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NOVEL POSSIBILITY SPHERICAL FUZZY SOFT SET MODEL AND ITS APPLICATION FOR A DECISION MAKING

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Abstract. We talk about possibility spherical fuzzy soft set (shortly PSFS set) has much stronger ability than possibility Pythagorean fuzzy soft set (shortly PPFS set) and intuitionistic fuzzy soft set. The PSFS soft set is a generalization of PPFS set and soft set. Here we talk through some operations consisting of complement, union, intersection, AND and OR. We verify that the De Morgan's laws, associate laws and distributive laws are satisfied in the case of PSFS sets. Also we discuss comparative analysis for the soft set model under the scheme of PSFS sets. Finally, an illustrative example is mentioned for the soft set model using PSFS set.

Keywords: PSFS set, SFS set, decision making problem.

AMS Subject Classification: 03E05, 06D72.

1. Introduction

A fuzzy set is introduced by Zadeh [35] and suggests that decision-makers are to be solving uncertain problems by considering membership degrees. The concept of an intuitionistic fuzzy set is introduced by Atanassov and is characterized by a degree of membership and non-membership satisfying the condition that sum of its membership degree and non-membership degree is not exceeding unity [4]. However, we may interact a problem in decision-making (DM) events where the sum of the degree of membership and non-membership of a particular attribute is exceeding unity. So Yager [33] was introduced by the concept of Pythagorean fuzzy sets and is characterized by the condition that the square sum of its degree of membership and non-membership is not exceeding unity. In 2018, spherical fuzzy sets were introduced by Kahraman and Gundogdu as an extension of Pythagorean, neutrosophic and picture fuzzy sets. Shahzaib Ashraf et al. discussed spherical fuzzy sets which is an advanced tool of the fuzzy sets, intuitionistic fuzzy sets and picture fuzzy sets [3].

In 2018, Garg et al. proposed the algorithm for T-spherical fuzzy multi-attribute

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decision making (MADM) based on improved interactive aggregation operators. Ashraf et al. proposed spherical aggregation operators and applied them in multi-attribute group decision making (MAGDM). Liu et al. extended the generalized Maclaurin symmetric mean (GMSM) operator to the T-spherical fuzzy environment and proposed the T-spherical fuzzy GMSM operator (T-SFGMSM) and the T-spherical fuzzy weighted GMSM operator (T-SFWGMSM). In 2019, Quek et al. developed some new operational laws for T-spherical fuzzy sets, and based on these new operations, proposed two types of Einstein aggregation operators, namely the Einstein interactive averaging aggregation operators and the Einstein interactive geometric aggregation operators under T-spherical fuzzy environment. In 2019, Gundogdu and Kahraman were introduced by spherical fuzzy sets, their operational laws, and the spherical fuzzy TOPSIS method. An extension of WASPAS with spherical fuzzy sets, VIKOR method using spherical fuzzy sets and correlation coefficients were presented by Gundogdu and Kahraman.

Molodtsov [22] proposed the theory of soft sets. Soft sets more accurately reflect the objectivity and complexity of DM during actual situations. Maji et al. proposed the concept of fuzzy soft set [20] and intuitionistic fuzzy soft set [21]. These two theories are applied to solve various DM problems. Alkhazaleh et al [1] defined the concept of possibility fuzzy soft sets. In recent years, Peng et al [24] has extended fuzzy soft set to Pythagorean fuzzy soft set. The soft set model solved a class of MADM consisting sum of the degree of membership and non-membership value is exceeding unity but the sum of the squares is equal or not exceeding unity. In general, the possibility degree of belongingness of the elements should be considered in MADM problems. However, Peng et al [24] failed to do it. The purpose of this paper is to extend the concept of PPFS set to the parameterization of PSFS set using the soft set model. The paper is organized into seven sections as follows. Section 1 is the introduction followed by Section 2 which is preliminaries of possibility fuzzy soft set and spherical fuzzy number. Section 3 presents the possibility spherical fuzzy soft sets of its properties with examples. Section 4 introduces the notion of similarity measure between PSFS sets. Section 5 is the application for the PSFS set. Comparative studies for PSFS set and SFS set in Section 6. Concluding and further investigation is provided in Section 7. Also, insert some numerical examples are given to evaluate the PSFS set.

