TWMS J. App. and Eng. Math. V.13, N.3, 2023, pp. 1005-1012

TOTAL ABSOLUTE DIFFERENCE EDGE IRREGULARITY STRENGTH OF SOME FAMILIES OF GRAPHS

A. LOURDUSAMY[1*], F. J. BEAULA[1], §

Abstract. A total labeling ξ is defined to be an edge irregular total absolute difference k-labeling of the graph G if for every two different edges e and f of G there is $wt(e) \neq wt(f)$ where weight of an edge $e = xy$ is defined as $wt(e) = |\xi(e) - \xi(x) - \xi(y)|$. The minimum k for which the graph G has an edge irregular total absolute difference labeling is called the total absolute difference edge irregularity strength of the graph G, $tades(G)$. In this paper, we determine the total absolute difference edge irregularity strength of the precise values for some families of graphs.

Keywords: Edge irregularity strength, total absolute difference edge irregularity strength, double fan, quadrilateral snake.

AMS Subject Classification: 05C78.

1. Introduction

Throughout this paper we consider only finite undirected graphs without loops or multiple edges. Chartrand et al. in [2] introduced edge k-labeling of a graph G such that $w(x) \neq w(y)$ for all vertices $x, y \in V(G)$ with $x \neq y$. Such labelings were called irregular assignments and the irregularity strength $s(G)$ of a graph G is known as the minimum k for which G has an irregular assignment using labels at most k. Baca et al. in [1] started to investigate the total edge irregularity strength of a graph, an invariant analogous to the irregularity strength for total labeling. Recently Ivanco and Jendrol [3] proved that for any tree T

$$tes(T) = max\left\{\left\lceil\frac{E(G)+2}{3}\right\rceil, \left\lceil\frac{\Delta(G)+1}{2}\right\rceil\right\}.$$

[1] Department of Mathematics, St. Xavier's College, Palayamkottai, 627002, Tamil Nadu, India.
e-mail: lourdusamy15@gmail.com; ORCID: https://orcid.org/0000-0001-5961-358X.
[*] Corresponding author.
e-mail: joybeaula@gmail.com; ORCID: https://orcid.org/0000-0002-5348-7567.
§ Manuscript received: April 19, 2021; accepted: September 30, 2021.
TWMS Journal of Applied and Engineering Mathematics, Vol.13, No.3

Moreover, they posed a conjecture that for an arbitrary graph G different from K_5 and having maximum degree $\Delta(G)$

$$tes(G) = max\left\{\left\lceil\frac{E(G)+2}{3}\right\rceil, \left\lceil\frac{\Delta(G)+1}{2}\right\rceil\right\}.$$

The Ivanco and Jendrol's conjecture has been verified for complete graphs and complete bipartite graphs in [4] and for categorical product of cycle and path in [6].

Motivated by the total edge irregularity strength of a graph and the graceful labeling, Ramalakshmi and Kathiresan introduced the total absolute difference edge irregularity strength of graphs to reduce the edge weights. For a graph $G = (V(G), E(G))$, the weight of an edge $e = xy$ under a total labeling ξ is $wt(e) = |\xi(e) - \xi(x) - \xi(y)|$. For a graph G we define a labeling $\xi : V(G) \bigcup E(G) \to \{1, 2, \ldots, k\}$ to be an edge irregular total absolute difference k-labeling of G if for every two different edges $e = xy$ and $f = x_0y_0$ of G one has $wt(e) \neq wt(f)$. The total absolute difference edge irregular strength, $tades(G)$, is defined as the minimum k for which G has an edge irregular total absolute difference k-labeling. In [5], they posed the following conjectures,

(1) For every tree T of maximum degree $\Delta(G)$ on p vertices,

$$tades(T) = max\left\{\frac{p}{2}, \frac{\Delta(G)+1}{2}\right\}$$

(2) For any graph G, $tes(G) \leq tades(G)$.

Theorem 1.1. [5] *Let $G = (V, E)$ be a graph with vertex set V and a non-empty edge set E. Then $\frac{|E|}{2} \leq tades(G) \leq |E| + 1$.*

In this paper we discuss with snake related graphs, wheel related graphs, lotus inside the circle and double fan graph. We determine the total absolute difference edge irregular strength for these families of graphs.

