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ON THE ORBITS OF SOME METABELIAN GROUPS

KARNIKA SHARMA[1], VIJAY KUMAR BHAT[1], §

Abstract. In this study, the orbits of non abelian metabelian groups of order 26, 28 and 30 are found using conjugation action. The non abelian metabelian groups considered in this study are some dihedral groups D_{13}, D_{14} and D_{15} as well as the semidirect products $D_7 \times Z_2$, $D_5 \times Z_3$ and $D_3 \times Z_5$. In order to obtain the orbit, the set of all pairs of commuting elements of the groups of size two is found. The results obtained from the orbits are then applied into an extension of commutativity degree which is the probability that an element of the groups fixes a set. Finally, the generalized conjugacy class graph is constructed and its chromatic number and clique number are also found.

Keywords: Metabelian group, conjugacy class graph, complete graph, group action.

AMS Subject Classification: 20E45, 05C69, 05C15, 20P05.

1. Introduction

In this paper, we give a brief description about non abelian metabelian groups, which include some dihedral groups and semidirect products of order 26, 28 and 30. Furthermore, we discuss the orbit and conjugacy class graphs of non abelian metabelian groups.

Throughout this paper, Γ denotes a simple undirected graph and G represents a non abelian metabelian group. A group G is said to be metabelian if there exist an abelian normal subgroup H of G such that G/H is abelian. In this study, G acts on itself by conjugation and the orbit of G is defined as the set of all conjugates of the elements. To obtain an orbit, a metabelian group G must act on a certain set. The set under this study is considered as the set of all pairs of commuting elements of a group of order two.

This work involves graph theory and group theory. In group theory, the orbits are used to calculate the extension of commutativity degree, namely the probability that a group elements fixes a set. The commutativity degree of a group is the probability that two random elements of a group commute. The concept of commutativity degree of a group

[1] School of Mathematics, Shri Mata Vaishno Devi University, India.
e-mail: karnika.sharma@smvdu.ac.in; ORCID: https://orcid.org/0000-0002-4160-237X.
e-mail: vijaykumarbhat2000@yahoo.com; ORCID: https://orcid.org/0000-0001-8423-9067.
Corresponding author.

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has been used in many studies. First, on some finite groups, it was introduced by Miller [4] in 1944. Another method of obtaining the commutativity degree using the number of conjugacy classes was introduced by Gustafon [6] in 1973. Beside that the concept of probability that a group element fixes a set was introduced by Omer et al. [11].

Graph theory is the study of vertices and edges. More precisely, it involves the ways in which set of points can be connected by edges. The notion was introduced by Leonard Euler in around 1736 while finding a solution to Konisbreg bridge problem. The results in appeared in Leonard Euler [5] in 1953. Various notions related to graph theory have been studied. Since its introduction include graph coloring in which n vertices of a graph Γ are colored in such a way that no two adjacent vertices have same color. However, the minimum number n required for which Γ is n- vertex colorable is known as chromatic number, denoted by $\chi(\Gamma)$. Furthermore, a clique is a complete subgraph in Γ and the size of largest clique in Γ is called clique number, denoted by $\omega(\Gamma)$.

In this article, the orbits are used to construct the generalized conjugacy class graph, where its vertices are non central orbits and two vertices are connected if their cardinalities are not coprime. The graph mentioned have been applied by various researchers for different type of groups. In 1967, Neumann [1] introduced a non commuting graph that gives the relation between non central elements of a group and the commutativity of the elements. In 1990, Bertram [8] introduced the generalized conjugacy class graph whose vertices are non central conjugacy classes and the vertices are adjacent if the cardinalities of the conjugacy classes are not coprime. Then Tolue and Erfanian [10] in 2013 generalized the concept of non commuting graph to the relative commutativity degree of a group. Omer et al. [13] in 2015 introduced the generalized conjugacy class graph whose vertices are non central orbits under group action on a set and the edges are formed if the cardinalities of the orbits are not coprime. Furthermore, in 2017 N. Zaid et al. [14] introduced the generalized conjugacy class graph of some dihedral groups. In 2018, S.N.A. Zamri et al. [15] introduced the orbits of some metabelian groups of order 24 and its applications. Recently, I. Gambo et al. [16] in 2019 introduced some graphs of finite metabelian groups of order less than 24.