2. Preliminaries

In this section, we recall and present some fundamental concepts in connection with the spherical fuzzy soft set, which are well known in literature.

Definition 2.1. [32, 33] *Let X be a non-empty set of the universe, Pythagorean fuzzy set A in X is an object having the form : $A = \{x, \mu_A(x), \nu_A(x) | x \in X\}$, where $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and degree of non-membership of A respectively. Consider the mapping $\mu_A : X \to [0,1]$, $\nu_A : X \to [0,1]$ and $0 \leq (\mu_A(x))^2 + (\nu_A(x))^2 \leq 1$. The degree of indeterminacy is determined as $\pi_A(x) = \left[\sqrt{1 - (\mu_A(x))^2 - (\nu_A(x))^2}\right]$. Since $A = \langle \mu_A, \nu_A \rangle$ is called a Pythagorean fuzzy number(PFN).*

Definition 2.2. [19] *Let X be a non-empty set, spherical set A in X is an object having the following form : $\overset{\sqcap}{A} = \{u, \nu_A(x), \omega_A(x), \zeta_A(x) | x \in X\}$, where $\nu_A(x)$, $\omega_A(x)$ $\zeta_A(x)$ represents the degree of positive membership, degree of neutral membership and degree of negative membership of A respectively. The mapping $\nu_A, \omega_A, \zeta_A : X \to [0,1]$ and $0 \leq (\nu_A(x))^2 + (\omega_A(x))^2 + (\zeta_A(x))^2 \leq 1$. The degree of refusal is determined as $r_A(x) =$*

$\left[\sqrt{1-(\nu_A(x))^2-(\omega_A(x))^2-(\zeta_A(x))^2}\right]$. *Since* $\overbracket{A}=\langle \nu_A,\omega_A,\zeta_A\rangle$ *is called a spherical fuzzy number(SFN).*

Definition 2.3. *Let* $\overbracket{\beta_1}=\langle \nu_{\beta_1},\omega_{\beta_1},\zeta_{\beta_1}\rangle$, $\overbracket{\beta_2}=\langle \nu_{\beta_2},\omega_{\beta_2},\zeta_{\beta_2}\rangle$ *and* $\overbracket{\beta_3}=\langle \nu_{\beta_3},\omega_{\beta_3},\zeta_{\beta_3}\rangle$ *are any three SFNs over* (X,E)*, then the following properties are holds:*

(1) $\overbracket{\beta_1^c}=\langle \zeta_{\beta_1},\omega_{\beta_1},\nu_{\beta_1}\rangle$
(2) $\overbracket{\beta_1}\sqcup\overbracket{\beta_2}=\left\langle \max(\nu_{\beta_1},\nu_{\beta_2}),\min(\omega_{\beta_1},\omega_{\beta_2}),\min(\zeta_{\beta_1},\zeta_{\beta_2})\right\rangle$
(3) $\overbracket{\beta_1}\sqcap\overbracket{\beta_2}=\left\langle \min(\nu_{\beta_1},\nu_{\beta_2}),\min(\omega_{\beta_1},\omega_{\beta_2}),\max(\zeta_{\beta_1},\zeta_{\beta_2})\right\rangle$
(4) $\overbracket{\beta_1}\leq\overbracket{\beta_2}$ *if and only if* $\nu_{\beta_1}\leq\nu_{\beta_2}$ *and* $\omega_{\beta_1}\leq\omega_{\beta_2}$ *and* $\zeta_{\beta_1}\geq\zeta_{\beta_2}$
(5) $\overbracket{\beta_1}=\overbracket{\beta_2}$ *if and only if* $\nu_{\beta_1}=\nu_{\beta_2}$ *and* $\omega_{\beta_1}=\omega_{\beta_2}$ *and* $\zeta_{\beta_1}=\zeta_{\beta_2}$.

Definition 2.4. *Let* X *be a non-empty set of the universe and* E *be a set of parameter. The pair* $(\overbracket{\mathcal{F}},A)$ *is called a spherical soft set on* X *if* $A\sqsubseteq E$ *and* $\overbracket{\mathcal{F}}:A\to\overbracket{S\mathcal{F}(X)}$*, where* $\overbracket{S\mathcal{F}(X)}$ *is the set of all spherical subsets of* X.

