TWMS J. App. and Eng. Math. V.12, N.4, 2022, pp. 1179-1188

PREVENTIVE REPLACEMENT FOR BELLIGERENT SYSTEMS

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ABSTRACT. A mortar is commonly used as an indirect firing weapon to support close fires with a variety of ammunition. There are mortar weapons with various shells. Each type of shells fired by mortars does damage to a weapon when the total damage on a mortar weapon reaches the tolerance limit, the mortar weapon either fails or explodes, leading to a compulsory replacement which is costly. In order to maintain the mortar weapons and archers in wars, a research was conducted to find the best number of mortar shells that will be fired until a preventive replacement for mortar weapons is implemented.

Keywords: Shock, Preventive replacement, Compulsory replacement, Expected costs per unit time, Tolerance limit.

AMS Subject Classification: 83-02, 99A00.

1. INTRODUCTION

Nowadays, there are many systems that are subjected to different shocks and these shocks tremendously affect the system which has a definite lifetime. The system is entirely affected by the shock. The summation impact of these shocks should not exceed the tolerance limit for the system.

The compulsory replacement costs of the system are extremely high, thereby scientists have always been looking for strategies to determine the number of shocks to replace the system before the system crashes.

A systematic survey was performed on maintenance policies based on the reliability theory (Nakagawa, 2005) [1]. Recently published books include (Osaki, (2002); Wang and Pham, (2007); Kobbacy and Morgatti (2008); Nakagawa and Ito (2008, 2011) [2–6]. The types of maintenance methods were examined regarding the theory of reliability and applied in industrial systems.

On the other hand, the damage done by shocks was studied by (Cox, (1962), Nagakawa and Osaki, (1974) [7, 8]. The first book in this field (Bagdenov and Koznin, (1985) [9], provided potential models associated with cumulative damage. Moreover, various types of preventive maintenance models have been widely studied by Wortman et al., (1994);

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§ Manuscript received: July 13, 2020; accepted: April 11, 2021.

Sheu and Griffith (1996); Sheu (1998); Sheu and Griffith (2002); Sheu and Chien (2004); Sheu et al. (2012) [10–15].

Nagakawa (2005) [1], explained preventive replacement in his book Maintenance Theory of Reliability, then Nagakawa (2007) [16], illustrated the damage to the system from the shock in his book (Shock and damage models in reliability theory). Furthermore, Nakagawa and Ito (2008) [5], showed maintenance policies for a system with multi-level risks. A number-dependent replacement policy for a system with continuous preventive maintenance and random lead times was introduced by Chien (2009) [24]. Taghipour (2012) [17], carried out an optimal inspection of a complex system subject to periodic and opportunistic inspections and preventive replacements. Zhao et al. (2013) [18], conducted research in optimal policies for cumulative damage models with maintenance last and first. Chang et al. (2013) [19] offered explanation concerning the optimal replacement model with age-dependent failure type based on a cumulative repair-cost limit. Chang (2014) [20] elucidated Optimum preventive maintenance policies for systems subject to random working times, replacement, and minimal repair. Chang (2015) [21] put forward the optimal preventive policies for imperfect maintenance models with a replacement first and last. Hamidi et al. (2016) [22], explicated new one cycle criteria for optimizing preventive replacement policies. Afrinaldi et al. (2017) [23], assessed minimizing economic and environmental impacts through an optimal preventive replacement schedule: Model and application. While, Chien (2018) [24], explains optimal age for preventive replacement of a system with GPP repair process, Sheu et al. (2019) [25], extended the optimal preventive replacement policy with random working cycles.

In this paper, we examine a reliability based system that was subjected to shocks over time.

