arbitrary functions of τ. Using Fredholm solvability condition we get the Ginzburg-Landau equation in the form:

$$P_1 \frac{dA_{10}}{d\tau} = P_2 A_{10} - P_3 A_{10}^3, \tag{35}$$

where, $P_1 = 1 + \dfrac{Ra_0 Pr \pi^2 \alpha^2}{k^6} - \dfrac{R_{S_1} Pr \pi^2 \alpha^2}{\tau_1 k^6} - \dfrac{R_{S_1} Pr \pi^2 \alpha^2}{\tau_2 k^6}$,

$P_2 = (Ra_2 - Ra_0 f)\dfrac{Pr\pi^2\alpha^2}{k^2}, P_3 = \left(Ra_0 - \dfrac{R_{S_1}}{\tau_1^2} - \dfrac{R_{S_2}}{\tau_2^2}\right)\dfrac{Pr\pi^3\alpha^4}{k^6}$,

6. Heat and Mass Transport at Lower Boundary

The expression for Nusselt number Nu and Sherwood number Sh which quantifies the amount of heat and mass transport is given by

$$Nu(\tau) = \frac{\left[\dfrac{\alpha}{2\pi}\int_0^{\frac{2\pi}{\alpha}} (1 - z + \theta)_z dx\right]_{z=0}}{\left[\dfrac{\alpha}{2\pi}\int_0^{\frac{2\pi}{\alpha}} (1 - z)_z dx\right]_{z=0}} = 1 - 2\pi C(\tau) = 1 + \epsilon^2 \frac{\pi^2\alpha^2}{4k^2} A_{10}^2, \tag{36}$$

$$Sh_i(\tau) = \frac{\left[\dfrac{\alpha}{2\pi}\int_0^{\frac{2\pi}{\alpha}} (1 - z + \phi_{S_i})_z dx\right]_{z=0}}{\left[\dfrac{\alpha}{2\pi}\int_0^{\frac{2\pi}{\alpha}} (1 - z)_z dx\right]_{z=0}} = 1 - 2\pi F_i(\tau) = 1 + \epsilon^2 \frac{\pi^2\alpha^2}{4k^2\tau_i^2} A_{10}^2, \ (i = 1, 2), \tag{37}$$

Average Nusselt number and average sheerwood number is considered in order to study effects of various parameters on heat and mass transport and it is given by:

$$\overline{Nu(\tau)} = \left(\frac{\omega}{2\pi}\right)\int_0^{\frac{2\pi}{\omega}} Nu \ d\tau, \quad \overline{Sh_i(\tau)} = \left(\frac{\omega}{2\pi}\right)\int_0^{\frac{2\pi}{\omega}} Sh_i \ d\tau, \ (i = 1, 2). \tag{38}$$

7. Results and Discussions

In this paper, the effects of sinusoidal (cosine) and non-sinusoidal (square and triangular) temperature modulations on the onset of triple diffusive convection heated and added solutes from below are analyzed with stress-free, isotemperature and isoconcentration boundaries. The influence of the parameters R_{S_1}, R_{S_2}, τ_1 and τ_2 on the onset and heat and mass transport in a triple diffusive convection are examined for sinusoidal and non-sinusoidal temperature modulations. The correction Rayleigh number R_{2_c} is plotted against frequency of modulation ω for in-phase temperature modulation where, $\varphi = 0$ and for out-of-phase temperature modulation where, $\varphi = \pi$. The graphs are plotted by taking moderate values of ω for a reason that when $\omega < 1$, the period of modulation is very high and makes finite amplitude important and $\omega \to \infty$ the modulations tends to zero. With $R_{S_1} = R_{S_2} = 0$, our results agree with those obtained by Venezian [12].

Figures (2)-(5) are the plots of R_{2_c} versus ω for different types of temperature modulations and for different values of R_{S_1}, R_{S_2}, τ_1 and τ_2. From these figures we observe that with an increase of ω, R_{2_c} decreases initially and reaches the minimum point (around $\omega = 5$). Further increase in ω, R_{2_c} increases and reaches the maximum value (around $\omega = 20$). For $\omega > 20$, R_{2_c} decreases with increase in ω and becomes zero for large values of ω. This indicates that the modulation effect is to destabilize the system for $\omega < 5$ and stabilize when $5 < \omega < 20$. The two peaks of R_{2_c} at $\omega = 10$ (approximately) and $\omega = 20$ (approximately) gives the maximum destabilizes and maximum stabilizes respectively.