Definition 2.5. [1] *Let* X *be a non-empty set of the universe and* E *be a set of parameter. The pair* (X,E) *is a soft universe. Consider the mapping* $\mathcal{F}:E\to\mathcal{F}(X)$ *and* ξ *be a fuzzy subset of* E*, ie.* $\xi:E\to\mathcal{F}(X)$. *Let* $\mathcal{F}_\xi:E\to\mathcal{F}(X)\times\mathcal{F}(X)$ *be a function defined as* $\mathcal{F}_\xi(e)=(\mathcal{F}(e)(x),\xi(e)(x)),\forall x\in X$. *Then* $\mathcal{F}_\xi$ *is called a possibility fuzzy soft set (PFS set) on* (X,E).

3. Possibility Spherical Fuzzy Soft Sets

We beginning the concept of possibility spherical fuzzy soft set(PSFS set).

Definition 3.1. *Let* X *be a non-empty set of the universe and* E *be a set of parameter. The pair* (X,E) *is called a soft universe. Suppose that* $\overbracket{\mathcal{F}}:E\to\overbracket{S\mathcal{F}(X)}$ *and* $\overbracket{p}$ *is a spherical subset of* E. *That is* $\overbracket{p}:E\to\overbracket{S\mathcal{F}(X)}$*, where* $\overbracket{S\mathcal{F}(X)}$ *denotes the collection of all spherical subsets of* X. *If* $\overbracket{\mathcal{F}_p}:E\to\overbracket{S\mathcal{F}(X)}\times\overbracket{S\mathcal{F}(X)}$ *is a function defined as* $\overbracket{\mathcal{F}_p}(e)=(\overbracket{\mathcal{F}(e)(x)},\overbracket{p(e)(x)}),x\in X$*, then* $\overbracket{\mathcal{F}_p}$ *is a PSFS set on* (X,E).
For each parameter e, $\overbracket{\mathcal{F}_p}(e)=\left\{\frac{x}{\left\langle(\nu_{\mathcal{F}(e)}(x),\omega_{\mathcal{F}(e)}(x),\zeta_{\mathcal{F}(e)}(x)),(\nu_{p(e)}(x),\omega_{p(e)}(x),\zeta_{p(e)}(x))\right\rangle},\ x\in X\right\}$.

To demonstrate the Definition 3.1, we provide a numerical example as follows:

Example 3.1. *A set of three patient's for cold infection* $X=\{x_1,x_2,x_3\}$ *and a set of parameter* $E=\{e_1=$ *Runny nose,* $e_2=$ *lung infection,* $e_3=$ *cough*$\}$. *Suppose that* $\overbracket{\mathcal{F}_p}:E\to\overbracket{S\mathcal{F}(X)}\times\overbracket{S\mathcal{F}(X)}$ *is given by*

$$\overbracket{\mathcal{F}_p(e_1)}=\left\{\begin{array}{c}\frac{x_1}{\langle(0.50,0.20,0.65),\ (0.65,0.30,0.35)\rangle}\\ \frac{x_2}{\langle(0.65,0.25,0.45),\ (0.55,0.20,0.45)\rangle}\\ \frac{x_3}{\langle(0.45,0.35,0.50),\ (0.40,0.35,0.30)\rangle}\end{array}\right\};\quad \overbracket{\mathcal{F}_p(e_2)}=\left\{\begin{array}{c}\frac{x_1}{\langle(0.45,0.25,0.60),\ (0.65,0.50,0.45)\rangle}\\ \frac{x_2}{\langle(0.55,0.45,0.50),\ (0.50,0.30,0.40)\rangle}\\ \frac{x_3}{\langle(0.65,0.35,0.55),\ (0.55,0.40,0.55)\rangle}\end{array}\right\};$$

$$\overbracket{\mathcal{F}_p(e_3)}=\left\{\begin{array}{c}\frac{x_1}{\langle(0.35,0.45,0.25),\ (0.55,0.25,0.45)\rangle}\\ \frac{x_2}{\langle(0.45,0.55,0.25),\ (0.45,0.35,0.55)\rangle}\\ \frac{x_3}{\langle(0.25,0.35,0.45),\ (0.55,0.45,0.50)\rangle}\end{array}\right\}$$