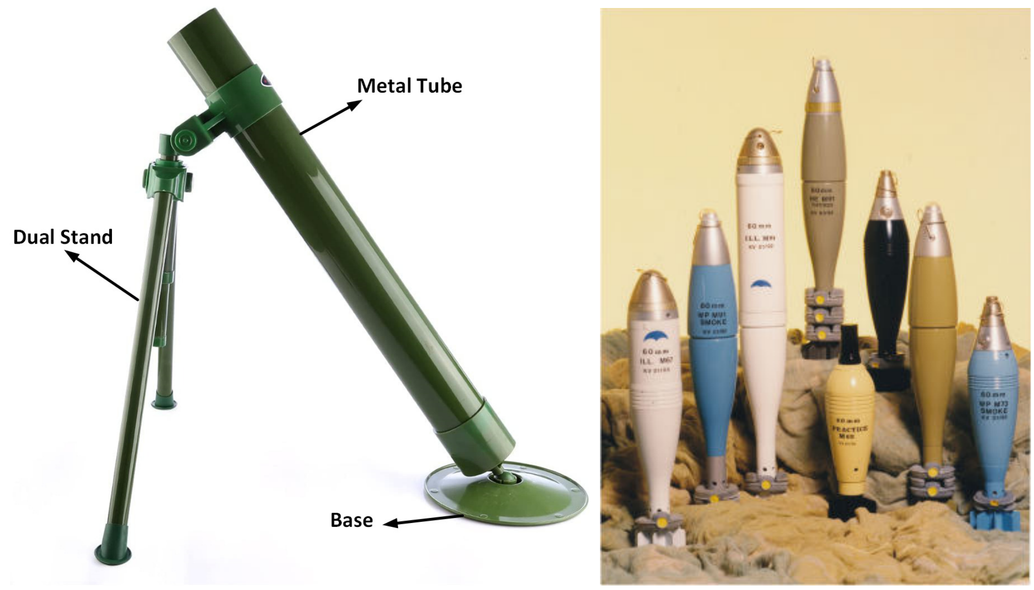
In this paper, on the one hand, a system that was subjected to shocks over time was examined based on reliability. When the total damage caused by shocks reaches the tolerance limit K, the system fails and must be replaced with coercion. We evaluated optimal preventive maintenance policies to minimize the costs of a system replacement before it failed. On the other hand, the optimal number of shocks for the preventive replacement of the system was determined. Furthermore, not only an example of a mortar weapon in which we throw shells is presented but also the optimal number of shells for a preventive replacement for the mortar weapon will be calculated, such actions lead to reduce the costs of failure and also protects the weapons and the archer as well.

2. Mortar (weapon)

The mortar weapon consists of a metal tube, a dual stand, and a base. The base is fixed in the ground, then we put the metal tube in the base and the dual stand to support the tube. As a result, when we want to throw the shell, it is placed in the metal tube.

Every shell thrown by the mortar inflicts damage to the metal tube, and if the total damage on the tube reaches the tolerance limit K of the metal tube, it fails. Failing can give rise to situations in which we place the mortar shell into the tube that cannot be thrown, or sometimes the tube itself can explode, or even the person who tosses it may be exposed to danger. The base is fixed to the ground, when we decide to replace the tube, another tube in the same base can be replaced which takes no time. Due to the fact that tube compulsive replacement is extremely costly, the preventive replacement is highly recommended which is much more cost-effective.

A comprehensive solution to the cost problem and avoids to seriously jeopardize the shooter and the weapon is to discover the best number of mortar shells that will be fired for the preventive replacement of the metal tube.



(a) (b)

FIGURE 1. (a)Mortar and (b)Mortar Shells

3. Model

The system that we studied is a system subjected to shocks because each mortar shell thrown by the mortar weapon causes considerable shocks that leave an impact on the mortar tube. Thereby we simulated a system that is subjected to shocks and makes an impact on the system.

however, X_j is the amount of damage resulted by the $(j)^{th}$ shock on the system, and $X_0 = 0$. And $Z_(n) = \sum_{j=1}^{n} X_j$ is the total damage caused by all shocks until the $(n)^{th}$ shock occurs to the system. X_j has an identical distribution is $G(x) = Pr\{X_j \leq x\}$ with mean μ.

$$G_j = Pr\{Z_j \leq x\}, \tag{1}$$

$$E(Z_j) = j\mu. \tag{2}$$

4. Replacement costs

The base of the mortar is fixed in the ground and never experienced any shocks, on the contrary, the metal tube is always exposed to shocks, so a spare tube is reserved in the case of replacement. The total damage resulted from shocks is originated from different types of mortar shells. Once the total damage reaches the tolerance limit K, the metal tube fails and sometimes the shooter face danger, so we will replace the mortar tube called compulsory replacement which is expensive. As a result, in order to avoid to enduring the compulsory replacement of the metal tube which is high-priced, we recommend the preventive replacement with a lower cost. There are two types of replacement costs:

C_k: Compulsory replacement cost,

C_N: Preventive replacement cost.

5. The probabilities of preventive replacement and Compulsory replacement

There are two possibilities for the mortar tube replacement. The first possibility is to replace it in case of failure and the second is the preventive substitution.