Figure (2(A)) is the plot of R_{2_c} versus ω, for in-phase; cosine, square and triangular temperature modulations and for different values of R_{S_1}. From the figure, we observe that, with increase in the values of R_{S_1}, the value of R_{2_c} also increases. This is because when the solutes are added from below, the system is not disturbed as the solute concentration settles down at the lower boundary and this expands when heated and the onset of convection is delayed. Thus, increase in R_{S_1} stabilizes the system. We also observed that the increase in R_{S_1} increases the peak for negative and positive values of R_{2_c}, which shows that maximum destabilizing and maximum stabilizing can be achieved by increasing R_{S_1}. Also R_{2_c} changes from negative to positive at around $\omega = 8$, which shows in-phase modulation destabilizes when $\omega < 8$ and stabilizes when $\omega > 8$.

Figure (3(A)) is the plot of R_{2_c} versus ω, for in-phase; cosine, square and triangular temperature modulations and for different values of R_{S_2}. From the figure, we observe that, with increase in the values of R_{S_2}, R_{2_c} increases. The physical reason is same as that of R_{S_1} as R_{S_2} is also added from below and hence makes the system stable. We notice from the graph that by increasing the value ofR_{S_2} increases, the maximum peak of R_{2_c} increases, thus by adding the third component to the system, the system becomes more stable.

Figure (4(A)) and (5(A)) are the plots of R_{2_c} versus ω, for in-phase; cosine, square and triangular temperature modulations and for different values of τ_1 and τ_2 respectively. From the figures, we observe that, with increase in the values of τ_1 and τ_2, the values of R_{2_c} decreases. τ_1 and τ_2 represents the ratio of solute diffusivity with heat diffusivity. The increase in τ_1 and τ_2 advances the onset of convection because heat diffuses faster than solute and therefore, salute gradient advances the onset of convection and hence destabilizes the system. Comparing the two graphs we observe that both maximum destabilizing peak and maximum stabilizing peak decreases in the case of increase in τ_2 compare to τ_1.

Figures (2(B)) - (5(B)) presents the plots of R_{2_c} with ω for out-of-phase temperature modulation respectively for different values of R_{S_1}, R_{S_2}, τ_1 and τ_2. The effect of these parameters is quantitatively similar to that of its effect in in-phase temperature modulation. Comparing the figure (2(B)) and figure (3(B)) we found that maximum peak of R_{2_c} is higher in figure (3(B)), which means that the addition of the third component R_{S_2} for a two component system, stabilizes the system, the same result as observed in the case of in-phase temperature modulation. Thus, the triple diffusive convection is more stable than single and two component convection. This result can also be anticipated from equation (18). We observe that in the case of out-of-phase temperature modulation R_{2_c} changes from negative to positive at $\omega = 3$. Thus, comparing two types of temperature modulation, R_{2_c} is more positive in out-of-phase temperature modulation in-phase temperature modulation and also from figures we note that $R_{2_c}^{\text{in-phase}} < R_{2_c}^{\text{out-of-phase}}$. Therefore, we can conclude that the out-of-phase temperature modulation is more stable than that of in-phase temperature modulation.

From the above figures following are true for all the three types of temperature modulation:
(i) For $\omega < \omega^*$, $R_{2_c}^S < R_{2_c}^C < R_{2_c}^T$ (ii) For $\omega > \omega^*$, $R_{2_c}^T < R_{2_c}^C < R_{2_c}^S$,
where $\omega^* = 8$ and 3 for in-phase and out-of-phase temperature modulation respectively.
From the above observation we can conclude that in general square wave (S) type of temperature modulation is more stable compared to the cosine (C) and triangular (T) wave type of temperature modulation.