Definition 3.2. *Let X be a non-empty set of the universe and E be a set of parameter. Suppose that $\overbracket{\mathcal{F}_p}$ and $\overbracket{\mathcal{G}_q}$ are two PSFS sets on (X, E). Now $\overbracket{\mathcal{F}_p}$ is a possibility spherical fuzzy soft subset of $\overbracket{\mathcal{G}_q}$ (denoted by $\overbracket{\mathcal{F}_p} \sqsubseteq \overbracket{\mathcal{G}_q}$) if and only if*

(1) $\overbracket{\mathcal{F}(e)(x)} \sqsubseteq \overbracket{\mathcal{G}(e)(x)}$ *if* $\nu_{\mathcal{F}(e)}(x) \leq \nu_{\mathcal{G}(e)}(x),\ \ \omega_{\mathcal{F}(e)}(x) \leq \omega_{\mathcal{G}(e)}(x),\ \ \zeta_{\mathcal{F}(e)}(x) \geq \zeta_{\mathcal{G}(e)}(x)$,

(2) $\overbracket{p(e)(x)} \sqsubseteq \overbracket{q(e)(x)}$ *if* $\nu_{p(e)}(x) \leq \nu_{q(e)}(x),\ \ \omega_{p(e)}(x) \leq \omega_{q(e)}(x),\ \ \zeta_{p(e)}(x) \geq \zeta_{q(e)}(x)$, $\forall\, e \in E$ *and* $\forall\, x \in X$.

It is easy to verify that these two conditions given in Definition 3.2. To illustrate the above Definition, we provide a numerical example as follows:

Example 3.2. *Consider the PSFS set $\overbracket{\mathcal{F}_p}$ over (X, E) in Example 3.1. Let $\overbracket{\mathcal{G}_q}$ be another PSFS set over (X, E) defined as:*

$$\overbracket{\mathcal{G}_q(e_1)} = \left\{ \begin{array}{c} \frac{x_1}{\langle(0.55,0.25,0.50),\ (0.70,0.35,0.20)\rangle} \\ \frac{x_2}{\langle(0.70,0.35,0.25),\ (0.60,0.30,0.35)\rangle} \\ \frac{x_3}{\langle(0.55,0.45,0.30),\ (0.50,0.40,0.20)\rangle} \end{array} \right\}; \quad \overbracket{\mathcal{G}_q(e_2)} = \left\{ \begin{array}{c} \frac{x_1}{\langle(0.50,0.35,0.40),\ (0.70,0.55,0.35)\rangle} \\ \frac{x_2}{\langle(0.65,0.55,0.30),\ (0.55,0.35,0.30)\rangle} \\ \frac{x_3}{\langle(0.70,0.40,0.45),\ (0.60,0.50,0.30)\rangle} \end{array} \right\};$$

$$\overbracket{\mathcal{G}_q(e_3)} = \left\{ \begin{array}{c} \frac{x_1}{\langle(0.40,0.50,0.20),\ (0.65,0.35,0.25)\rangle} \\ \frac{x_2}{\langle(0.50,0.60,0.20),\ (0.55,0.45,0.25)\rangle} \\ \frac{x_3}{\langle(0.35,0.45,0.40),\ (0.60,0.50,0.35)\rangle} \end{array} \right\}$$

Definition 3.3. *Let X be a non-empty set of the universe and E be a set of parameter. Suppose that $\overbracket{\mathcal{F}_p}$ and $\overbracket{\mathcal{G}_q}$ are two PSFS sets on (X, E). These two PSFS sets are equal (denoted by $\overbracket{\mathcal{F}_p} = \overbracket{\mathcal{G}_q}$) if and only if $\overbracket{\mathcal{F}_p} \sqsubseteq \overbracket{\mathcal{G}_q}$ and $\overbracket{\mathcal{F}_p} \sqsupseteq \overbracket{\mathcal{G}_q}$.*

Definition 3.4. *Let X be a non-empty set of the universe and E be a set of parameter. Let $\overbracket{\mathcal{F}_p}$ be a PSFS set on (X, E). The complement of $\overbracket{\mathcal{F}_p}$ is denoted by $\overbracket{\mathcal{F}_p^c}$ and is defined by $\overbracket{\mathcal{F}_p^c} = \left\langle \overbracket{\mathcal{F}^c(e)(x)}, \overbracket{p^c(e)(x)} \right\rangle$, where $\overbracket{\mathcal{F}^c(e)(x)} = \left\langle \zeta_{\mathcal{F}(e)}(x), \omega_{\mathcal{F}(e)}(x), \nu_{\mathcal{F}(e)}(x) \right\rangle$, $\overbracket{p^c(e)(x)} = \left\langle \zeta_{p(e)}(x), \omega_{p(e)}(x), \nu_{p(e)}(x) \right\rangle$. Also true that $\left(\overbracket{\mathcal{F}_p^c}\right)^c = \overbracket{\mathcal{F}_p}$*