N is the planned number of shocks for the preventive replacement of the system. The probability of a preventive replacement of system when we reach the number N of shocks before the total damage has reached the tolerance limit K.

$$P_N = Pr\{\sum_{j=0}^{N} X_j \leq K\} = G_N(K). \tag{3}$$

As for the probability of replacing when the system fails, i.e. the probability of compulsory replacement, once the total damage reaches the tolerance limit K and the number of shocks does not reach the number N,

$$P_K = \sum_{j=0}^{N-1} Pr\{\sum_{i=0}^{j} X_i < K \leq \sum_{i=0}^{j+1} X_i\} = \sum_{j=0}^{N-1}(G_j(K) - G_{j+1}(K)). \tag{4}$$

See (4) in appendix.

With the note that $P_N + P_K = 1$.

6. Expected cost rate

The time interval between the first replacement and the next is called a cycle. Time and cost for each cycle is independent, the expected cost per unit of time is calculated and named the expected cost rate:

$$C(N) = \frac{Expected\ cost\ of\ one\ cycle}{Mean\ time\ of\ one\ cycle}$$

6.1. Calculation of $C(N)$.

To calculate the meantime for the replacement of one cycle, we need the time for the replacement and also the probability of the replacement.

$E(T)$: is the meantime for a shock occurs to the system.

The preventive substitution is carried out when an N shock occurs in the system, so the time for its substitution is the meantime $E(T)$ multiplied by N, the probability of a preventive replacement isP_N.

When the total damage reaches tolerance limit K, then the compulsory replacement happens. The time for the compulsory substitution is $E(T)$ multiplied by $(j+1)$ which is the number of shocks until total damage reaches tolerance limit K. The probability of a compulsory replacement is P_k.

$$\begin{cases} NE(T) \quad \textit{with probability} \quad P_N \\ (j+1)E(T) \quad \textit{with probability} \quad P_K \end{cases}$$

The mean time for replacement of a cycle is equal to

$$= E(T)\sum_{j=0}^{N-1} G_j(K). \tag{5}$$

(5) is calculated in appendix.

In order to calculate the expected cost of a cycle, we need the cost of replacement and also the probability of replacement.

$$\begin{cases} C_N & \textit{with probability} \quad P_N \\ C_K & \textit{with probability} \quad P_K \end{cases}$$

The expected cost of a cycle is equal to

$$\begin{aligned} & C_N \times P_N + C_K \times P_K \\ & = C_K + (C_N - C_K)P_N. \end{aligned} \tag{6}$$

So,

$$C(N) = \frac{C_K + (C_N - C_K)G_N(K)}{E(T)\sum_{j=0}^{N-1} G_j(K)}. \tag{7}$$

7. Optimum number

Mortars work irregularly particularly in wars, depending on the number of shells in the mortar tube, they are replaced before the failure occurs. This method is more optimal than replacing the mortar based on the time. So the number of shells needed for the preventive replacement was calculated; N* represents the optimum number of shells for a preventive replacement of the mortar. The costs of replacing a mortar at the beginning of its life are great, then costs decrease during its working life of the mortar. Correspondingly, the costs will rise at the end of its lifetime as it becomes increasingly more vulnerable to failure.

Our goal is to replace the mortar before the failure occurs since the costs begin to increase at the end of mortar life. Thereby, we choose the shells when the costs are at the lowest rate on the mortar. Shells are thrown sporadically, but N does not continue. To calculate N*, the minimum C (N) was determined by the following method:

$$\begin{cases} C(N+1) - C(N) \geq 0 \\ C(N) - C(N-1) \leq 0 \end{cases} \tag{8}$$

$$\begin{cases} L(N) \geq \frac{1}{1-\frac{C_N}{C_K}} \\ L(N-1) \leq \frac{1}{1-\frac{C_N}{C_K}} \end{cases} \tag{9}$$

where $L(N)$ is equal

$$\frac{[G_N(K) - G_{N+1}(K)]\sum_{j=0}^{N-1} G_j(K)}{G_N(K)} + G_N(K).$$

We will compute (8) in the appendix to get (9).

In equation (9) there is no average shock time within it. This means that the optimum number of shocks for a preventive replacement of the system is not affected by the time of the projectile.

