

QUADRIPARTITIONED SINGLE VALUED NEUTROSOPHIC ROUGH SETS AND THEIR APPLICATIONS IN DECISION MAKING

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ABSTRACT. This paper introduces the notion of quadripartitioned single valued neutrosophic rough set. Some basic set metaphysical terminologies, operations and properties of quadripartitioned single valued neutrosophic rough set are given here. Conjointly different kinds of distances and similarity measures are mentioned here. Finally a decision making problem using the similarity measure technique of quadripartitioned single valued neutrosophic rough sets has been solved.

Keywords: Single valued neutrosophic set; Single valued neutrosophic rough sets; Quadripartitioned single valued neutrosophic sets; Quadripartitioned single valued neutrosophic rough set.

AMS Subject Classification: 54F99, 68P01.

1. INTRODUCTION

To solve uncertainty-based real and scientific problems, Professor Zadeh [35] introduced the fuzzy set as a constructive tool. Later Prof. Atanassov [1] extended the concept of fuzzy set theory to the intuitionistic fuzzy set (IFS), in which each element has both a degree of membership and a degree of non-membership. It is very clear that IFS is more helpful than fuzzy set theory for solving different types of uncertainty model. In 2005, Smarandache [21] introduced the concept of the neutrosophic set (NS) as a further generalization of the IFS from a philosophical perspective. Gradually, neutrosophic sets become a more powerful technology for representing imperfect, inconsistent and uncertain information in our real world. In neutrosophic set, truth membership functions (T_A), indeterminacy membership functions (I_A), and falsity membership functions (F_A) are represented independently. But in case of NS, all components lie in $]0^-, 1^+[$. Thus it is very inconvenient to apply the non standard the unit interval in real life problems. To solve this problem Wang et al. [28] introduced single valued NS sets in 2010. Gradually single valued neutrosophic (SVN) sets become an important tool and it was applied to many practical problems [3, 14, 20, 27, 30, 31, 32, 33, 34]. Quadripartitioned single valued neutrosophic

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set was introduced by Smarandache [23] in 2013. Smarandache also extended the neutrosophic set to refined [n -valued] neutrosophic set, i.e. the truth value T is refined/split into types of sub-truths such as T_1, T_2, \dots , indeterminacy I is refined/split into types of sub-indeterminacies I_1, I_2, \dots , and the falsehood F is refined/split into sub-falsehood F_1, F_2, \dots in [24]. He also exemplified into this paper that a quadruple neutrosophic set is obtain when only indeterminacy (I) is refined/split into $I_1 = \text{Contradiction}$ and $I_2 = \text{Uncertainty}$. But the I as T , as F can be refined/split into many more sub-components (as many as needed into each application). Again quadripartitioned single valued neutrosophic set was further studied by R. Chatterjee et al. [9] in 2016. It is four valued logic set A over a universal set X consisting truth-membership T_A , a contradiction membership C_A , an ignorance-membership U_A and a falsity membership F_A for each $x \in X$. Now quadripartitioned single valued neutrosophic set becomes an important tool in solving various types of decision making problems, medical diagnosis problems, clustering problems etc [4, 8].

On the other hand, Prof. Pawlak introduced the idea of Rough set to process the incomplete information in a more formal way [19]. Crisp set and the equivalence relations, which is the mathematical basis of rough sets are the main elements of Rough set theory. Later on several researchers have studied different aspects of rough sets and applied it to different socio-economic model [10, 11, 26, 29, 36]. In this paper we have introduced the quadripartitioned single valued neutrosophic rough (QSVNR) set for the first time. The organization of this paper have been done as follows: In section 2, preliminary concepts regarding the paper have been discussed with proper citations. We have defined QSVNR set and it's various types of operations in section 3. Section 4 introduces the idea of different type of similarity measures of QSVNR sets. In section 5, we have tried to solve a real life problem using QSVNR set. Section 6 concludes the paper.

2. PRELIMINARIES

The term 'neutrosophy' was introduced by Prof. Smarandache as a branch of philosophy. There are many different types of neutrosophic sets which have many applications [2, 6, 13, 22, 28]. However for our purposes, we have the following:

Definition 2.1. [21] *Suppose U be a universe. A neutrosophic sets (NS) A in U is characterized by a truth-membership function T_A , an indeterminacy membership function I_A and a falsity-membership function F_A . Here for each $x \in U$, $T_A(x), I_A(x)$ and $F_A(x)$ are real non-standard elements of $]0^-, 1^+[$. A can be written as:*

$$A = \{ \langle (x, T_A(x), I_A(x), F_A(x)) : x \in U, T_A(x), I_A(x), F_A(x) \in]0^-, 1^+[\rangle \}.$$

There is no restriction on the sum of $T_A(x), I_A(x), F_A(x)$, i.e.,

$$0^- \leq T_A(x), I_A(x), F_A(x) \leq 3^+.$$

Definition 2.2. [28] *Let U be a universe. A single valued neutrosophic set (SVNS) A in U is characterized by a truth-membership function T_A , an indeterminacy membership function I_A and a falsity-membership function F_A . $T_A(x), I_A(x)$ and $F_A(x)$ are real standard elements of $[0, 1]$. It can be written as*

$$A = \{ \langle (x, T_A(x), I_A(x), F_A(x)) : x \in U, T_A(x), I_A(x), F_A(x) \in [0, 1] \rangle \}.$$

Definition 2.3. [22] *Suppose X be a non-empty set. A quadripartitioned single valued neutrosophic set (QSVNS) A over X characterizes each element x in X by a truth-membership function T_A , a contradiction membership function C_A , an ignorance-membership*

function U_A and a falsity membership function F_A such that for each $x \in X$

$$T_A(x), C_A(x), U_A(x), F_A(x) \in [0, 1], 0 \leq T_A(x) + C_A(x) + U_A(x) + F_A(x) \leq 4$$

On the other hand Prof. Pawlak defined the rough set for the first time in his paper [19]. Broumi et. al [7] introduced the Rough Neutrosophic sets for the first time in 2015.