With the clear knowledge of effects of parameters and temperature modulations on the onset of convection, we move on to discuss their effects on heat and mass transfer which represents post onset regime of the convection. In the case of nonlinear analyses of Ginzburg-Landau equation (35) is derived from seventh order non -autonomous Lorenz model equations (24)- (28) using multi-scale method. The heat and mass transfer is quantified by Nusselt number (see equation (36)) and mass transports are quantified by Sherwood numbers Sh_1 and Sh_2 (see equation (37)). Evaluating Ginzburg-Landau equation (35) numerically, the average Nusselt number ($\overline{Nu}$) and average Sherwood numbers ($\overline{Sh_1}$ and $\overline{Sh_2}$) are calculated for different parameters and for different temperature modulations. The results obtained in this case are depicted in the Figures (6) - (9).
From the figures following observations are made:

i. $\overline{Nu}_{R_{S_1}=R_{S_2}=50} < \overline{Nu}_{R_{S_1}=R_{S_2}=100}$
ii. $\overline{Sh1}_{R_{S_1}=R_{S_2}=50} < \overline{Sh1}_{R_{S_1}=R_{S_2}=100}$
iii. $\overline{Sh2}_{R_{S_1}=R_{S_2}=50} < \overline{Sh2}_{R_{S_1}=R_{S_2}=100}$
Thus, increase in R_{S_1} and R_{S_2} increases $\overline{Nu}, \overline{Sh_1}$ and $\overline{Sh_2}$ and therefore increases the heat and mass transport. This is true for in-phase and out-of-phase modulation.
iv. $\overline{Nu}\ _{\tau_1=0.3/\tau_2=0.7} > \overline{Nu}\ _{\tau_1=0.5/\tau_2=0.9}$
v. $\overline{Sh_1}\ _{\tau_1=0.3/\tau_2=0.7} > \overline{Nu}\ _{\tau_1=0.5/\tau_2=0.9}$
vi. $\overline{Sh_2}\ _{\tau_1=0.3/\tau_2=0.7} > \overline{Nu}\ _{\tau_1=0.5/\tau_2=0.9}$
Thus, increase in diffusivity ratios decreases the heat and mass transport. This is true for in-phase and out-of-phase modulation.
vii. $\overline{Nu}^{\text{ in-phase}} > \overline{Nu}^{\text{ out-of-phase}}$, $\overline{Sh_1}^{\text{ in-phase}} > \overline{Sh_1}^{\text{ out-of-phase}}$, $\overline{Sh_2}^{\text{ in-phase}} > \overline{Sh_2}^{\text{ out-of-phase}}$,
Thus, heat and mass transport decreases in out-of-phase compare to in-phase.
viii. $\overline{Nu}^{\ T} < \overline{Nu}^{\ C} < \overline{Nu}^{\ S}$, $\overline{Sh_1}^{\ T} < \overline{Sh_1}^{\ C} < \overline{Sh_1}^{\ S}$, $\overline{Sh_2}^{\ T} < \overline{Sh_2}^{\ C} < \overline{Sh_2}^{\ S}$
Thus, heat and mass transport is maximum in square wave type of temperature modulation and least in triangular wave type of modulation in case of out-of-phase modulation, but in the case of in-phase modulation $\overline{Nu}$, $\overline{Sh_1}$ and $\overline{Sh_2}$ for all three types of modulations coincides because in in-phase modulation the phase angle is zero ($\varphi = 0$).
ix. In genral $\overline{Sh_2} < \overline{Sh_1}$ for all three types of temperature modulation.
x. Mass transfer is more compare to heat transfer.

8. Conclusions

The following are the conclusions drawn from the study:

(1) The temperature modulation can either stabilize or destabilise depending on the value of frequency of modulation.
(2) Compare to two types of temperature modulation, out-of-phase temperature modulation is more stable than that of in-phase.

(3) Square wave form is more stable compare to the cosine and triangular wave form in general.
(4) The addition of third component stabilizes the system more compare to single and two component convection.
(5) Solute Rayleigh numbers R_{S_1} and R_{S_2} stabilize the system, whereas ratio of diffusivities of solute to heat diffusivity τ_1 and τ_2 destabilize the system.
(6) It is possible to control the onset of triple diffusive convection and also heat and mass transfer by appropriately choosing the type of temperature modulation and also choosing the appropriate wave form of modulation.