The join of two graphs G_1 and G_2 is denoted by $G_1 + G_2$ and whose vertex set is $V(G_1 + G_2) = V(G_1) \bigcup V(G_2)$ and edge set is $E(G_1 + G_2) = E(G_1) \bigcup E(G_2) \bigcup \{uv : u \in V(G_1), v \in V(G_2)\}$. The double fan DF_n is defined as $P_n + 2K_1$. The wheel W_n is defined as the join $C_n + K_1$. The vertex K_1 is the apex vertex and the vertices on the underlying cycle are called rim vertices. The edges of the underlying cycle are called the rim edges and the edges joining the apex and the rim vertices are called spoke edges. The gear graph G_n is obtained from the wheel W_n by adding a vertex between every pair of adjacent vertices of the cycle C_n. The helm H_n is obtained from a wheel W_n by attaching a pendant edge at each vertex of the cycle C_n. The flower graph Fl_n is the graph obtained from a Helm by joining each pendant vertex to the central vertex of the Helm. The closed helm CH_n is a graph obtained from a Helm H_n by joining each pendant vertex to form a cycle. The web Wb_n is the graph obtained by joining the pendant vertices of a helm H_n to form a cycle and then adding a pendant edge to each vertex of outer cycle.

The lotus inside a circle LC_n is a graph obtained from the cycle $C_n : b_1b_2 \cdots b_nb_1$ and the star $K_{1,n}$ with central vertex u and the end vertices $a_1, a_2, a_3, \ldots, a_n$ by joining each b_i to a_i and a_{i+1} $(mod\ n)$.

A K_n-snake is defined as a connected graph in which all blocks are isomorphic to K_n and the block-cut point graph is a path. A K_3-snake is called triangular snake.

The quadrilateral snake is obtained from a path $a_1a_2\cdots a_{n+1}$ by joining a_i, a_{i+1} to new vertices b_i, c_i respectively and joining b_i and c_i.

2. Snake Related Graphs

In this section we discuss the total absolute difference edge irregular strength for snake related graphs.

Theorem 2.1. *For T_n, $n \geq 1$, $tades(T_n) = \lceil \frac{3n}{2} \rceil$.*

Proof. Let T_n be a triangular snake with n blocks. Since $|V(T_n)| = 2n+1$ and $|E(T_n)| = 3n$. Let $k = \lceil \frac{3n}{2} \rceil$. From Theorem (1.1), $tades(T_n) \geq \lceil \frac{3n}{2} \rceil$. It is enough to prove that $tades(T_n) \leq \lceil \frac{3n}{2} \rceil$. Define the labeling ξ as follows:

$\xi(u_1) = 1;$
$\xi(u_{2i}) = 3i-1,\ 1 \leq i \leq \lceil \frac{n}{2} \rceil;$
$\xi(u_{2i+1}) = 3i,\ 1 \leq i \leq \lfloor \frac{n}{2} \rfloor;$
$\xi(v_{2i-1}) = 3i-2,\ 1 \leq i \leq \lceil \frac{n}{2} \rceil;$
$\xi(v_{2i}) = 3i,\ 1 \leq i \leq \lfloor \frac{n}{2} \rfloor;$
$\xi(u_1u_2) = 2;$
$\xi(u_iu_{i+1}) = 1,\ 1 \leq i \leq n;$
$\xi(u_1v_1) = 2;$
$$\xi(u_iv_i) = \begin{cases} 2 & \text{if } i \text{ is even and } 2 \leq i \leq n \\ 1 & \text{if } i \text{ is odd and } 2 \leq i \leq n; \end{cases}$$
$\xi(v_iu_{i+1}) = 1,\ 1 \leq i \leq n.$

Now,
$$max\{\{\xi(u)|u \in V(T_n)\}, \{\xi(e)|e \in E(T_n)\}\} = \lceil \tfrac{3n}{2} \rceil$$
and we observe that,

$wt(u_iv_i) = 3i-3,\ 1 \leq i \leq n;$
$wt(v_iu_{i+1}) = 3i-1,\ 1 \leq i \leq n;$
$wt(u_iu_{i+1}) = 3i-2,\ 1 \leq i \leq n.$

The weights are distinct. Hence $tades(T_n) = \lceil \frac{3n}{2} \rceil$. □

Theorem 2.2. *For Q_n, $n \geq 1$, $tades(Q_n) = 2n$.*

Proof. Let Q_n be a quadrilateral snake with $V(Q_n) = \{a_i|1 \leq i \leq n+1\} \bigcup \{b_i, c_i|1 \leq i \leq n\}$ and $E(Q_n) = \{a_ia_{i+1}, a_ib_i, a_{i+1}c_i, b_ic_i|1 \leq i \leq n\}$. Therefore, $|V(Q_n)| = 3n+1$ and $|E(Q_n)| = 4n$. From Theorem (1.1), $tades(Q_n) \geq 2n$. For the reverse inequality, we define the labeling ξ as follows.