Definition 2.4. [19] Suppose U be any non-empty set . Suppose R is an equivalence relation over U . For any non-null subset X of U , the sets $A_1(x) = \{x : [x]_R \subseteq X\}$ and $A_2(x) = \{x : [x]_R \cap X \neq \phi\}$ are called the lower approximation and upper approximation respectively of X , where the pair $S = (U, R)$ is called an approximation space. This equivalent relation R is called indiscernibility relation. The pair $A(X) = (A_1(x), A_2(x))$ is called the rough set of X in S . Here $[x]_R$ denotes the equivalence class of R containing x .

The definitions, set theoretic properties and many other properties i.e. entropy, various types of distances, similarity measures of a neutrosophic sets, single valued neutrosophic sets, rough neutrosophic sets etc can be found in any of the monograph say [7, 21, 22, 25].

3. QUADRIPARTITIONED SINGLE VALUED NEUTROSOPHIC ROUGH (QSVNR) SET

In this section, we will introduce the concept of QSVNR set and define some algebraic operations on them.

Definition 3.1. Suppose X be a non-empty set and R be an equivalence relation on X . Let A be quadripartitioned single valued neutrosophic set in X with truth-membership function T_A , a contradiction membership function C_A , an ignorance membership function U_A and a falsity-membership function F_A . The lower and the upper approximations of A in the approximation (X, R) denoted by $\underline{N}(A)$ and $\overline{N}(A)$ are respectively defined as follows: $\underline{N}(A) = \{\langle x, T_{\underline{A}}(x), C_{\underline{A}}(x), U_{\underline{A}}(x), F_{\underline{A}}(x) \rangle | x \in [x]_R \subseteq X\}$

$$\overline{N}(A) = \{\langle x, T_{\overline{A}}(x), C_{\overline{A}}(x), U_{\overline{A}}(x), F_{\overline{A}}(x) \rangle | x \in [x]_R \subseteq X\},$$

where

$$\begin{aligned} T_{\underline{A}}(x) &= \wedge_{x \in [x]_R} T_A(x), C_{\underline{A}}(x) = \wedge_{x \in [x]_R} C_A(x), U_{\underline{A}}(x) = \vee_{x \in [x]_R} U_A(x), \\ F_{\underline{A}}(x) &= \vee_{x \in [x]_R} F_A(x), T_{\overline{A}}(x) = \vee_{x \in [x]_R} T_A(x), C_{\overline{A}}(x) = \vee_{x \in [x]_R} C_A(x), \\ U_{\overline{A}}(x) &= \wedge_{x \in [x]_R} U_A(x), F_{\overline{A}}(x) = \wedge_{x \in [x]_R} F_A(x), \end{aligned}$$

where $0 \leq T_{\underline{A}}(x) + C_{\underline{A}}(x) + U_{\underline{A}}(x) + F_{\underline{A}}(x) \leq 4$, $0 \leq T_{\overline{A}}(x) + C_{\overline{A}}(x) + U_{\overline{A}}(x) + F_{\overline{A}}(x) \leq 4$, where \wedge, \vee mean “min” and “max” operators respectively, $T_A(x), C_A(x), U_A(x), F_A(x)$ are the respective membership function of x with respect to A . $\underline{N}(A)$ and $\overline{N}(A)$ are two quadripartitioned neutrosophic sets in U . The pair $(\underline{N}(A), \overline{N}(A))$ is called QSVNR set in (X, R) .

Example 3.1. Consider the case where eight doctors $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ were asked to give their opinion on a diabetic patient report. Each of the eight doctors gave their opinion in terms of degree of agreement, agreement or disagreement both, neither agreement nor disagreement, disagreement respectively. As per their opinion, they are classified into the following equivalent class R as following:

$$X/R = \{\{x_1, x_8\}, \{x_2, x_3, x_5\}, \{x_6\}, \{x_4, x_7\}\}$$

Consider a QSVN set A based on the aggregate of some of their opinion as follows:

$$\begin{aligned} A = \{ & (x_1, (0.8, 0.6, 0.2, 0.2)), (x_4, (0.6, 0.7, 0.4, 0.1)), (x_6, (0.5, 0.5, 0.7, 0.1)), \\ & (x_8, (0.4, 0.6, 0.6, 0.8)) \}. \end{aligned}$$

Here according to x_1 , the degree of agreement with the statement is 0.8, the degree of both agreement and disagreement is 0.6, the degree of neither agreement nor disagreement is 0.2, while the degree of disagreement with the statement is 0.2. Now by Definition 3.1, we have,

$$\begin{aligned}\underline{N}(A) &= \{(x_1, (0.4, 0.6, 0.6, 0.8)), (x_6, (0.5, 0.5, 0.7, 0.1)), (x_8, (0.4, 0.6, 0.6, 0.8))\} \\ \overline{N}(A) &= \{(x_1, (0.8, 0.6, 0.2, 0.2)), (x_4, (0.6, 0.7, 0.4, 0.1)), (x_6, (0.5, 0.5, 0.7, 0.1)), \\ &\quad (x_7, (0.6, 0.7, 0.4, 0.1)), (x_8, (0.8, 0.6, 0.2, 0.2))\}\end{aligned}$$