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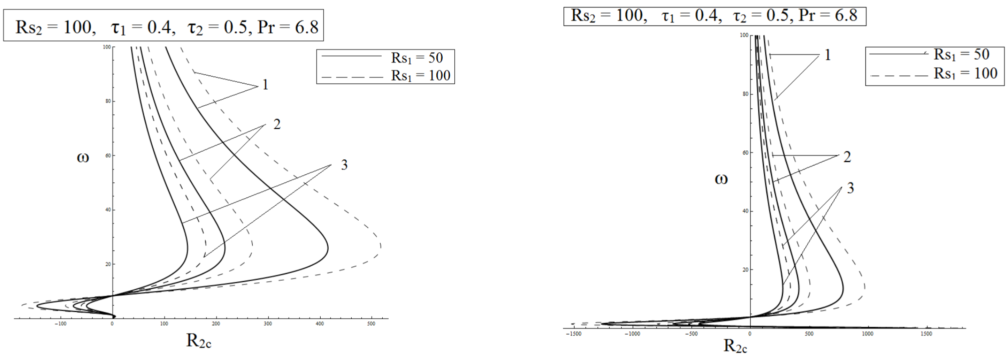
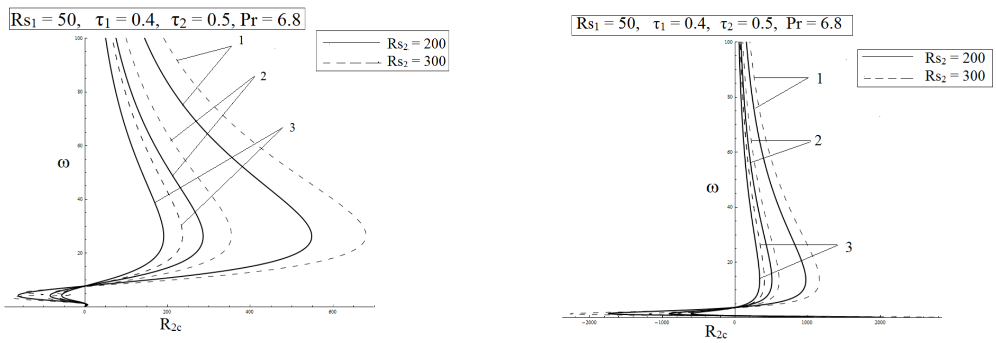


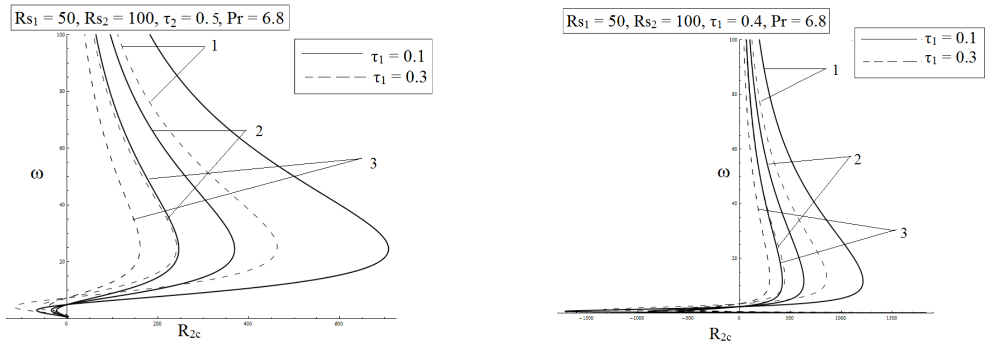
(A) (B)

FIGURE 2. Plot of variation of R_{2_c} with ω for different values of R_{S_1} for (A) in-phase and (B) out-of-phase temperature modulation and for (1) square (2) cosine and (3) triangular waveforms.



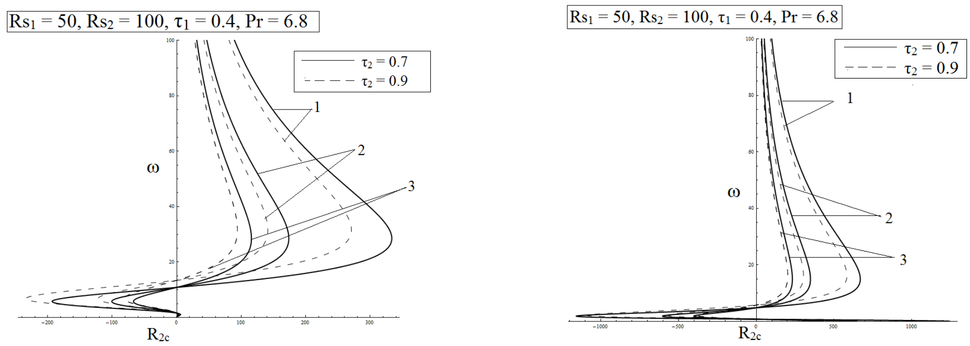
(A) (B)

FIGURE 3. Variation of R_{2_c} vs ω for different values of R_{S_2} for (A) in-phase and (B) out-of-phase temperature modulation and for (1) square (2) cosine and (3) triangular waveforms.