$\xi(a_1) = 1;$
$\xi(a_{2i}) = 4i-2,\ 1 \leq i \leq \lceil \frac{n}{2} \rceil;$
$\xi(a_{2i+1}) = 4i,\ 1 \leq i \leq \lfloor \frac{n}{2} \rfloor;$
$\xi(b_i) = 2i-1,\ 1 \leq i \leq n;$
$\xi(c_i) = 2i,\ 1 \leq i \leq n;$
$\xi(a_1a_2) = 2;$
$\xi(a_ia_{i+1}) = 1,\ 2 \leq i \leq n;$
$\xi(a_1b_1) = 2;$
$\xi(b_ic_i) = 1,\ 1 \leq i \leq n;$
$\xi(a_ib_i) = 1,\ 2 \leq i \leq n;$

$\xi(a_{i+1}c_i) = 1, \ 1 \le i \le n.$

Now,

$$max\{\{\xi(a)|a \in V(Q_n)\}, \{\xi(e)|e \in E(Q_n)\}\} = 2n$$

and we observe that,

$$wt(a_ia_{i+1}) = 4i - 3, \ 1 \le i \le n;$$
$$wt(a_ib_i) = 4i - 4, \ 1 \le i \le n;$$
$$wt(b_ic_i) = 4i - 2, \ 1 \le i \le n;$$
$$wt(a_{i+1}c_i) = 4i - 1, \ 1 \le i \le n.$$

The weights are distinct. Hence $tades(Q_n) = 2n$. □

3. Wheel Related Graphs

In this section we investigate the total absolute difference edge irregular strength for wheel related graphs.

Theorem 3.1. *For $H_n, \ n \ge 3$, $tades(H_n) = \lceil \frac{3n}{2} \rceil$.*

Proof. Let $V(H_n) = \{a, x_i, y_i|1 \le i \le n\}$ and $E(H_n) = \{ax_i, x_iy_i|1 \le i \le n\} \bigcup \{x_ix_{i+1}, x_nx_1|1 \le i \le n-1\}$. Since $|V(H_n)| = 2n+1$ and $|E(H_n)| = 3n$. Let $k = \lceil \frac{3n}{2} \rceil$. By Theorem (1.1), we have $tades(H_n) \ge \lceil \frac{3n}{2} \rceil$. It is enough to prove that $tades(H_n) \le \lceil \frac{3n}{2} \rceil$. Define the labeling $\xi : V \bigcup E \to \{1, 2, 3, \ldots, \lceil \frac{3n}{2} \rceil\}$ as follows:

Case 1. n is odd.

$\xi(a) = k$; $\xi(x_i) = \lfloor \frac{n}{2} \rfloor + i, \ 1 \le i \le n$; $\xi(y_i) = 1, \ 1 \le i \le n$; $\xi(ax_i) = 1, \ 1 \le i \le n$; $\xi(x_ix_{i+1}) = i + 1, \ 1 \le i \le n - 1$; $\xi(x_nx_1) = 1$; $\xi(x_iy_i) = \lfloor \frac{n}{2} \rfloor + 2, \ 1 \le i \le n$.

Case 2. n is even.

$\xi(a) = k$; $\xi(x_i) = \frac{n}{2} + i, \ 1 \le i \le n$; $\xi(y_i) = 1, \ 1 \le i \le n$; $\xi(ax_i) = 1, \ 1 \le i \le n$; $\xi(x_ix_{i+1}) = i + 2, \ 1 \le i \le n - 1$; $\xi(x_nx_1) = 2$; $\xi(x_iy_i) = \frac{n}{2} + 2, \ 1 \le i \le n$.