Definition 3.2. Suppose X be a non-empty set and R be an equivalence relation on X . Let A be QSVN set in X with truth-membership function T_A , a contradiction membership function C_A , an ignorance membership function U_A and a falsity-membership function F_A . The lower and the upper approximations of A in the approximation (X, R) is denoted by $\underline{N}(A)$ and $\overline{N}(A)$ are respectively. If

- (i) $\underline{N}(A) = \overline{N}(A)$, then the pair $(\underline{N}(A), \overline{N}(A))$ is called constant QSVNR set in (X, R) .
- (ii) $\forall x \in \underline{N}(A)$ (and $\overline{N}(A)$), $T_A(x) = 1, C_A(x) = U_A(x) = F_A(x) = 0$, then the pair $(\underline{N}(A), \overline{N}(A))$ is called unit QSVNR set in (X, R) .
- (iii) $\forall x \in \underline{N}(A)$ (and $\overline{N}(A)$), $T_A(x) = C_A(x) = U_A(x) = 0, F_A(x) = 1$, then the pair $(\underline{N}(A), \overline{N}(A))$ is called zero QSVNR set in (X, R) and it is denoted by Φ .

Definition 3.3. If $N(A) = (\underline{N}(A), \overline{N}(A))$ is a QSVNR set in (X, R) , then the complement of $N(A)$ is also a QSVNR set which is defined as follows: $N^c(A) = ((\underline{N}(A))^c, (\overline{N}(A))^c)$, where $(\underline{N}(A))^c, (\overline{N}(A))^c$ are the complements of $\underline{N}(A), \overline{N}(A)$ respectively.

$$\begin{aligned}(\underline{N}(A))^c &= \{\langle x, F_{\underline{A}}(x), 1 - U_{\underline{A}}(x), 1 - C_{\underline{A}}(x), T_{\underline{A}}(x) \rangle | x \in X\} \\ (\overline{N}(A))^c &= \{\langle x, F_{\overline{A}}(x), 1 - U_{\overline{A}}(x), 1 - C_{\overline{A}}(x), T_{\overline{A}}(x) \rangle | x \in X\},\end{aligned}$$

Now we will introduce some set-theoretic operations on QSVNR set over a common universe X and study some of their basic properties.

Definition 3.4. Consider two QSVNR sets $N(A_1), N(A_2)$ in (X, R) . Then $N(A_1)$ is equal to a set $N(A_2)$ if $\underline{N}(A_1) = \underline{N}(A_2)$ and $\overline{N}(A_1) = \overline{N}(A_2)$.

Definition 3.5. Consider two QSVNR sets $N(A_1), N(A_2)$ in (X, R) . Then $N(A_1)$ is said to be a subset of $N(A_2)$ if $\underline{N}(A_1) \subseteq \underline{N}(A_2)$ and $\overline{N}(A_1) \subseteq \overline{N}(A_2)$ i.e.

$$\begin{aligned}T_{\underline{A}_1}(x) &\leq T_{\underline{A}_2}(x), C_{\underline{A}_1}(x) \leq C_{\underline{A}_2}(x), U_{\underline{A}_1}(x) \geq U_{\underline{A}_2}(x), \\ F_{\underline{A}_1}(x) &\geq F_{\underline{A}_2}(x), T_{\overline{A}_1}(x) \geq T_{\overline{A}_2}(x), C_{\overline{A}_1}(x) \geq C_{\overline{A}_2}(x), \\ U_{\overline{A}_1}(x) &\leq U_{\overline{A}_2}(x), F_{\overline{A}_1}(x) \leq F_{\overline{A}_2}(x) \forall x \in X.\end{aligned}$$

Definition 3.6. Suppose $N(A_1), N(A_2)$ are two QSVNR sets in (X, R) . Then the union of $N(A_1)$ and $N(A_2)$ i.e. $N(A) = N(A_1) \cup N(A_2)$ is defined as:

$$\begin{aligned}\underline{N}(A) &= \{\langle x, T_{\underline{A}_1}(x) \vee T_{\underline{A}_2}(x), C_{\underline{A}_1}(x) \vee C_{\underline{A}_2}(x), U_{\underline{A}_1}(x) \wedge U_{\underline{A}_2}(x), \\ &\quad F_{\underline{A}_1}(x) \wedge F_{\underline{A}_2}(x) \rangle | x \in X\} \\ \overline{N}(A) &= \{\langle x, T_{\overline{A}_1}(x) \wedge T_{\overline{A}_2}(x), C_{\overline{A}_1}(x) \wedge C_{\overline{A}_2}(x), U_{\overline{A}_1}(x) \vee U_{\overline{A}_2}(x), \\ &\quad F_{\overline{A}_1}(x) \vee F_{\overline{A}_2}(x) \rangle | x \in X\}\end{aligned}$$

Definition 3.7. Suppose $N(A_1), N(A_2)$ are two QSVNR sets in (X, R) . Then the intersection of $N(A_1)$ and $N(A_2)$ i.e. $N(A) = N(A_1) \cap N(A_2)$ is defined as:

$$\begin{aligned} \underline{N}(A) &= \{ \langle x, T_{\underline{A_1}}(x) \wedge T_{\underline{A_2}}(x), C_{\underline{A_1}}(x) \wedge C_{\underline{A_2}}(x), U_{\underline{A_1}}(x) \vee U_{\underline{A_2}}(x), \\ &\quad F_{\underline{A_1}}(x) \vee F_{\underline{A_2}}(x) \rangle | x \in X \} \\ \overline{N}(A) &= \{ \langle x, T_{\overline{A_1}}(x) \vee T_{\overline{A_2}}(x), C_{\overline{A_1}}(x) \vee C_{\overline{A_2}}(x), U_{\overline{A_1}}(x) \wedge U_{\overline{A_2}}(x), \\ &\quad F_{\overline{A_1}}(x) \wedge F_{\overline{A_2}}(x) \rangle | x \in X \} \end{aligned}$$

Proposition 3.1. The set-theoretic axioms are satisfied by any QSVNR set as it can be easily verified. Consider QSVNR sets $N(A_1), N(A_2), N(A_3)$ in (X, R) . Then the following properties hold all for QSVNR sets over X .