In this study, the orbits of a non abelian metabelian groups are found using conjugation action. The groups we consider are some dihedral groups D_{13}, D_{14} and D_{15} as well as the semidirect products $D_7 \times Z_2$, $D_5 \times Z_3$ and $D_3 \times Z_5$ of order 26, 28 and 30. The results obtained from the orbits are then applied to find out the probability of a group element fixes the set. The set considered under this study is the set of all pairs of commuting elements of a group in the form of (a, b) where $\text{lcm}(|a|, |b|) = 2$. After that the generalized conjugacy class graph is constructed for all those six metabelian groups. Finally, some properties of graph which are chromatic number and clique number are obtained.

2. Preliminaries

In this section, some fundamental concepts and definitions related to this topic are presented. The set that is used throughout this study is defined as below.

Definition 2.1. (Rotman, 2002)[2] Group acting on set:
Let G be a finite group and S be a set. G acts on S if there is a function which maps $G \times S \to S$ such that it satisfies the following axioms:

1. **Identity:** $e.s = s.e, \forall s \in S$.
2. **Compatibility:** $(gh)s = g(hs), \forall s \in S, g, h \in G$.

Definition 2.2. (Goodman, 2003)[3] Orbit:
Let G be a finite group that acts on a set Υ and $\upsilon \in \Upsilon$. The orbit of υ, denoted by $O(\upsilon)$ is the subset $O(\upsilon) = \{g\upsilon \mid g \in G, \upsilon \in \Upsilon\}$. In this study, the group action is considered as a conjugation action. Hence, the orbit is defined as

$$O(\upsilon) = \{g\upsilon g^{-1} \mid g \in G, \upsilon \in \Upsilon\}.$$

Definition 2.3. (Miller, 1944)[4] Commutativity degree:
The probability that two random element (a, b) in a group G commute is called the commutativity degree and is defined as

$$P(G) = \frac{|\{(a, b) \in G \times G \mid ab = ba\}|}{|G|^2}.$$

Another method of computing commutativity degree is by using number of conjugacy classes which was introduced by Gustafson[6] in 1973 and defined as follows:

Definition 2.4. (Gustafson, 1973)[6] Commutativity degree:
Let G be a finite group and $K(G)$ is the number of conjugacy classes of the group. Then the commutativity degree of a group, denoted by $P(G)$, is given as

$$P(G) = \frac{K(G)}{|G|}.$$

Definition 2.5. (Sherman, 1975)[7] Let G be a group. Let X be a non-empty set of G (i.e., G is a group of permutations of X). Then the probability of an automorphism of a group fixes a random element from X is defined as follows

$$P_G(X) = \frac{|(g, x)|gx = x \forall g \in G, x \in X|}{|X||G|}.$$

In 2011, Moghaddam et al. [9] explored Sherman [7] definition in which an automorphism of a group fixes a subgroup element of a finite group. The probability is defined as follows:

$$P_{A_G}(H, G) = \frac{|(\alpha, h)|h^{\alpha}, h \in H, \alpha \in A_G|}{|H||G|},$$

where A_G is the group of automorphisms of a group G. It is clear that if $H = G$, then $P_{A_G}(G, G) = P_{A_G}(G)$.

Definition 2.6. (Omer et al., 2013)[11] The set Υ:
The set Υ is the set of all pairs of commuting elements in the form of (a, b) where a and b are the elements of the non abelian metabelian groups and the least common multiple of the order of the elements is two. Symbolically, it is represented as

$$\Upsilon = \{(a, b) \in G \times G | ab = ba, a \neq b, lcm(|a|, |b|) = 2\}.$$

In 2013, Omer et al.[11] extended the concept of commutativity degree in order to determine the probability that an element of a group fixes a set and is defined as below:

Definition 2.7. (Omer et al., 2013)[11] Probability that an element of a group fixes a set:

Let G be a finite group and H be the set of elements of G of order two in the form of (a, b) where a and b commute. Let Υ be the set of all subsets of commuting elements of order two in G. If G acts on Υ by conjugation. Then the probability of an element of a group fixes a set is given as

$$P_G(\Upsilon) = \frac{|\{(g, \upsilon) \mid g\upsilon = \upsilon, \forall g \in G, \upsilon \in \Upsilon\}|}{|G||\Upsilon|}.$$

In terms of number of orbits, the probability that an element of a group fixes a set is written as below.

Definition 2.8. (Omer et al., 2013)[11] Probability that an element of a group fixes a set:
Let G be a finite group and H be the set of elements of G of order two in the form of (a, b) where a and b are commute. Let Υ be the set of all subsets of commuting elements of order two in G, where G acts on Υ by conjugation. Then the probability of an element of a group fixes a set is given as

$$P_G(\Upsilon) = \frac{K}{|\Upsilon|},$$

where K is the number of orbits of Υ in G.