Now,

$$max\{\{\xi(x)|x \in V(H_n)\}, \{\xi(e)|e \in E(H_n)\}\} = \lceil \tfrac{3n}{2} \rceil$$

and the edge weights are as follows:

$$wt(ax_i) = 2n - 1 + i, \ 1 \le i \le n;$$
$$wt(x_ix_{i+1}) = n + i - 1, \ 1 \le i \le n - 1;$$
$$wt(x_iy_i) = i - 1, \ 1 \le i \le n;$$
$$wt(x_nx_1) = 2n - 1.$$

Hence, the weights are distinct. Therefore, $tades(H_n) = \lceil \frac{3n}{2} \rceil$. □

Theorem 3.2. *For $CH_n, \ n \ge 3$, $tades(CH_n) = 2n$.*

Proof. Let $V(CH_n) = \{a, x_i, y_i|1 \le i \le n\}$ and $E(CH_n) = \{ax_i, x_iy_i|1 \le i \le n\} \bigcup \{x_ix_{i+1}, x_nx_1, y_iy_{i+1}, y_ny_1|1 \le i \le n-1\}$. Define the labeling $\xi : V \bigcup E \to \{1, 2, 3, \ldots, 2n\}$ by

$$\xi(a) = 2n;$$
$$\xi(x_i) = n + i, \ 1 \le i \le n;$$
$$\xi(y_i) = i, \ 1 \le i \le n;$$
$$\xi(ax_i) = 1, \ 1 \le i \le n;$$
$$\xi(x_ix_{i+1}) = i + 2, \ 1 \le i \le n - 1;$$
$$\xi(x_iy_i) = i + 1, \ 1 \le i \le n;$$
$$\xi(y_iy_{i+1}) = i + 2, \ 1 \le i \le n - 1;$$
$$\xi(x_nx_1) = \xi(y_ny_1) = 2.$$

Now,

$$max\{\{\xi(x)|x \in V(CH_n)\}, \{\xi(e)|e \in E(CH_n)\}\} = 2n$$

and we observe that,

$wt(ax_i) = 3n - 1 + i,\ 1 \leq i \leq n;$
$wt(x_i x_{i+1}) = 2n + i - 1,\ 1 \leq i \leq n - 1;$
$wt(x_i y_i) = n + i - 1,\ 1 \leq i \leq n.$
$wt(y_i y_{i+1}) = i - 1,\ 1 \leq i \leq n - 1;$
$wt(x_n x_1) = 3n - 1;$
$wt(y_n y_1) = n - 1.$

The weights are distinct. Then we have $tades(CH_n) \leq 2n$. However by Theorem (1.1), $tades(CH_n) \geq \lceil \frac{4n}{2} \rceil = 2n$, that is $tades(CH_n) \geq 2n$. This completes the proof. □

Theorem 3.3. *For $Wb_n,\ n \geq 3$, $tades(Wb_n) = \lceil \frac{5n}{2} \rceil$.*

Proof. Let $V(Wb_n) = \{a, x_i, y_i, z_i | 1 \leq i \leq n\}$ and $E(Wb_n) = \{ax_i, x_i y_i, y_i z_i | 1 \leq i \leq n\} \bigcup \{x_i x_{i+1}, x_n x_1, y_i y_{i+1}, y_n y_1 | 1 \leq i \leq n - 1\}$. Let $k = \lceil \frac{5n}{2} \rceil$. By Theorem (1.1), we have $tades(Wb_n) \geq \lceil \frac{5n}{2} \rceil$. It is enough to prove that the reverse inequality. We define the function ξ by considering the following two cases.

Case 1. n is odd.

$\xi(a) = k;$
$\xi(x_i) = k - n + i - 1,\ 1 \leq i \leq n;$
$\xi(y_i) = k - 2n + i - 1,\ 1 \leq i \leq n;$
$\xi(z_i) = 1,\ 1 \leq i \leq n;$
$\xi(ax_i) = 1,\ 1 \leq i \leq n;$
$\xi(x_i x_{i+1}) = \xi(y_i y_{i+1}) = i + 1,\ 1 \leq i \leq n - 1;$
$\xi(x_n x_1) = \xi(y_n y_1) = 1;$
$\xi(x_i y_i) = i,\ 1 \leq i \leq n;$
$\xi(y_i z_i) = \lfloor \frac{n}{2} \rfloor + 2,\ 1 \leq i \leq n.$

Case 2. n is even.