- (i) $N(A_1) \cup N(A_2) = N(A_2) \cup N(A_1); N(A_1) \cap N(A_2) = N(A_2) \cap N(A_1)$.
- (ii) $N(A_1) \cup (N(A_2) \cup N(A_3)) = (N(A_1) \cup N(A_2)) \cup N(A_3); N(A_1) \cap (N(A_2) \cap N(A_3)) = (N(A_1) \cap N(A_2)) \cap N(A_3)$
- (iii) $N(A_1) \cap (N(A_1) \cup N(A_2)) = N(A_1); N(A_1) \cup (N(A_1) \cap N(A_2)) = N(A_1)$.
- (iv) $((N(A_1))^c)^c = N(A_1)$.
- (v) $(N(A_1) \cup N(A_2))^c = N(A_1)^c \cap N(A_2)^c; (N(A_1) \cap N(A_2))^c = N(A_1)^c \cup N(A_2)^c$
- (vi) $N(A_1) \cup N(A_1) = N(A_1); N(A_1) \cap N(A_1) = N(A_1)$.
- (vii) $N(A_1) \cup \Phi = N(A_1); N(A_1) \cap \Phi = \Phi$.

4. QSVNR SET AND IT'S DIFFERENT SIMILARITY MEASURES

Consider the non-empty universe of discourse $X \neq \Phi$ and denote the set of QSVNR set over X by $\mathbf{N}(X)$.

Definition 4.1. A mapping $s : \mathbf{N} \times \mathbf{N} \rightarrow [0, 1]$ is said to be similarity measure if and only if for $N(A), N(B) \in \mathbf{N}$ the following properties are satisfied:

- (i) $s(N(A), N(B)) = s(N(B), N(A))$
- (ii) $0 \leq s(N(A), N(B)) < 1$ and $s(N(A), N(B)) = 1$ if and only if $N(A) = N(B)$.
- (iii) for any $N(A), N(B), N(C) \in \mathbf{N}(X), N(A) \subset N(B) \subset N(C)$, $s(N(A), N(C)) \leq s(N(A), N(B)) \wedge s(N(B), N(C))$.

Although in Definition 4.1 the condition (iii) exists but most well-known similarity measure methods such as weight similarity measure, cosine similarity measures etc. do not satisfy the above condition. On the other hand methods such as weighted similarity measure, cosine similarity measure have huge application in solving a variety of real life problems. So it is natural to introduce the definition of the similarity measure that omits the condition (iii) of the Definition 4.1. In that case we redefine quasi-similarity measure between two QSVNR sets over X as a measure of similarity.

Definition 4.2. Consider the non-empty universe of discourse $X \neq \Phi$ and refer to the set of QSVNR set over X by $\mathbf{N}(X)$. Then a mapping $s' : \mathbf{N} \times \mathbf{N} \rightarrow [0, 1]$ is said to be quasi similarity measure if and only if for $N(A), N(B) \in \mathbf{N}(X)$, the following properties are satisfied:

- (i) $s(N(A), N(B)) = s(N(B), N(A))$
- (ii) $0 \leq s(N(A), N(B)) < 1$ and $s(N(A), N(B)) = 1$ if and only if $N(A) = N(B)$.

4.1. Different types of distances between two QSVNR sets.

Definition 4.3. A mapping $d_N : \mathbf{N} \times \mathbf{N} \rightarrow \mathbb{R}^+$, where \mathbb{R}^+ is the set of all positive real numbers

is said to be a distance measure for QSVNR sets if and only if for $N(A), N(B), N(C) \in \mathcal{N}(X)$, it satisfies the following properties:

- (i) $d_N(N(A), N(B)) = d_N(N(B), N(A))$
- (ii) $d_N(N(A), N(B)) \geq 0$ and $d_N(N(A), N(B)) = 0$ if and only if $N(A) = N(B)$.
- (iii) $d_N(N(A), N(B)) \leq d_N(N(A), N(C)) + d_N(N(C), N(B))$.

Then d_N is a metric on $\mathcal{N}(X)$. Consider two QSVNR sets $N(H), N(K) \in \mathcal{N}(X)$ over an universal set $X = \{x_1, x_2, \dots, x_n\}$.

Definition 4.4. The Hamming distance between $N(H)$ and $N(K)$ is defined as

$$h(N(H), N(K)) = \min \{ \{h(\underline{N}(H), \underline{N}(K)), \{h(\overline{N}(H), \overline{N}(K))\} \}, \text{ where,}$$

$$h(\underline{N}(H), \underline{N}(K)) = \left\{ \sum_{j=1}^n (|T_{\underline{H}}(x_j) - T_{\underline{K}}(x_j)| + |C_{\underline{H}}(x_j) - C_{\underline{K}}(x_j)| + |U_{\underline{H}}(x_j) - U_{\underline{K}}(x_j)| + |F_{\underline{H}}(x_j) - F_{\underline{K}}(x_j)|) \right\}$$

$$h(\overline{N}(H), \overline{N}(K)) = \left\{ \sum_{j=1}^n (|T_{\overline{H}}(x_j) - T_{\overline{K}}(x_j)| + |C_{\overline{H}}(x_j) - C_{\overline{K}}(x_j)| + |U_{\overline{H}}(x_j) - U_{\overline{K}}(x_j)| + |F_{\overline{H}}(x_j) - F_{\overline{K}}(x_j)|) \right\},$$

where $x_j \in X$.