Definition 2.9. (Omer et al., 2015)[13] Generalized conjugacy class graph, Γ_G^{Υ}:
Let G be a metabelian group and Υ be a set. Then generalized conjugacy class graph, Γ_G^{Υ} is defined as a graph whose vertices are non central orbits under group action on a set that is $V(\Gamma_G^{\Upsilon}) = K(\Upsilon) - A$, where $K(\Upsilon)$ is non central orbits under group action on set Υ and $A = \{\upsilon \in \Upsilon \mid g\upsilon = \upsilon g,\, g \in G\}$. Two vertices are connected if the cardinality of the orbits are not coprime.

Definition 2.10. Complete graph, K_n:
A graph G is said to be a complete graph of n vertices, if there exists an edge between every pair of vertices and is denoted by K_n.

3. Main Results

This section provides our main results which is the computation of an orbit of some non abelian metabelian groups of order 26, 28 and 30. Then based on the orbits of a group, probability that an element of a group fixes a set is obtained. Finally, conjugacy class graph of a group are constructed and its chromatic number and clique number are also found.

Proposition 3.1. Let $G = D_{13}$ and $\Upsilon = \{(a, b) \in G \times G | ab = ba, a \neq b, lcm(|a|, |b|) = 2\}$. Then $|\Upsilon| = 26$. If G acts on Υ by conjugation, then $|k(\Upsilon)| = 2$.

Proof. Suppose $G = D_{13}$ and $\Upsilon = \{(a,b) \in G \times G | ab = ba, a \neq b, lcm(|a|,|b|) = 2\}$. The group D_{13} has 13 elements of size two, which are in the form of $a^i b$, $0 \leq i \leq n-1$.

There are 26 elements in the form of $(1, a^i b)$, $0 \leq i \leq n-1$. Therefore, $|\Upsilon| = 26$. Since D_{13} acts on Υ by conjugation. So there are two orbits in the form of $(1, a^i b)$, $0 \leq i \leq n-1$ of size 13. Hence $|k(\Upsilon)| = 2$. □

Proposition 3.2. Let $G = D_{14}$ and $\Upsilon = \{(a,b) \in G \times G | ab = ba, a \neq b, lcm(|a|,|b|) = 2\}$. Then $|\Upsilon| = 72$. If G acts on Υ by conjugation, then $|k(\Upsilon)| = 12$.

Proof. Suppose $G = D_{14}$ and $\Upsilon = \{(a,b) \in G \times G | ab = ba, a \neq b, lcm(|a|,|b|) = 2\}$. The group D_{14} has 15 elements of size two, which are in the form of $a^i b$, $0 \leq i \leq n-1$.

There are 2 elements in the form of $(1, a^7)$, 28 elements in the form of $(1, a^i b)$, $0 \leq i \leq n-1$, 28 elements in the form of $(a^7, a^i b)$, $0 \leq i \leq n-1$, and 14 elements in the form of $(a^i b, a^j b)$, $0 \leq i \leq 6$, $7 \leq j \leq n-1$. Therefore, $|\Upsilon| = 72$.

Since D_{14} acts on Υ by conjugation. So there are two orbits in the form of $(1, a^7)$ of size 1, two orbits in the form of $(1, a^{2i} b)$, $0 \leq i \leq 6$ of size 7, two orbits in the form of $(1, a^{2i+1} b)$, $0 \leq i \leq 6$ of size 7, two orbits in the form of $(a^7, a^{2i} b)$, $0 \leq i \leq 6$ of size 7, two orbits in the form of $(a^7, a^{2i+1} b)$, $0 \leq i \leq 6$ of size 7 and two orbits in the form of $(a^i b, a^j b)$, $0 \leq i \leq 6$, $7 \leq j \leq n-1$ of size 7. Hence $|k(\Upsilon)| = 12$. □

Proposition 3.3. Let $G = D_7 \times Z_2$ and $\Upsilon = \{(a,b) \in G \times G | ab = ba, a \neq b, lcm(|a|,|b|) = 2\}$. Then $|\Upsilon| = 72$. If G acts on Υ by conjugation, then $|k(\Upsilon)| = 12$.

Proof. Suppose $G = D_7 \times Z_2$ and $\Upsilon = \{(a,b) \in G \times G | ab = ba, a \neq b, lcm(|a|,|b|) = 2\}$. The group $D_7 \times Z_2$ has 15 elements of order two, which are in the form $a^i b$, $a^i bc$, c, where $0 \leq i \leq n-1$, $n = 7$.

