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FUNCTIONAL VARIABLE METHOD TO THE CHIRAL NONLINEAR SCHRODINGER EQUATION

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Abstract. In this paper, we study the different types of new soliton solutions to the Chiral nonlinear Schrodinger equation with the aid of the functional variable method. Then, we get some special soliton solutions for Chiral nonlinear Schrodinger equation. The parameters of the soliton envelope are obtained as a function of the dependent model coefficients.

Keywords: Chiral nonlinear Schrodinger equation, Functional variable method, Soliton. solution

AMS Subject Classification: 35A09.

1. Introduction

The nonlinear equations are prevalently used as models to identify numerous physical occurrences and have a very serious role in many natural sciences such as mathematics, mechanics and other fields. Nonlinear evolution equations (NEEs) which describe many physical phenomena are often illustrated by nonlinear partial differential equations. So, the exact solutions of NLPDE are explored in detail in order to understand the physical structure of natural phenomena that are described by such equations. Searching for explicit, exact solutions of NLPDE by many different methods is the main goal of this active research area. Some of these methods, the Riccati Equation method [1], Hirota's bilinear operators [2], Hirota's dependent variable transformation [3], the Jacobi elliptic function expansion [4], the homogeneous balance method [5], the tanh-function expansion [6], first integral method [7,8], the sub-equation method [9], the exp-function method [10], the Backlund transformation, and similarity reduction [11-17] are used to obtain the exact solutions of NLPDE.

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2. Analysis of the method

In order, we describe the functional variable method. Consider a given NLPDE for u (x, t) in the form

$$H\left(u, u_x, u_t, u_{xx}, ...\right) = 0, \tag{1}$$

here H is a polynomial of its arguments. Using the transformation

$$u(x,\ t) = U\left(\xi\right),\ \xi = x \pm ct \tag{2}$$

here c is the wave speed, then we get ordinary differential equation (ODE) like

$$Q\left(P, P_\xi, P_{\xi\xi}, ...\right) = 0, \tag{3}$$

Now make a transformation in which the unknown function P is considered as a functional variable in the form $P_\xi = G\left(P\right)$and another derivatives of P are

$$P_{\xi\xi} = \frac{1}{2}\left(G^2\right)', \tag{4}$$

$$P_{\xi\xi\xi} = \frac{1}{2}\left(G^2\right)''\sqrt{G^2}, \tag{5}$$

$$P_{\xi\xi\xi\xi} = \frac{1}{2}\left[\left(G^2\right)''G^2 + \left(G^2\right)''\left(G^2\right)'\right]. \tag{6}$$

Eq. (1) can be written with respect to P, G and its derivatives upon using the statement of (2) into (1) gives

$$R\left(U, G', G'', G''', ...\right) = 0, \tag{7}$$

by integrating of Eq. (7), Eq. (7) can be written with respect to G, and it is found the appropriate solutions by using Eq. (3) for the investigated problem.

3. FVM to the Chiral nonlinear Schrödinger equation

The Chiral nonlinear Schrödinger equation is given by (see [1-3])

$$i\frac{\partial\Psi}{\partial t} + \varepsilon(\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial x^2}) + i\left(c_1(\Psi\frac{\partial\Psi^*}{\partial x} - \Psi^*\frac{\partial\Psi}{\partial x}) + c_2(\Psi\frac{\partial\Psi^*}{\partial y} - \Psi^*\frac{\partial\Psi}{\partial y})\right)\Psi = 0, \tag{8}$$

here Ψ is the complex function of x, y and t, ε is the coefficient of the dispersion terms. Also, c_1 and c_2 are the coefficients of nonlinear coupling terms.
In this case for solving Eq. (8), it is assumed that the soliton solution to Eq. (8) is given by

$$\Psi(x, y, t) = e^{i\phi(x,y,t)}P(x, y, t), \tag{9}$$

where $P(x, y, t)$is the amplitude portion of the soliton, while the phase portion of the soliton is given by

$$\phi(x, y, t) = \kappa_1 x + \kappa_2 y + \omega t + \theta. \tag{10}$$

by using of conformable fractional derivatives [4-17]. Here in Eq. (10), κ_1 and κ_2 are the frequencies in the x-and y-directions, ω is the soliton frequency while θ is the phase constant. Thus, from Eqs. (9) and (10),