Definition 4.5. The Normalized Hamming distance between $N(H)$ and $N(K)$ is defined as $h_N(N(H), N(K)) = \frac{1}{8n}(h(N(H), N(K)))$.

Definition 4.6. The Euclidean distance $E(N(H), N(K))$ is defined as follows:

$$E(N(H), N(K)) = \min \{ \{E(\underline{N}(H), \underline{N}(K)), \{E(\overline{N}(H), \overline{N}(K))\} \}, \text{ where,}$$

$$E(\underline{N}(H), \underline{N}(K)) = \left\{ \sum_{j=1}^n (|T_{\underline{H}}(x_j) - T_{\underline{K}}(x_j)|^2 + |C_{\underline{H}}(x_j) - C_{\underline{K}}(x_j)|^2 + |U_{\underline{H}}(x_j) - U_{\underline{K}}(x_j)|^2 + |F_{\underline{H}}(x_j) - F_{\underline{K}}(x_j)|^2) \right\}^{\frac{1}{2}}$$

$$E(\overline{N}(H), \overline{N}(K)) = \left\{ \sum_{j=1}^n (|T_{\overline{H}}(x_j) - T_{\overline{K}}(x_j)|^2 + |C_{\overline{H}}(x_j) - C_{\overline{K}}(x_j)|^2 + |U_{\overline{H}}(x_j) - U_{\overline{K}}(x_j)|^2 + |F_{\overline{H}}(x_j) - F_{\overline{K}}(x_j)|^2) \right\}^{\frac{1}{2}} \forall x_j \in X.$$

Definition 4.7. The normalized Euclidean distance $Q(N(H), N(K))$ is defined as follows:

$$Q(N(H), N(K)) = \frac{1}{2\sqrt{2n}} E(N(H), N(K))$$

In Definition 4.4 and Definition 4.6 both the distances are in real numbers thus we have taken the minimum value. Also minimum values measures the minimum distances between two QSVNR sets which in future gives us the minimum value of similarity measures. Also in Definition 4.4 and Definition 4.6 we may get value higher than 1. To make the value of Hamming distance as well as Euclidian distance in between 0 and 1 we have done the normalization of both the distances in Definition 4.5 and Definition 4.7. It is seen from recent papers in this area that distance measurement between two sets becomes the most attractive research topic among researchers. There is a large practical application based on the distance between two sets. Many researchers have used a variety of distance-based measures to solve uncertainty-based real-life problems containing uncertainties [12, 13]. In addition different types of distance operators i.e. induced weighted aggregation distance (IOWAD) operators, an extended version of common OWA operators, are used to find distance measures to solve decision-making problems [16, 17, 18]. However, we are concerned only with the study of distance-based similarity measures in this paper.

4.2. Distance based similarity measure between two QSVNR sets. We have defined several types of distances between a pair of QSVNR sets $N(H)$ and $N(K)$ over the set $N(X)$ in the previous section. Now using these distances we can also define similarity measures for QSVNR sets. In the following we now define two similarity measure based on Hamming Distance.

$$S_1^{d_N}(N(H), N(K)) = \frac{1}{1 + h(N(H), N(K))},$$

$$S_2^{d_N}(N(H), N(K)) = e^{-\alpha \cdot h(N(H), N(K))}$$

where α is a positive real number (parameter) called the *steepness measure*. Similarly using Euclidian distance, another type of similarity measures can be defined as follows:

$$S'_1(N(H), N(K)) = \frac{1}{1 + E(N(H), N(K))},$$

$$S'_2(N(H), N(K)) = e^{-\beta \cdot E(N(H), N(K))}$$

where β is a positive real number (parameter) called the *steepness measure*.

4.3. Cosine similarity measure between two QSVNR sets. The cosine similarity measure is obtained when the internal product of the two vectors is divided by the product of their length. This is the cosine of the angle between the vector representations of two QSVNR set. The cosine similarity measure is the primary measure used in information technology. Therefore, a new cosine similarity measure is proposed between QSVNR sets as follows:

Definition 4.8. Consider $N(H), N(K) \in N(X)$. Then for each $x_i \in X, i = 1, 2, \dots, n$, a cosine similarity measure between QSVNR sets $N(H)$ and $N(K)$ is defined as follows:

$$C_{QSVNR}(N(H), N(K)) = \frac{1}{n} \sum_{i=1}^n \frac{S_1}{S_2 \cdot S_3}, \text{ where,}$$

$$S_1 = \partial T_{N(H)}(x_i) \partial T_{N(K)}(x_i) + \partial C_{N(H)}(x_i) \partial C_{N(K)}(x_i) + \partial U_{N(H)}(x_i) \partial U_{N(K)}(x_i) + \partial F_{N(H)}(x_i) \partial F_{N(K)}(x_i),$$

$$S_2 = \sqrt{\partial T_{N(H)}(x_i)^2 + \partial C_{N(H)}(x_i)^2 + \partial U_{N(H)}(x_i)^2 + \partial F_{N(H)}(x_i)^2},$$

$$S_3 = \sqrt{\partial T_{N(K)}(x_i)^2 + \partial C_{N(K)}(x_i)^2 + \partial U_{N(K)}(x_i)^2 + \partial F_{N(K)}(x_i)^2},$$

where $\partial T_{N(K)}(x_i) = \frac{T_{\underline{K}}(x_i) + T_{\overline{K}}(x_i)}{2}, \partial C_{N(K)}(x_i) = \frac{C_{\underline{K}}(x_i) + C_{\overline{K}}(x_i)}{2},$