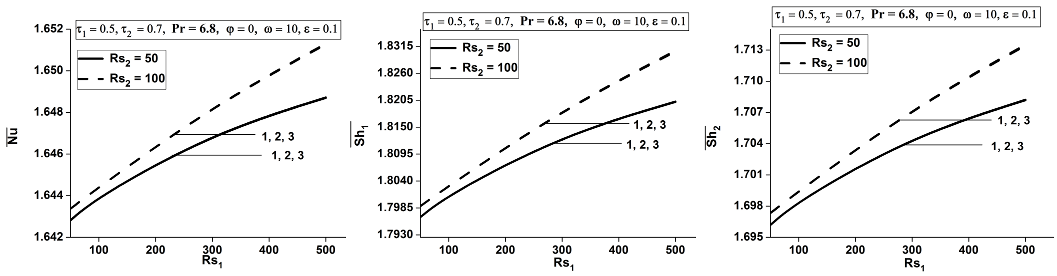
(A) (B)

FIGURE 4. Plot showing the effect of τ_1 on R_{2_c} for (A) in-phase and (B) out-of-phase temperature modulation and for (1) square (2) cosine and (3) triangular waveforms.



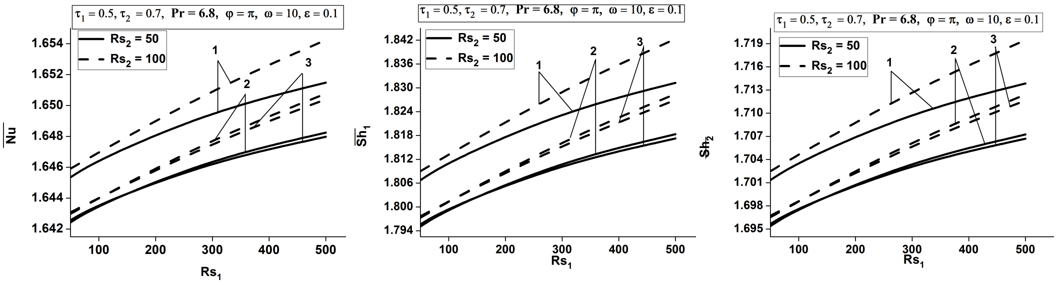
(A) (B)

FIGURE 5. Plot showing the effect of τ_2 on R_{2_c} for (A) in-phase and (B) out-of-phase temperature modulation and for (1) square (2) cosine and (3) triangular waveforms.



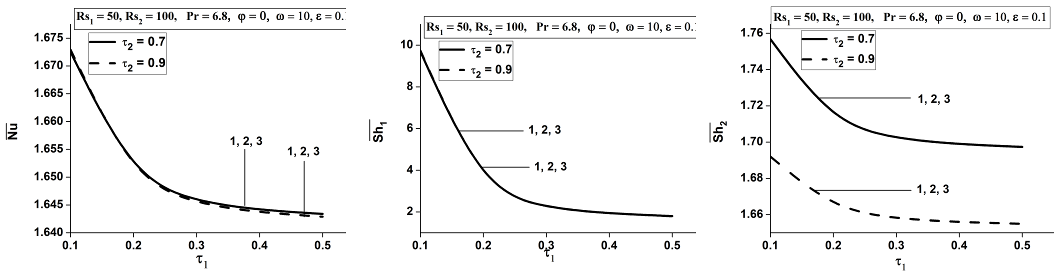
(A) (B) (C)

FIGURE 6. Variation of (A) $\overline{Nu}$ vs R_{S_1} (B) $\overline{Sh_1}$ vs R_{S_1} and (C) $\overline{Sh_2}$ vs R_{S_1} for different values of R_{S_2} and for for (A) in-phase and (B) out-of-phase temperature modulation and for (1) square (2) cosine and (3) triangular in-of-phase temperature modulation