$$i\frac{\partial\Psi}{\partial t} = \left(i\frac{\partial P}{\partial t} - \omega P\right)e^{i\phi}, \tag{11}$$

$$\frac{\partial^2\Psi}{\partial x^2} = \left(\frac{\partial^2 P}{\partial x^2} + 2i\kappa_1\frac{\partial P}{\partial x} - \kappa_1^2 P\right)e^{i\phi}, \tag{12}$$

$$\frac{\partial^2\Psi}{\partial y^2} = \left(\frac{\partial^2 P}{\partial y^2} + 2i\kappa_2\frac{\partial P}{\partial y} - \kappa_2^2 P\right)e^{i\phi}, \tag{13}$$

$$(\Psi\frac{\partial\Psi^*}{\partial x}-\Psi^*\frac{\partial\Psi}{\partial x})\Psi=-2i\kappa_1P^3e^{i\phi}. \tag{14}$$

and

$$(\Psi\frac{\partial\Psi^*}{\partial y}-\Psi^*\frac{\partial\Psi}{\partial y})\Psi=-2i\kappa_2P^3e^{i\phi}. \tag{15}$$

By substituting Eqs. (11)–(15) into Eq. (8) and decomposing into real and imaginary parts yields, respectively,

$$\omega P-\varepsilon\left\{\left(\frac{\partial^2P}{\partial x^2}-\kappa_1^2P\right)+\left(\frac{\partial^2P}{\partial y^2}-\kappa_2^2P\right)\right\}-2(\kappa_1c_1+\kappa_2c_2)P^3=0, \tag{16}$$

$$\frac{\partial P}{\partial t}+2a(\kappa_1\frac{\partial P}{\partial x}+\kappa_2\frac{\partial P}{\partial y})=0. \tag{17}$$

This pair of equations will be analyzed further depending on the type of soliton solution which is fetched. Under the traveling wave transformation

$$P(x,y,t)=U(\xi),\qquad \xi=B_1x+B_2y-\upsilon t \tag{18}$$

we have

$$\varepsilon\left(B_1^2+B_2^2\right)\frac{d^2U}{d\xi^2}-\left(\varepsilon(\kappa_1^2+\kappa_2^2)+\omega\right)U+\left(2(\kappa_1c_1+\kappa_2c_2)\right)U^3=0, \tag{19}$$

$$\left(-\upsilon+2\varepsilon(\kappa_1B_1+\kappa_2B_2)\right)\frac{dU}{d\xi}=0. \tag{20}$$

from Eq. (20), we get

$$\upsilon=2\varepsilon(\kappa_1B_1+\kappa_2B_2).$$

$$\varepsilon\left(B_1^2+B_2^2\right)\frac{d^2U}{d\xi^2}-\left(\varepsilon(\kappa_1^2+\kappa_2^2)+\omega\right)U+\left(2(\kappa_1c_1+\kappa_2c_2)\right)U^3=0, \tag{21}$$

Substituting (4) into Eq. (21) leads to the following equation

$$G\left(P\right)=P'=\sqrt{\frac{\left(\varepsilon(\kappa_1^2+\kappa_2^2)+\omega\right)}{\varepsilon\left(B_1^2+B_2^2\right)}P^2-\frac{(\kappa_1c_1+\kappa_2c_2)}{\varepsilon\left(B_1^2+B_2^2\right)}P^4+h_0} \tag{22}$$

here h_0 is an integration constant. We have solutions of Eq. (22) as following:

Case 1: For $h_0=0$, $\frac{\left(\varepsilon(\kappa_1^2+\kappa_2^2)+\omega\right)}{\varepsilon\left(B_1^2+B_2^2\right)}<0$, and $\frac{(\kappa_1c_1+\kappa_2c_2)}{\varepsilon\left(B_1^2+B_2^2\right)}>0$, we know that Eq. (21) have triangular soliton solutions as following

$$P\left(x,t\right)=\pm\sqrt{\frac{\varepsilon(\kappa_1^2+\kappa_2^2)+\omega}{(\kappa_1c_1+\kappa_2c_2)}}\sec\left[\sqrt{-\frac{\left(\varepsilon(\kappa_1^2+\kappa_2^2)+\omega\right)}{\varepsilon\left(B_1^2+B_2^2\right)}}\left(B_1x+B_2y-\upsilon t\right)\right]$$