$$\partial U_{N(K)}(x_i) = \frac{U_{\underline{K}}(x_i) + U_{\overline{K}}(x_i)}{2}, \partial F_{N(K)}(x_i) = \frac{F_{\underline{K}}(x_i) + F_{\overline{K}}(x_i)}{2},$$

and $\partial T_{N(H)}(x_i) = \frac{T_{\underline{H}}(x_i) + T_{\overline{H}}(x_i)}{2}, \partial C_{N(H)}(x_i) = \frac{C_{\underline{H}}(x_i) + C_{\overline{H}}(x_i)}{2},$

$$\partial U_{N(H)}(x_i) = \frac{U_{\underline{H}}(x_i) + U_{\overline{H}}(x_i)}{2}, \partial F_{N(H)}(x_i) = \frac{F_{\underline{H}}(x_i) + F_{\overline{H}}(x_i)}{2},$$

Theorem 4.1. $C_{QSVNR}(N(H), N(K))$ is a similarity measure between two QSVNR sets $N(H), N(K) \in N(X)$.

Proof. Proof is very straight forward. □

4.4. Similarity measure of QSVNR sets based on membership values. Consider $N(H), N(K) \in \mathbf{N}(X)$. For each $x_i \in X, \forall i = 1, 2, \dots, n$ and for $j = 1, 2, \dots, 4$ we define the functions $\underline{h}_j^{N(H),N(K)}, \overline{h}_j^{N(H),N(K)} : X \rightarrow [0, 1]$ respectively as follows:

$$\begin{aligned}\underline{h}_1^{N(H),N(K)}(x_i) &= |T_{\underline{H}}(x_i) - T_{\underline{K}}(x_i)|, \\ \underline{h}_2^{N(H),N(K)}(x_i) &= |F_{\underline{H}}(x_i) - F_{\underline{K}}(x_i)|, \\ \underline{h}_3^{N(H),N(K)}(x_i) &= \frac{1}{3}(h_1^{N(H),N(K)}(x_i) + h_2^{N(H),N(K)}(x_i) + |C_{\underline{H}}(x_i) - C_{\underline{K}}(x_i)|), \\ \underline{h}_4^{N(H),N(K)}(x_i) &= |U_{\underline{H}}(x_i) - U_{\underline{K}}(x_i)|, \\ \overline{h}_1^{N(H),N(K)}(x_i) &= |T_{\overline{H}}(x_i) - T_{\overline{K}}(x_i)|, \\ \overline{h}_2^{N(H),N(K)}(x_i) &= |F_{\overline{H}}(x_i) - F_{\overline{K}}(x_i)|, \\ \overline{h}_3^{N(H),N(K)}(x_i) &= \frac{1}{3}(h_1^{N(H),N(K)}(x_i) + h_2^{N(H),N(K)}(x_i) + |C_{\overline{H}}(x_i) - C_{\overline{K}}(x_i)|), \\ \overline{h}_4^{N(H),N(K)}(x_i) &= |U_{\overline{H}}(x_i) - U_{\overline{K}}(x_i)|\end{aligned}$$

The functions defined above measure the difference between different membership values corresponding to the two sets $N(H)$ and $N(K)$ w.r.t. each $x_i, \forall i = 1, 2, \dots, n$. Based on the above functions we now define a new similarity measure function $\tilde{S}(N(H), N(K))$ as follows:

$$\tilde{S}(N(H), N(K)) = 1 - \frac{1}{4n} \left[\sum_{i=1}^n \sum_{j=1}^4 \overline{h}_j^{N(H),N(K)}(x_i) + \sum_{i=1}^n \sum_{j=1}^4 \underline{h}_j^{N(H),N(K)}(x_i) \right].$$

As a consequence we have the following theorem:

Theorem 4.2. $\tilde{S}(N(H), N(K))$ is a measure of similarity between the two QSVNR sets $N(H), N(K) \in \mathbf{N}(X)$.

Proof. All the values of $\underline{T}, \underline{C}, \underline{U}, \underline{F}, \overline{T}, \overline{C}, \overline{U}, \overline{F}$ for a QSVNR set $N(H)$ or $N(K)$ lie between $[0, 1]$. Among all these quantities, all has maximum value 1 and the minimum value 0. As a result $0 \leq \tilde{S}(N(H), N(K)) \leq 1$. Again $\tilde{S}(N(H), N(K)) = 1$ implies that $\sum_{i=1}^n \sum_{j=1}^4 \overline{h}_j^{N(H),N(K)}(x_i) + \sum_{i=1}^n \sum_{j=1}^4 \underline{h}_j^{N(H),N(K)}(x_i) = 0 \forall x_i \in X$ which is only possible if and only if

$$\begin{aligned}T_{\underline{H}}(x_i) &= T_{\underline{K}}(x_i), C_{\underline{H}}(x_i) = C_{\underline{K}}(x_i), U_{\underline{H}}(x_i) = U_{\underline{K}}(x_i), F_{\underline{H}}(x_i) = F_{\underline{K}}(x_i), \\ T_{\overline{H}}(x_i) &= T_{\overline{K}}(x_i), C_{\overline{H}}(x_i) = C_{\overline{K}}(x_i), U_{\overline{H}}(x_i) = U_{\overline{K}}(x_i), F_{\overline{H}}(x_i) = F_{\overline{K}}(x_i) \\ \forall x_i \in X, i.e. N(H) &= N(K).\end{aligned}$$