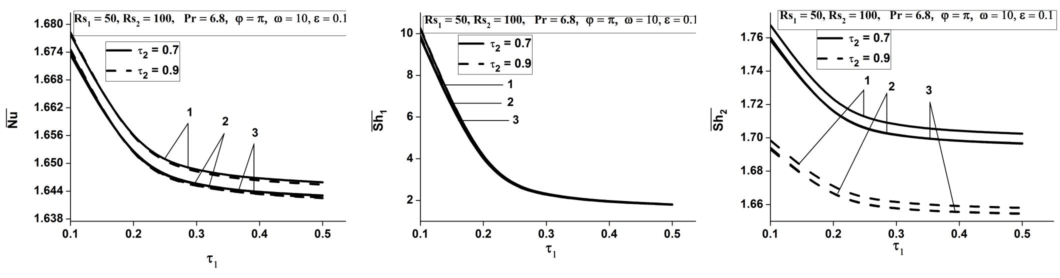
(A) (B) (C)

FIGURE 7. Variation of (A) $\overline{Nu}$ vs R_{S_1} (B) $\overline{Sh_1}$ vs R_{S_1} and (C) $\overline{Sh_2}$ vs R_{S_1} for different values of R_{S_2} and for for (1) square (2) cosine and (3) triangular out-of-phase temperature modulation



(A) (B) (C)

FIGURE 8. Variation of (A) $\overline{Nu}$ vs τ_1 (B) $\overline{Sh_1}$ vs τ_1 and (C) $\overline{Sh_2}$ vs τ_1 for different values of τ_2 and for (1) square (2) cosine and (3) triangular in-phase temperature modulation



(A) (B) (C)

FIGURE 9. Variation of (A) $\overline{Nu}$ vs τ_1 (B) $\overline{Sh_1}$ vs τ_1 and (C) $\overline{Sh_2}$ vs τ_1 for different values of τ_2 and for (1) square (2) cosine and (3) triangular out-of-phase temperature modulation

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References

[1] Bénard, H., (1901), Les tourbillons cellulaires dans une nappe liquide.- Methodes optiques dobservation et d'enregistrement, J. Phys. Arch., 10(1), pp. 254-266.

[2] Rayleigh, L., (1916), On convection currents in a horizontal layer of fluid, when the higher temperature is on the underside, Philos. Mag. , 32(192), pp. 529-546.

[3] Pearson, R. A., (1958), On convection cells induced by surface tension, J. Fluid Mech., 4, pp. 489-500.

[4] Stern, M. E., (1960), The salt-fountain and thermohaline convection, Tellus, 12(2), pp. 172-175.

[5] Turner, J. S., (1985), Multicomponent convection, Annu. Rev. Fluid Mech., 17, pp. 11-44.

[6] Griffiths, R. W., (1979), The influence of a third diffusing component upon the onset of convection, J. Fluid Mech., 92(4), pp. 659-670.

[7] Poulikakos, D., (1985), The effect of a third diffusing component on the onset of convection in a horizontal porous layer, Phys. Fluids, 28(10), pp. 3172-3174.

[8] Tarannum, S., (2017), Heat and mass transfer of triple diffusive convection in Boussinesq-Stokes suspension using Ginzburg-Landau model, JP J. Heat Mass Transf., 14(1), pp. 131-147.

[9] Tarannum, S. and Pranesh, S., (2017), Heat and mass transfer of triple diffusive convection in a rotating couple stress liquid using Ginzburg-Landau model, World Acad. Sci. Eng. Technol. - Int. J. Mech. Mechatronics Eng., 11(3), pp. 583-588.

[10] Tarannum, S. and Pranesh, S., (2018), Synchronous and asynchronous boundary temperature modulations on triple diffusive convection in couple stress liquid using Ginzburg-Landau model, Int. J. Eng. Technol., 7(4), pp. 645-652.

[11] Raghunatha, K. R., Shivakumara, I. S. and Swamy, M. S., (2019), Effect of cross-diffusion on the stability of a triple-diffusive Oldroyd-B fluid layer, J. Appl. Math. Phys., 70(100), pp. 1-21.

[12] Venezian, G., (1969), Effect of modulation on the onset of thermal convection, J. Fluid Mech., 35(2), pp. 243-254.

[13] Siddheshwar, P. G., Uma, D. and Bhavya, S., (2019), Effects of variable viscosity and temperature modulation on linear Rayleigh-Bénard convection in Newtonian dielectric liquid, Appl. Math. Mech., 40(11), pp. 1601-1614.

[14] Siddheshwar, P. G. and Pranesh, S., (1999), Effect of temperature / gravity modulation on the onset of magneto-convection in weak electrically conducting fluids with internal angular momentum, J. Magn. Magn. Mater., 192, pp. 159–176.

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