And

$$P\left(x,t\right)=\pm\sqrt{\frac{\varepsilon(\kappa_1^2+\kappa_2^2)+\omega}{(\kappa_1c_1+\kappa_2c_2)}}\csc\left[\sqrt{-\frac{\left(\varepsilon(\kappa_1^2+\kappa_2^2)+\omega\right)}{\varepsilon\left(B_1^2+B_2^2\right)}}\left(B_1x+B_2y-\upsilon t\right)\right]$$

Case 2: For $h_0=0$, $\frac{\left(\varepsilon(\kappa_1^2+\kappa_2^2)+\omega\right)}{\varepsilon\left(B_1^2+B_2^2\right)}>0$, and $\frac{(\kappa_1c_1+\kappa_2c_2)}{\varepsilon\left(B_1^2+B_2^2\right)}<0$, we know that Eq. (21) have triangular soliton solutions as following

$$P\left(x,t\right)=\pm\sqrt{\frac{\varepsilon(\kappa_1^2+\kappa_2^2)+\omega}{(\kappa_1c_1+\kappa_2c_2)}}\sec h\left[\sqrt{\frac{\left(\varepsilon(\kappa_1^2+\kappa_2^2)+\omega\right)}{\varepsilon\left(B_1^2+B_2^2\right)}}\left(B_1x+B_2y-\upsilon t\right)\right]$$

And

$$P\left(x,t\right)=\pm\sqrt{\frac{\varepsilon(\kappa_1^2+\kappa_2^2)+\omega}{(\kappa_1c_1+\kappa_2c_2)}}\operatorname{csc}h\left[\sqrt{\frac{\left(\varepsilon(\kappa_1^2+\kappa_2^2)+\omega\right)}{\varepsilon\left(B_1^2+B_2^2\right)}}\left(B_1x+B_2y-\upsilon t\right)\right]$$

Case 3: For $h_0\neq 0$, $\frac{\left(\varepsilon(\kappa_1^2+\kappa_2^2)+\omega\right)}{\varepsilon\left(B_1^2+B_2^2\right)}<0$, and $\frac{(\kappa_1c_1+\kappa_2c_2)}{\varepsilon\left(B_1^2+B_2^2\right)}>0$, we know that Eq. (21) have triangular soliton solutions as following

$$P\left(x,t\right)=\pm\sqrt{\frac{\varepsilon(\kappa_1^2+\kappa_2^2)+\omega}{2(\kappa_1c_1+\kappa_2c_2)}}\tanh\left[\sqrt{-\frac{\left(\varepsilon(\kappa_1^2+\kappa_2^2)+\omega\right)}{2\varepsilon\left(B_1^2+B_2^2\right)}}\left(B_1x+B_2y-\upsilon t\right)\right]$$

And

$$P\left(x,t\right)=\pm\sqrt{\frac{\varepsilon(\kappa_1^2+\kappa_2^2)+\omega}{2(\kappa_1c_1+\kappa_2c_2)}}\coth\left[\sqrt{-\frac{\left(\varepsilon(\kappa_1^2+\kappa_2^2)+\omega\right)}{2\varepsilon\left(B_1^2+B_2^2\right)}}\left(B_1x+B_2y-\upsilon t\right)\right]$$

Case 4: For $h_0\neq 0$, $\frac{\left(\varepsilon(\kappa_1^2+\kappa_2^2)+\omega\right)}{\varepsilon\left(B_1^2+B_2^2\right)}>0$, and $\frac{(\kappa_1c_1+\kappa_2c_2)}{\varepsilon\left(B_1^2+B_2^2\right)}>0$we know that Eq. (21) have triangular soliton solutions as following

$$P\left(x,t\right)=\pm\sqrt{-\frac{\varepsilon(\kappa_1^2+\kappa_2^2)+\omega}{2(\kappa_1c_1+\kappa_2c_2)}}\tan\left[\sqrt{\frac{\left(\varepsilon(\kappa_1^2+\kappa_2^2)+\omega\right)}{2\varepsilon\left(B_1^2+B_2^2\right)}}\left(B_1x+B_2y-\upsilon t\right)\right]$$