Lastly for three BQSVNR sets $N(H), N(K), N(J) \in \mathbf{N}(X)$ we suppose that $N(H) \subseteq N(K) \subseteq N(J)$. Now by the Definition 3.5 we have $\forall x_i \in X, \forall j = 1, 2, \dots, 4$ Thus,

$$\tilde{S}(N(H), N(J)) < \tilde{S}(N(H), N(K)) \wedge \tilde{S}(N(K), N(J)).$$

Hence the result follows. \square

4.5. Weighted similarity measure between two QSVNR sets. The weighted similarity measure between two QSVNR sets $N(H), N(K) \in \mathbf{N}(X)$ are defined as follows:

$$S^w(N(H), N(K)) = \left[1 - \frac{1}{8n} \left\{ \sum_{i=1}^n \sum_{j=1}^4 w_i \overline{h}_j^{N(H),N(K)}(x_i) + \sum_{i=1}^n \sum_{j=1}^4 w_i \underline{h}_j^{N(H),N(K)}(x_i) \right\} \right]^{\frac{1}{k}}$$

where k is any integer defined to be the order of similarity, w_i are the weights associated to the elements x_i of the universe, $i = 1, 2, \dots, n$ such that $\sum_{i=1}^n w_i = 1$. Using the same

proof procedure of the Theorem 4.2, one can easily see that $S^w(N(H), N(K))$ is also a measure of similarity between the two QSVNR sets $N(H), N(K) \in \mathcal{N}(X)$.

4.6. Correlation coefficient based similarity measure between two QSVNR sets.

Definition 4.9. The correlation coefficient based similarity measure between two QSVNR sets $N(H), N(K) \in \mathcal{N}(X)$ can be defined as follows:

$$S^C(N(H), N(K)) = \frac{[\sum_{i=1}^n \frac{T_{\underline{H}}(x_i).T_{\underline{K}}(x_i)+C_{\underline{H}}(x_i).C_{\underline{K}}(x_i)+U_{\underline{H}}(x_i).U_{\underline{K}}(x_i)+F_{\underline{H}}(x_i).F_{\underline{K}}(x_i)}{T_{\underline{H}}(x_i).T_{\underline{K}}(x_i)+C_{\underline{H}}(x_i).C_{\underline{K}}(x_i)+U_{\underline{H}}(x_i).U_{\underline{K}}(x_i)+F_{\underline{H}}(x_i).F_{\underline{K}}(x_i)}]}{[\sum_{i=1}^n \frac{(T_{\underline{H}}(x_i)^2+C_{\underline{H}}(x_i)^2+U_{\underline{H}}(x_i)^2+F_{\underline{H}}(x_i)^2)}{(T_{\underline{H}}(x_i)^2+C_{\underline{H}}(x_i)^2+U_{\underline{H}}(x_i)^2+F_{\underline{H}}(x_i)^2)}][\sum_{i=1}^n \frac{(T_{\underline{K}}(x_i)^2+C_{\underline{K}}(x_i)^2+U_{\underline{K}}(x_i)^2+F_{\underline{K}}(x_i)^2)}{(T_{\underline{K}}(x_i)^2+C_{\underline{K}}(x_i)^2+U_{\underline{K}}(x_i)^2+F_{\underline{K}}(x_i)^2)}]}$$

Remark 4.1. $S^C(N(H), N(K))$ is a quasi-similarity between $N(H), N(K)$.

5. A DECISION MAKING PROBLEM USING QSVNR SETS

For an example of the above proposed approaches we consider the problem of medical diagnosis from a practical perspective. Uncertainties in medical diagnosis and the amount of information available to physicians from new medical technologies are a great obstacle for treatment of a patient. The process of classifying a variety of symptoms under the same name of a disease is a difficult task. In some practical situations, it is seen that each element lies between the lower and upper approximation of the QSVNR set. Proposed similarity measurement in patients versus symptoms and symptoms versus diseases provide appropriate medical diagnosis. The main feature of this proposed approach is that it considers the true membership, ignorance, contradiction and false membership of each element between two approximations of the QSVNR set by checking for a single diagnosis.

Now, an example of a medical diagnosis of diabetic patients is presented. Let $C = \{C_1, C_2, C_3\}$ be a set of patients, $D = \{\text{Fatigueness, Kidney problem, Heart problem, Low vision problem}\}$ be a set of diseases and $S = \{\text{Weakness, Headache, Urinal pain, Cough, Chest pain}\}$ be a set of symptoms. Also suppose that d_1, d_2, d_3, d_4 denote the diseases fatigue ness, kidney problem, heart problem, low vision problem respectively and x_1, x_2, x_3, x_4, x_5 denote the symptoms weakness, headache, urinal pain, cough, chest pain in the body respectively. Our solution is to examine the patient and to determine the disease of the patient in rough quadripartitioned neutrosophic environment. We have represented the relation R_1 between Patients and Symptoms as follows:

$$\begin{aligned} C_1 &= \{(0.8,0.6,0.2,0.5), (0.1,0.3,0.2,0.5)\} / x_1 + \{(0.5,0.4,0.4,0.3), (0.6,0.4,0.3,0.3)\} / x_2 + \\ & \{(0.8,0.6,0.6,0.9), (0.2,0.5,0.5,0.4)\} / x_3 + \{(0.7,0.6,0.4,0.5), (0.2,0.3,0.4,0.1)\} / x_4 + \\ & \{(0.3,0.5,0.5,0.6), (0.1,0.4,0.5,0.6)\} / x_5. \\ C_2 &= \{(0.4,0.3,0.1,0.6), (0.4,0.5,0.6,0.2)\} / x_1 + \{(0.8,0.3,0.9,0.1), (0.3,0.5,0.4,0.4)\} / x_2 + \\ & \{(0.6,0.7,0.3,0.5), (0.4,0.6,0.6,0.3)\} / x_3 + \{(0.6,0.5,0.4,0.4), (0.3,0.6,0.5,0.2)\} / x_4 + \\ & \{(0.7,0.6,0.6,0.3), (0.4,0.5,0.6,0.2)\} / x_5. \\ C_3 &= \{(0.7,0.5,0.5,0.2), (0.2,0.4,0.4,0.4)\} / x_1 + \{(0.5,0.3,0.6,0.4), (0.5,0.5,0.4,0.3)\} / x_2 + \\ & \{(0.5,0.4,0.3,0.2), (0.2,0.4,0.6,0.8)\} / x_3 + \{(0.4,0.8,0.5,0.6), (0.4,0.7,0.4,0.4)\} / x_4 + \\ & \{(0.5,0.6,0.4,0.5), (0.6,0.4,0.4,0.6)\} / x_5. \end{aligned}$$