There are 2 elements in the form of $(1, c)$, 14 elements in the form of $(1, a^i b)$, 14 elements in the form of $(1, a^i bc)$, 14 elements in the form of $(c, a^i b)$, 14 elements in the form of $(c, a^i bc)$, and 14 elements in the form of $(a^i b, a^i bc)$. Therefore, $|\Upsilon| = 72$.

Since $D_7 \times Z_2$ acts on Υ by conjugation. So there are two orbits in the form of $(1, c)$ of size 1, two orbits in the form of $(1, a^i b)$ of size 7, two orbits in the form of $(1, a^i bc)$ of size 7, two orbits in the form of $(c, a^i b)$ of size 7, two orbits in the form of $(c, a^i bc)$ of size 7 and two orbits in the form of $(a^i b, a^i bc)$ of size 7. Hence $|k(\Upsilon)| = 12$. □

Proposition 3.4. Let $G = D_{15}$ and $\Upsilon = \{(a,b) \in G \times G | ab = ba, a \neq b, lcm(|a|,|b|) = 2\}$. Then $|\Upsilon| = 30$. If G acts on Υ by conjugation, then $|k(\Upsilon)| = 2$.

Proof. Suppose $G = D_{15}$ and $\Upsilon = \{(a,b) \in G \times G | ab = ba, a \neq b, lcm(|a|,|b|) = 2\}$. The group D_{15} has 15 elements of size two, which are in the form of $a^i b$, $0 \leq i \leq n-1$.

There are 30 elements in the form of $(1, a^i b)$, $0 \leq i \leq n-1$. Therefore, $|\Upsilon| = 30$. Since D_{15} acts on Υ by conjugation. So there are two orbits in the form of $(1, a^i b)$, $0 \leq i \leq n-1$ of size 15. Hence $|k(\Upsilon)| = 2$. □

Proposition 3.5. Let $G = D_5 \times Z_3$ and $\Upsilon = \{(a,b) \in G \times G | ab = ba, a \neq b, lcm(|a|,|b|) = 2\}$. Then $|\Upsilon| = 10$. If G acts on Υ by conjugation, then $|k(\Upsilon)| = 2$.

Proof. Suppose $G = D_5 \times Z_3$ and $\Upsilon = \{(a,b) \in G \times G | ab = ba, a \neq b, lcm(|a|,|b|) = 2\}$. The group $D_5 \times Z_3$ has 5 elements of size two, which are in the form of $a^i b$, where

$0 \leq i \leq n-1$, $n = 5$.

There are 10 elements in the form of $(1, a^i b)$. Therefore, $|\Upsilon| = 10$. Since $D_5 \times Z_3$ acts on Υ by conjugation. So there are two orbits in the form of $(1, a^i b)$. Hence $|k(\Upsilon)| = 2$. □

Proposition 3.6. Let $G = D_3 \times Z_5$ and $\Upsilon = \{(a,b) \in G \times G | ab = ba, a \neq b, lcm(|a|,|b|) = 2\}$. Then $|\Upsilon| = 6$. If G acts on Υ by conjugation, then $|k(\Upsilon)| = 2$.

Proof. Suppose $G = D_3 \times Z_5$ and $\Upsilon = \{(a,b) \in G \times G | ab = ba, a \neq b, lcm(|a|,|b|) = 2\}$. The group $D_3 \times Z_5$ has 3 elements of size two, which are in the form of $a^i b$, where $0 \leq i \leq n-1$, $n = 3$.

There are 6 elements in the form of $(1, a^i b)$. Therefore, $|\Upsilon| = 6$. Since $D_3 \times Z_5$ acts on Υ by conjugation. So there are two orbits in the form of $(1, a^i b)$. Hence $|k(\Upsilon)| = 2$. □

Proposition 3.7. Let G be a non abelian metabelian groups of order $26, 28$ or 30 and $\Upsilon = \{(a,b) \in G \times G | ab = ba, a \neq b, lcm(|a|,|b|) = 2\}$. If G acts on Υ by conjugation, then

$$P_G(\Upsilon) = \begin{cases} \frac{1}{n}, & \text{if } G = D_n, n = 13, 15 \\ \frac{1}{6}, & \text{if } G = D_{14}, D_7 \times Z_2 \\ \frac{1}{5}, & \text{if } G = D_5 \times Z_3 \\ \frac{1}{3}, & \text{if } G = D_3 \times Z_5 \end{cases}$$

Proof. Let $G = D_{13}$. Using Proposition 3.1, we have $|\Upsilon| = 26$ and $|K(\Upsilon)| = 2$. Therefore, $P_G(\Upsilon) = \frac{2}{26} = \frac{1}{13}$.