$\xi(a) = k;$
$\xi(x_i) = k - n + i,\ 1 \leq i \leq n;$
$\xi(y_i) = k - 2n + i,\ 1 \leq i \leq n;$
$\xi(z_i) = 1,\ 1 \leq i \leq n;$
$\xi(ax_i) = 1,\ 1 \leq i \leq n;$
$\xi(x_i x_{i+1}) = \xi(y_i y_{i+1}) = i + 2,\ 1 \leq i \leq n - 1;$
$\xi(x_n x_1) = \xi(y_n y_1) = 2;$
$\xi(x_i y_i) = i + 1,\ 1 \leq i \leq n;$
$\xi(y_i z_i) = \frac{n}{2} + 2,\ 1 \leq i \leq n.$

Now,

$$max\{\{\xi(x) | x \in V(Wb_n)\}, \{\xi(e) | e \in E(Wb_n)\}\} = \lceil \frac{5n}{2} \rceil$$

and we observe that,

$wt(ax_i) = 4n - 1 + i,\ 1 \leq i \leq n;$
$wt(x_i x_{i+1}) = 3n + i - 1,\ 1 \leq i \leq n - 1;$
$wt(x_i y_i) = 2n + i - 1,\ 1 \leq i \leq n.$
$wt(y_i y_{i+1}) = n + i - 1,\ 1 \leq i \leq n - 1;$
$wt(y_i z_i) = i - 1,\ 1 \leq i \leq n.$
$wt(x_n x_1) = 4n - 1;$
$wt(y_n y_1) = 2n - 1.$

The weights are distinct. Hence $tades(Wb_n) \leq \lceil \frac{5n}{2} \rceil$. □

Theorem 3.4. *For $Fl_n,\ n \geq 3$, $tades(Fl_n) = 2n$.*

Proof. Let $V(Fl_n) = \{a, x_i, y_i | 1 \leq i \leq n\}$ and $E(Fl_n) = \{ax_i, ay_i, x_i y_i | 1 \leq i \leq n\} \bigcup \{x_i x_{i+1}, x_n x_1 | 1 \leq i \leq n - 1\}$. Define the labeling $\xi : V \bigcup E \to \{1, 2, 3, \ldots, 2n\}$ by

$\xi(a) = 2n;$
$\xi(x_i) = i,\ 1 \leq i \leq n;$
$\xi(y_i) = n + i,\ 1 \leq i \leq n;$
$\xi(ax_i) = 1,\ 1 \leq i \leq n;$
$\xi(ay_i) = 1,\ 1 \leq i \leq n;$
$\xi(x_iy_i) = i + 1,\ 1 \leq i \leq n;$
$\xi(x_ix_{i+1}) = i + 2,\ 1 \leq i \leq n - 1;$
$\xi(x_nx_1) = 2.$

Now,

$$max\{\{\xi(x)|x \in V(Fl_n)\}, \{\xi(e)|e \in E(Fl_n)\}\} = 2n$$

and we observe that,

$wt(ax_i) = 2n - 1 + i,\ 1 \leq i \leq n;$
$wt(ay_i) = 3n - 1 + i,\ 1 \leq i \leq n;$
$wt(x_iy_i) = n + i - 1,\ 1 \leq i \leq n.$
$wt(x_ix_{i+1}) = i - 1,\ 1 \leq i \leq n - 1;$
$wt(x_nx_1) = n - 1.$

The weights are distinct. Then we have $tades(Fl_n) \leq 2n$. However by Theorem (1.1), $tades(Fl_n) \geq \lceil \frac{4n}{2} \rceil = 2n$, that is $tades(Fl_n) \geq 2n$. This completes the proof. □

Theorem 3.5. *For $G_n,\ n \geq 3$, $tades(G_n) = \lceil \frac{3n}{2} \rceil$.*

Proof. Let $V(G_n) = \{u, a_i, b_i|1 \leq i \leq n\}$ and $E(G_n) = \{ua_i, a_ib_i|1 \leq i \leq n\} \bigcup \{b_ia_{i+1}, b_na_1|1 \leq i \leq n - 1\}$. Let $k = \lceil \frac{3n}{2} \rceil$. From Theorem (1.1), $tades(G_n) \geq \lceil \frac{3n}{2} \rceil$. It is enough to prove that $tades(G_n) \leq \lceil \frac{3n}{2} \rceil$. Define the labeling $\xi : V \bigcup E \rightarrow \{1, 2, 3, \ldots, \lceil \frac{3n}{2} \rceil\}$ by

Case 1. n is odd.