We have represented the relation R_2 between Patients and Symptoms as follows:

$$\begin{aligned}
 x_1 &= \{(0.4, 0.5, 0.6, 0.2), (0.6, 0.9, 0.4, 0.3)\} / d_1 + \{(0.4, 0.5, 0.2, 0.1), (0.8, 0.5, 0.7, 0.5)\} / d_2 + \\
 &\quad \{(0.4, 0.5, 0.4, 0.4), (0.4, 0.6, 0.8, 0.1)\} / d_3 + \{(0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.4)\} / d_4. \\
 x_2 &= \{(0.3, 0.4, 0.2, 0.1), (0.3, 0.3, 0.4, 0.6)\} / d_1 + \{(0.8, 0.7, 0.8, 0.8), (0.5, 0.6, 0.4, 0.3)\} / d_2 + \\
 &\quad \{(0.4, 0.5, 0.5, 0.9), (0.4, 0.3, 0.3, 0.6)\} / d_3 + \{(0.4, 0.4, 0.5, 0.6), (0.3, 0.6, 0.5, 0.4)\} / d_4. \\
 x_3 &= \{(0.3, 0.4, 0.1, 0.5), (0.5, 0.6, 0.6, 0.4)\} / d_1 + \{(0.2, 0.3, 0.6, 0.5), (0.7, 0.7, 0.5, 0.8)\} / d_2 + \\
 &\quad \{(0.3, 0.4, 0.5, 0.9), (0.3, 0.6, 0.6, 0.7)\} / d_3 + \{(0.4, 0.5, 0.5, 0.6), (0.4, 0.5, 0.4, 0.4)\} / d_4. \\
 x_4 &= \{(0.4, 0.6, 0.5, 0.3), (0.1, 0.4, 0.2, 0.7)\} / d_1 + \{(0.7, 0.3, 0.7, 0.4), (0.8, 0.4, 0.8, 0.8)\} / d_2 + \\
 &\quad \{(0.2, 0.2, 0.5, 0.5), (0.7, 0.8, 0.8, 0.4)\} / d_3 + \{(0.5, 0.7, 0.7, 0.5), (0.8, 0.6, 0.5, 0.3)\} / d_4. \\
 x_5 &= \{(0.8, 0.6, 0.2, 0.5), (0.1, 0.3, 0.2, 0.5)\} / d_1 + \{(0.5, 0.4, 0.4, 0.3), (0.6, 0.4, 0.3, 0.3)\} / d_2 + \\
 &\quad \{(0.8, 0.6, 0.6, 0.9), (0.2, 0.5, 0.5, 0.4)\} / d_3 + \{(0.5, 0.3, 0.3, 0.6), (0.7, 0.7, 0.7, 0.8)\} / d_4.
 \end{aligned}$$

Now we calculate the QSVNR cosine similarity measure between R_1 and R_2 as follows:

$$\begin{aligned}
 C_1 &= \{\langle d_1, 0.785 \rangle, \langle d_2, 0.951 \rangle, \langle d_3, 0.876 \rangle, \langle d_4, 0.966 \rangle\} \\
 C_2 &= \{\langle d_1, 0.766 \rangle, \langle d_2, 0.865 \rangle, \langle d_3, 0.801 \rangle, \langle d_4, 0.920 \rangle\} \\
 C_3 &= \{\langle d_1, 0.859 \rangle, \langle d_2, 0.867 \rangle, \langle d_3, 0.847 \rangle, \langle d_4, 0.958 \rangle\}
 \end{aligned}$$

Although there always exist some similarity between two sets, so we take only the case of highest similarity measure among all pairs. Here d_4 has highest similarity with all the sets $C_i, i = 1, 2, 3$. Form the above result we can say that all the patients will have a chance of headache in body than other symptoms due to diabetic. One can easily try another methods of similarity measure between the QSVNR sets obtained from relations R_1 and R_2 respectively.

6. CONCLUSION

In the real world, there exist many examples where four valued arguments work. In 1977 Prof. N.D. Belknap showed modern uses of many valuable arguments in his paper [5]. So it quite natural to apply four valued logic to rough set theory. Through out this paper we have tried to develop the idea of a new hybrid set, say QSVNR set which is a combination of QSVNR set and a rough set. We have also discussed several measures of similarity for QSVNR sets and applied it in solving a decision making problem. SVN sets which are generalization of IFS have greater powers in handling uncertainty data including incomplete or even inconsistent data. Rough sets are used to model uncertain situations where the data set has vague boundaries. Combining these two features in QSVNR set give it additional powers in handling uncertain situations. On the other hand similarity measure is widely used technique in soft computing with many applications in areas like pattern recognition, medical diagnosis, decision making etc. In the future papers we wish to highlight several applications of our newly developed sets in solving many real life problems under uncertain environment and extend it to other multi-dimensional decision making problems.

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