Let $G = D_{15}$. Using Proposition 3.4, we have $|\Upsilon| = 72$ and $|K(\Upsilon)| = 2$. Therefore, $P_G(\Upsilon) = \frac{2}{30} = \frac{1}{15}$

Let $G = D_{14}$ and $D_7 \times Z_2$. By Propositions 3.2, 3.3, we get $|\Upsilon| = 72$ and $|K(\Upsilon)| = 12$ for both the groups. Therefore, $P_G(\Upsilon) = \frac{12}{72} = \frac{1}{6}$.

Let $G = D_5 \times Z_3$ and $D_3 \times Z_5$. By Propositions 3.5, 3.6, we get $|\Upsilon| = 10$ for $D_5 \times Z_3$ and $|\Upsilon| = 6$ for $D_3 \times Z_5$ and $|K(\Upsilon)| = 2$ for both the groups. Hence $P_{D_5 \times Z_3}(\Upsilon) = \frac{2}{10} = \frac{1}{5}$ and $P_{D_3 \times Z_5}(\Upsilon) = \frac{2}{6} = \frac{1}{3}$. □

Proposition 3.8. Let G be a non abelian metabelian groups of order $26, 28$ or 30 and $\Upsilon = \{(a,b) \in G \times G | ab = ba, a \neq b, lcm(|a|,|b|) = 2\}$. If G acts on Υ by conjugation, then

$$\Gamma_G^{\Upsilon} = \begin{cases} Completegraph, K_2, & \text{if } G = D_{13}, D_{15}, D_3 \times Z_5, D_5 \times Z_3 \\ Completegraph, K_{10}, & \text{if } G = D_{14}, D_7 \times Z_2 \end{cases}$$

Proof. Suppose $G = D_{13}$, D_{15}, $D_5 \times Z_3$ and $D_3 \times Z_5$. Now using Propositions 3.1, 3.4, 3.5 and 3.6, we get $|K(\Upsilon)| = 2$, where none of the orbits are central orbits. Then, $|V(\Gamma_G^{\Upsilon})| = 2 - 0 = 2$.

Since both the orbits have 13 elements in D_{13}, 15 elements in D_{15}, 5 elements in $D_5 \times Z_3$ and 3 elements in $D_3 \times Z_5$ making them not coprime with each other. Hence, the vertices are connected and, therefore, Γ_G^{Υ} is a complete graph of two vertices, K_2. The graph is as

FIGURE 1. complete graph, K_2

in the following figure:

Suppose $G = D_{14}$ and $D_7 \times Z_2$. Now using Propositions 3.2, 3.3, we get $|K(\Upsilon)| = 12$, where two of the orbits are central orbits. Meanwhile, the other 10 orbits are non central orbits. Then, $|V(\Gamma_G^{\Upsilon})| = 12 - 2 = 10$.

Since all the non central orbits in D_{14} and $D_7 \times Z_2$ have seven elements with cardinalities are not coprime. Therefore, all the vertices are adjacent to each other. Hence, Γ_G^{Υ} is a complete graph with 10 vertices, K_{10}. The graph is as in the following figure: □

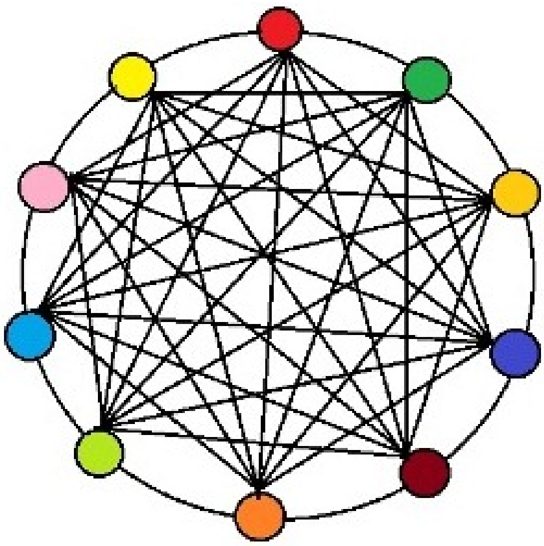

FIGURE 2. complete graph, K_{10}

Theorem 3.9. *Let G be a non abelian metabelian groups of order $26, 28$ or 30 and $\Upsilon = \{(a,b) \in G \times G | ab = ba, a \neq b, lcm(|a|,|b|) = 2\}$. If G acts on Υ by conjugation, then*

$$\chi(\Gamma_G^{\Upsilon}) = \begin{cases} 2, & if\, G = D_{13}, D_{15}, D_3 \times Z_5, D_5 \times Z_3 \\ 10, & if\, G = D_{14}, D_7 \times Z_2 \end{cases}$$