And

$$P\left(x,t\right)=\pm\sqrt{-\frac{\varepsilon(\kappa_1^2+\kappa_2^2)+\omega}{2(\kappa_1c_1+\kappa_2c_2)}}\cot\left[\sqrt{\frac{\left(\varepsilon(\kappa_1^2+\kappa_2^2)+\omega\right)}{2\varepsilon\left(B_1^2+B_2^2\right)}}\left(B_1x+B_2y-\upsilon t\right)\right]$$

4. Graphical behavior:

The graphical behavior of the solutions for different values are represented below in the following figures by using computation software Maple (see Figs. 1-3).

Figure 1:

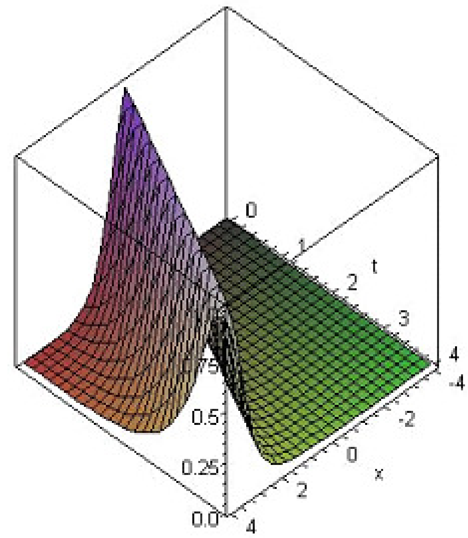


Figure 2:

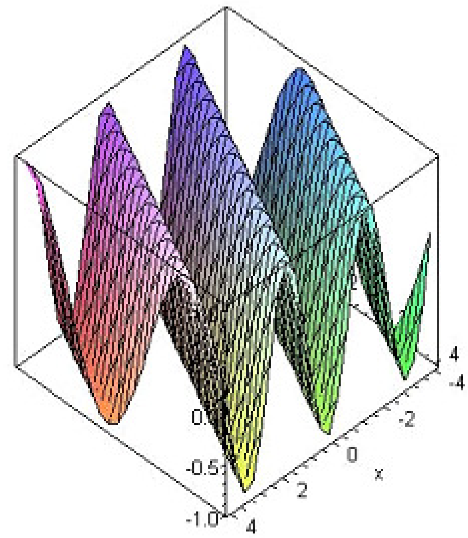
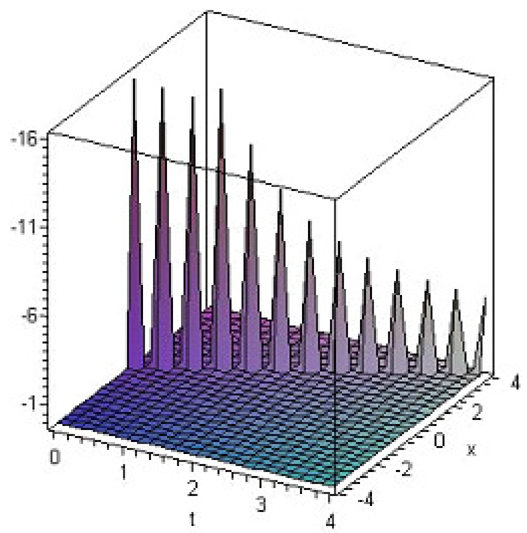


Figure 3:



5. Conclusions

This paper derived new exact soliton solutions of nonlinear Schrodinger equations, namely,the Chiral nonlinear Schrodinger equation which describe the propagation of ultra short pulses in nonlinear optical fibers by using the Functional variable method. We boldly say that the work here is valuable and may be beneficial for studying in other nonlinear science. The exact solutions obtained from the model equations provide important insight into the dynamics of solitary waves. The solutions obtained in this paper have not been reported in the old research.

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