$\xi(u) = k$; $\xi(a_i) = k-n+i,\ 1 \leq i \leq n$; $\xi(b_i) = k-n+i-2,\ 1 \leq i \leq n$; $\xi(ua_i) = 2,\ 1 \leq i \leq n$; $\xi(a_ib_i) = n + 1,\ 1 \leq i \leq n$; $\xi(b_ia_{i+1}) = n + 1,\ 1 \leq i \leq n - 1$; $\xi(b_na_1) = 1$.

Case 2. n is even.

$\xi(u) = k$; $\xi(a_i) = k-n+i,\ 1 \leq i \leq n$; $\xi(b_i) = k-n+i-1,\ 1 \leq i \leq n$; $\xi(ua_i) = 1,\ 1 \leq i \leq n$; $\xi(a_ib_i) = n + 1,\ 1 \leq i \leq n$; $\xi(b_ia_{i+1}) = n + 1,\ 1 \leq i \leq n - 1$; $\xi(b_na_1) = 1$.

Now,

$$max\{\{\xi(a)|a \in V(G_n)\}, \{\xi(e)|e \in E(G_n)\}\} = \lceil \tfrac{3n}{2} \rceil$$

and we observe that,

$wt(ua_i) = 2n - 1 + i,\ 1 \leq i \leq n;$
$wt(a_ib_i) = 2i - 2,\ 1 \leq i \leq n;$
$wt(b_ia_{i+1}) = 2i - 1,\ 1 \leq i \leq n - 1;$
$wt(b_na_1) = 2n - 1.$

The weights are distinct. Hence $tades(G_n) = \lceil \frac{3n}{2} \rceil$. □

4. Some Families of Graphs

In this section we determine the total absolute difference edge irregular strength for lotus inside the circle and double fan graph.

Theorem 4.1. *For $LC_n,\ n \geq 3$, $tades(LC_n) = 2n$.*

Proof. Let $V(LC_n) = \{u, a_i, b_i : 1 \leq i \leq n\}$ and $E(LC_n) = \{ua_i, a_ib_i|1 \leq i \leq n\} \bigcup \{a_{i+1}b_i, b_ib_{i+1}, a_1b_n, b_nb_1|1 \leq i \leq n - 1\}$. Let $k = 2n$, then from (1.1) it follows that, $tades(LC_n) \geq 2n$. We define a total labeling ξ as follows.

$\xi(u) = 2n;$
$\xi(a_i) = n + i,\ 1 \leq i \leq n;$

$\xi(b_i) = i + 1,\ 1 \le i \le n - 1;$
$\xi(b_n) = 1;$
$\xi(ua_i) = 1,\ 1 \le i \le n;$
$\xi(a_i b_i) = 1,\ 1 \le i \le n - 1;$
$\xi(a_n b_n) = n + 1;$
$\xi(a_{i+1} b_i) = 1,\ 1 \le i \le n - 1;$
$\xi(a_1 b_n) = 1;$
$\xi(b_i b_{i+1}) = i + 3,\ 1 \le i \le n - 2;$
$\xi(b_{n-1} b_n) = 2;$
$\xi(b_n b_1) = 3.$

Now,

$$max\{\{\xi(a)|a \in V(LC_n)\}, \{\xi(e)|e \in E(LC_n)\}\} = 2n$$

and the edge weights are as follows:

$wt(ua_i) = 3n + i - 1,\ 1 \le i \le n;$
$wt(a_i b_i) = n + 2i,\ 1 \le i \le n - 1;$
$wt(a_n b_n) = n;$
$wt(a_{i+1} b_i) = n + 2i + 1,\ 1 \le i \le n - 1;$
$wt(a_1 b_n) = n + 1;$
$wt(b_i b_{i+1}) = i,\ 1 \le i \le n - 1;$
$wt(b_1 b_n) = 0.$

The weights are distinct. Hence $tades(LC_n) = 2n$. □

Theorem 4.2. *For $DF_n,\ n \ge 2,\ tades(DF_n) = \lceil \frac{3n-1}{2} \rceil$.*