Proof. Let $G = D_{13}$, D_{15}, $D_5 \times Z_3$ and $D_3 \times Z_5$. Using Proposition 3.8, $\Gamma_G^{\Upsilon} = K_2$, a complete graph of two vertices. Since both of its vertices are connected to each other and hence each of the vertices needed to have different colors so that both adjacent vertices are not sharing the same color. Therefore, $|\chi(\Gamma_G^{\Upsilon})| = 2$.

Let $G = D_{14}$ and $D_7 \times Z_2$. Using Proposition 3.8, $\Gamma_G^{\Upsilon} = K_{10}$, a complete graph of ten vertices. Since both of its vertices are connected to each other and hence each of the vertices needed to have different colors so that no two adjacent vertices share the same color. Hence $|\chi(\Gamma_G^{\Upsilon})| = 10$. □

Theorem 3.10. *Let G be a non abelian metabelian groups of order $26, 28$ or 30. If G acts on Υ by conjugation, then*

$$\omega(\Gamma_G^{\Upsilon}) = \begin{cases} 0, & if\, G = D_{13}, D_{15}, D_3 \times Z_5, D_5 \times Z_3 \\ 10, & if\, G = D_{14}, D_7 \times Z_2 \end{cases}$$

Proof. Assume $G = D_{13}$, D_{15}, $D_3 \times Z_5$ and $D_5 \times Z_3$. By Proposition 3.8, we have $\Gamma_G^{\Upsilon} = K_2$, a complete graph of two vertices. Since graph has only two vertices, so there is no complete subgraph in graph. Hence, $|\omega(\Gamma_G^{\Upsilon})| = 0$

Assume $G = D_{14}$ and $D_7 \times Z_2$. By Proposition 3.8, we have $\Gamma_G^{\Upsilon} = K_{10}$, a complete graph of ten vertices. Therefore, the largest complete subgraph that can be obtained is a complete graph of ten vertices itself. Hence $|\omega(\Gamma_G^{\Upsilon})| = 10$. □

4. Conclusion

In this paper, the orbits of non abelian metabelian groups of order 26, 28 and 30 which are dihedral groups D_{13}, D_{14} and D_{15} as well as the semidirect products $D_7 \times Z_2$, $D_5 \times Z_3$ and $D_3 \times Z_5$ are found using conjugation action. These orbits are then used to obtain the probability that a group element fixes a set as well as the generalized conjugacy class graphs. We also find out the chromatic and clique number of a graphs. The results are summarized in following tables:

Table 1. Summary of the Main Results

Metabelian groups	Number of orbits	Probability	Generalized Conjugacy Class graphs	Chromatic number	Clique number
D_{13}	2	$\frac{1}{3}$	Complete graph, K_2	2	0
D_{14}	12	$\frac{1}{6}$	Complete graph, K_{10}	10	10
D_{15}	2	$\frac{1}{15}$	Complete graph, K_2	2	0
$D_7 \times Z_2$	12	$\frac{1}{6}$	Complete graph, K_{10}	10	10
$D_5 \times Z_3$	2	$\frac{1}{5}$	Complete graph, K_2	2	0
$D_3 \times Z_5$	2	$\frac{1}{3}$	Complete graph, K_2	2	0

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Karnika Sharma completed her M.Sc. in Mathematics at Shri Mata Vaishno Devi University, Katra, India in 2018. She has been awarded with certificate of distinction for excelling in academics and co-curricular and extra-curricular activities. She has also received INSPIRE fellowship from Department of Science and Technology. Currently, she is pursuing her Ph. D. at school of mathematics, Shri Mata Vaishno Devi University, Katra, India.

Vijay Kumar Bhat completed his Ph.D. in Mathematics in 2000 from University of Jammu, India. Currently, he is a professor of mathematics at Shri Mata Vaishno Devi University. His research interest is Algebra (group theory, skew polynomial rings) and graph theory.