8. Numerical example

Depending on the target, the mortar weapon applies various types of mortar shells. When a mortar shell is thrown, it causes damage to the mortar tube. The damage resulting from different types of mortar shells is only measured when the mortar shells fired. The damage rate caused by firing these shells is measured when firing mortar shells is $\frac{1}{\mu} = \frac{1}{50}$. Once the total damage caused by the mortar shells on the mortar tube reaches tolerance limit $K = 15$, the mortar tube fails (i.e. when we place the projectile into the tube, it does not fire or it may explode the tube and even can murder the person who is carrying out the firing), consequently, we are obliged to replace the mortar tube. As the cost of the compulsory replacement is $C_K = 5000, Z_n$ has Erlanger distribution, so $G_n(x) = 1 - \sum_{i=0}^{n-1} e^{-\mu x} \frac{(\mu x)^i}{i!}$. Seeking an ultimate solution to the problem, preventive replacement for mortar tubes is recommended. This is conducted by specifying the number N^* of mortar shells (i.e. once we fire this number N^* of mortar shells, then we implement the preventive replacement of the mortar tube). Preventive replacement for mortar tubes protects the mortar tube against failure, reducing financial and human loss. Hence, the price of a preventive replacement $C_N = 500$ is less than a compulsory replacement for the mortar tube. We put all the values in the equation (9) to obtain N^*

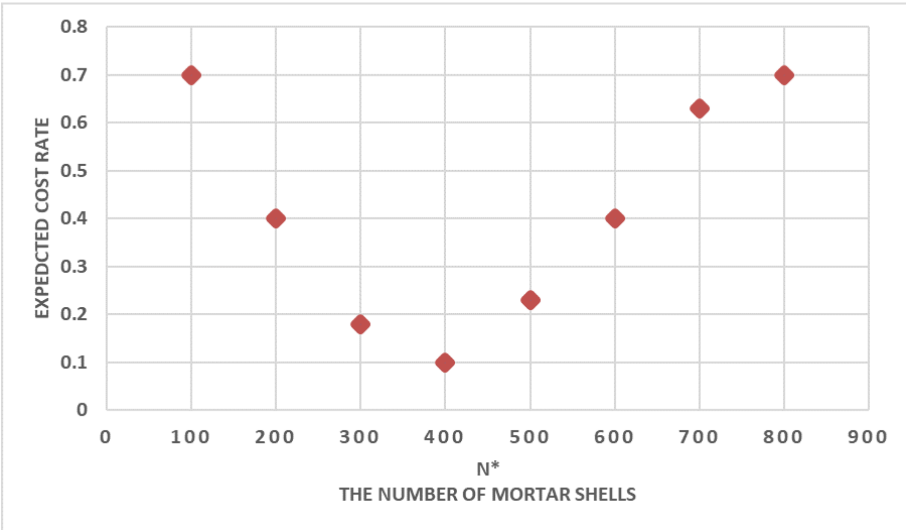


FIGURE 2. Diagram between the expected cost rate and The number of mortar shells

The result is 400 mortar shells which is the number of shells for preventive replacement of the mortar tube.

We will change both tolerance limit K and cost ratios $\frac{C_K}{C_N}$ indicated in table 1.

TABLE 1. the optimum number of mortar shells for preventive replacement for mortar tube

K	5	10	15	20
$\frac{C_K}{C_N}$	N^*	N^*	N^*	N^*
5	150	300	450	650
10	100	250	400	600
20	100	200	350	500
50	50	200	300	450

If tolerance limit K increases, the mortar tube will be more durable which implies that the optimum number of mortar shells for preventive replacement of the mortar tube increases. However, once the ratio $\frac{C_K}{C_N}$ rises, note us a decrease in the optimum number of mortar shells for preventive replacement of the mortar tube will be noted.

9. Conclusion

In this article, mortar weapons that throw various shells were thoroughly studied. Each type of shell leads to different damage to the mortar tube comparing to other shells. It was the major aim of the present study to discover the optimal number of mortar shells for preventive replacement of the mortar tube.

We studying a system that was subjected to shocks, and we had to anticipate the number of shocks for a preventive replacement of a system in order to avoid failure. As a result, we developed the formula (9) in which we put the values, providing us the optimum number of shocks for preventive replacement for the system.

Then a mortar weapon was set up and throw the shells, thereafter, the optimal number of shells for a preventive replacement for mortar tube was calculated through equation (9). The results indicated that when the tolerance limit increases, the optimal number of shells for preventive replacement of the mortar tube witnessed a dramatic increase.