Proof. The vertex set of DF_n is $V(DF_n) = \{x_i, a, b|1 \le i \le n\}$ and edge set of DF_n is $E(DF_n) = \{ax_i, bx_i|1 \le i \le n\} \bigcup \{x_i x_{i+1}|1 \le i \le n - 1\}$. Therefore, $|V(DF_n)| = n + 2$ and $|E(DF_n)| = 3n - 1$. By Theorem (1.1), we have $tades(DF_n) \ge \lceil \frac{3n-1}{2} \rceil$. For the reverse inequality, we define the labeling $\xi : V \bigcup E \to \{1, 2, 3, \ldots, \lceil \frac{3n-1}{2} \rceil\}$ by considering the following two cases.

Case 1. n is odd.
$\xi(a) = 1;\ \xi(b) = \lceil \frac{3n-1}{2} \rceil;\ \xi(x_i) = k - n + i,\ 1 \le i \le n;\ \xi(ax_i) = \frac{n+3}{2},\ 1 \le i \le n;$ $\xi(x_i x_{i+1}) = i + 1,\ 1 \le i \le n - 1;\ \xi(bx_i) = 1,\ 1 \le i \le n.$

Case 2. n is even.
$\xi(a) = 1;\ \xi(b) = \lceil \frac{3n-1}{2} \rceil;\ \xi(x_i) = k - n + i - 1,\ 1 \le i \le n;\ \xi(ax_i) = \frac{n}{2} + 1,\ 1 \le i \le n;$ $\xi(x_i x_{i+1}) = i,\ 1 \le i \le n - 1;\ \xi(bx_i) = 1,\ 1 \le i \le n.$

Now,
$max\{\{\xi(x)|x \in V(DF_n)\}, \{\xi(e)|e \in E(DF_n)\}\} = \lceil \frac{3n-1}{2} \rceil$
and the edge weights are as follows:

$wt(ax_i) = i - 1,\ 1 \le i \le n;$
$wt(x_i x_{i+1}) = n + i - 1,\ 1 \le i \le n - 1;$
$wt(bx_i) = 2n + i - 2,\ 1 \le i \le n.$

Hence, the weights are distinct. Therefore, $tades(DF_n) = \lceil \frac{3n-1}{2} \rceil$. □

5. Conclusions

In this paper, we have determined the edge irregular total absolute difference k-labeling for snake related graphs, wheel related graphs, lotus inside the circle and double fan graph. We are further investigating Transformed tree related graphs, super subdivision of graphs, ladder and bistar related graphs admit edge irregular total absolute difference k-labeling.

References

[1] Baca, M., Jendrol, S., Miller M. and Ryan, J., (2007), On irregular total labeling, Discrete Math. 307, pp. 1378-1388.

[2] Chartrand, G., Jacobson, M. S., Lehel, J., Oellermann, O. R., Ruiz, S. and Saba, F., (1988), Irregular networks, Congr. Numer., 64, pp. 187–192.

[3] Ivanco, J. and Jendrol, S., (2006), Total edge irregularity strength for trees, Discuss. Math. Graph Theory 26, pp. 449-456.

[4] Jendrol, S., Miskuf, J. and Sotak, R., (2010), Total edge irregularity strength of complete graphs and complete bipartite graphs, Discrete Math. 310, pp. 400-407.

[5] Ramalakshmi, R. and Kathiresan, KM., (2022), Total absolute difference edge irregularity strength of the graph, Kragujevac Journal of Mathematics, 46 (6), pp. 895-903.

[6] Siddiqui, M. K., (2012), On total edge irregularity strength of categorical product of cycle and path, AKCE Int. J. Graphs Comb. 9 (1), pp. 43–52.

A. Lourdusamy received his M.Sc. from St. Joseph's College, Trichy, India and his Ph.D from Manonmaniam Sundaranar University, Tirunelveli. At present he is an Associate Professor and IQAC Coordinator of St. Xavier's College, Palayamkottai.

F. Joy Beaula graduated from Holy Cross College, Trichy, India. She received her M.Sc. from St. Xavier's College, Palayamkottai. At present, she is pursuing her Ph.D. at Manonmaniam Sundaranar University under the guidance of Dr. A. Lourdusamy. Her area of interest is Graph Labeling and Graph Pebbling.