10. Analysis (4): in appendix

Assume that $(A) = (\sum_{i=}^{j} X_i < K)$ and $(B) = (\sum_{i=0}^{j} X_i + X_{j+1} \geq K)$. So $(\bar{B}) = (\sum_{i=0}^{j} X_i + X_{j+1} < K)$.

$$\begin{aligned}
P_K &= \sum_{j=0}^{N-1} P(\sum_{i=0}^{j} X_i < K \leq \sum_{i=0}^{j+1} X_i) \\
&= \sum_{j=0}^{N-1} P(\sum_{i=0}^{j} X_i < K \leq \sum_{i=0}^{j} X_i + X_{j+1}) \\
&= \sum_{j=0}^{N-1} P(\sum_{i=0}^{j} X_i < K, \sum_{i=0}^{j} X_i + X_{j+1} \geq K)
\end{aligned}$$

According the law: $P(A \text{ and } B) = P(A) - P(A \text{ and } \bar{B})$

$$\sum_{j=0}^{N-1} (P(\sum_{i=0}^{j} X_i < K) - P(\sum_{i=0}^{j} X_i < K, \sum_{i=0}^{j} X_i + X_{j+1} < K))$$

Applying the law which says: $(\bar{B}) \subset (A) \Rightarrow P((A) \text{ and } (\bar{B})) = P((\bar{B})$

$$\begin{aligned}
&= \sum_{j=0}^{N-1} P(\sum_{i=0}^{j} X_i < K) - P(\sum_{i=0}^{j+1} X_i < K)) \\
&= \sum_{j=0}^{N-1} G_j(K) - G_{j+1}(K).
\end{aligned}$$

10.1. **Calculate (5),**

$$\begin{cases} NE(T) & P_N \\ (j+1)E(T) & P_K \end{cases}$$

The mean time for replacement of one cycle is equal to

$$
\begin{aligned}
&NE(T) \times P_N + (j+1)E(T) \times P_K \\
&= NE(T)G_N(K) + \sum_{j=0}^{N-1}(j+1)E(T)(G_j(K) - G_{j+1}(K)). \\
&= E(T)\sum_{j=0}^{N-1} G_j(K).
\end{aligned}
$$

10.2. **Analysis (9):**

$$C(N+1) - C(N) \geq 0$$

$$\frac{C_K + (C_N - C_K)G_{N+1}(K)}{E(T)\sum_{j=0}^{N} G_j(K)} - \frac{C_K + (C_N - C_K)G_N(K)}{E(T)\sum_{j=0}^{N-1} G_j(K)} \geq 0$$

$$\frac{\binom{C_K\sum_{j=0}^{N-1} G_j(K) + (C_N - C_K)G_{N+1}(K)\sum_{j=0}^{N-1} G_j(K)}{-C_K\sum_{j=0}^{N} G_j(K) - (C_N - C_K)G_N(K)\sum_{j=0}^{N} G_J(K)}}{E(T)\sum_{j=0}^{N} G_j(K)\sum_{j=0}^{N-1} G_j(K)} \geq 0$$

$$\frac{(C_N - C_K)G_{N+1}(K)\sum_{j=0}^{N-1} G_j(K)}{G_N(K)} - (C_N - C_K)\sum_{j=0}^{N} G_j(K) \geq C_K$$

$$\frac{-G_(N+1)(K)\sum_{j=0}^{N-1} G_j(K)}{G_N(K)} + \sum_{j=0}^{N} G_j(K) \geq \frac{C_K}{C_K - C_N}$$

$$\frac{G_N(K)\sum_{j=0}^{N} G_j(K) - G_{N+1}(K)\sum_{j=0}^{N-1} G_j(K)}{G_N(K)} \geq \frac{C_K}{C_K - C_N}$$

$$\frac{[G_N(K) - G_{N+1}(K)]\sum_{j=0}^{N-1} G_j(K)}{G_N(K)} + G_N(K) \geq \frac{C_K}{C_K - C_N}$$

$$\frac{[G_N(K) - G_{N+1}(K)]\sum_{j=0}^{N-1} G_j(K)}{G_j(K)} + G_N(K) \geq \frac{1}{1 - \frac{C_N}{C_K}}$$

$$L(N) \geq \frac{1}{1 - \frac{C_N}{C_K}}$$

In the same way

$$C(N) - C(N-1) \leq 0$$

$$L(N-1) \leq \frac{1}{1 - \frac{C_N}{C_K}}$$

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