Definition 3.5. *Let X be a non-empty set of the universe and E be a set of parameter. Let $\overbracket{\mathcal{F}_p}$ and $\overbracket{\mathcal{G}_q}$ be two PSFS sets on (X, E). The union and intersection of $\overbracket{\mathcal{F}_p}$ and $\overbracket{\mathcal{G}_q}$ over (X, E) are denoted by $\overbracket{\mathcal{F}_p} \sqcup \overbracket{\mathcal{G}_q}$ and $\overbracket{\mathcal{F}_p} \sqcap \overbracket{\mathcal{G}_q}$ respectively and is defined by $\overbracket{J_j} : E \to \overbracket{\mathcal{SF}(X)} \times \overbracket{\mathcal{SF}(X)}$, $\overbracket{I_i} : E \to \overbracket{\mathcal{SF}(X)} \times \overbracket{\mathcal{SF}(X)}$ such that $\overbracket{J_j}(e)(x) = \left\langle \overbracket{J(e)(x)}, \overbracket{j(e)(x)} \right\rangle$, $\overbracket{I_i(e)(x)} = \left\langle \overbracket{I(e)(x)}, \overbracket{i(e)(x)} \right\rangle$, where $\overbracket{J(e)(x)} = \overbracket{\mathcal{F}(e)(x)} \sqcup \overbracket{\mathcal{G}(e)(x)}$, $\overbracket{j(e)(x)} = \overbracket{p(e)(x)} \sqcup \overbracket{q(e)(x)}$, $\overbracket{I(e)(x)} = \overbracket{\mathcal{F}(e)(x)} \sqcap \overbracket{\mathcal{G}(e)(x)}$ and $\overbracket{i(e)(x)} = \overbracket{p(e)(x)} \sqcap \overbracket{q(e)(x)}$, for all $x \in X$.*

Example 3.3. *Let $\overbracket{\mathcal{F}_p}$ and $\overbracket{\mathcal{G}_q}$ be the two PSFS sets on (X, E) is defined by*

$$\overbracket{\mathcal{F}_p(e_1)} = \left\{ \begin{array}{c} \frac{x_1}{\langle(0.5,0.4,0.6),(0.4,0.3,0.7)\rangle} \\ \frac{x_2}{\langle(0.5,0.6,0.4),(0.6,0.3,0.5)\rangle} \\ \frac{x_3}{\langle(0.7,0.5,0.3),(0.8,0.4,0.3)\rangle} \end{array} \right\}; \quad \overbracket{\mathcal{F}_p(e_2)} = \left\{ \begin{array}{c} \frac{x_1}{\langle(0.6,0.1,0.7),(0.4,0.3,0.6)\rangle} \\ \frac{x_2}{\langle(0.5,0.2,0.6),(0.6,0.1,0.5)\rangle} \\ \frac{x_3}{\langle(0.7,0.3,0.4),(0.5,0.4,0.3)\rangle} \end{array} \right\};$$

$$\overbracket{\mathcal{F}_p(e_3)} = \left\{ \begin{array}{c} \frac{x_1}{\langle(0.3,0.2,0.7),(0.3,0.2,0.8)\rangle} \\ \frac{x_2}{\langle(0.5,0.4,0.6),(0.6,0.3,0.4)\rangle} \\ \frac{x_3}{\langle(0.7,0.5,0.3),(0.4,0.6,0.5)\rangle} \end{array} \right\}$$

and

$$\overline{\mathcal{G}_q(e_1)} = \left\{ \begin{array}{c} \frac{x_1}{\langle(0.4,0.3,0.5),(0.2,0.6,0.7)\rangle} \\ \frac{x_2}{\langle(0.3,0.1,0.8),(0.3,0.7,0.4)\rangle} \\ \frac{x_3}{\langle(0.6,0.4,0.3),(0.4,0.2,0.8)\rangle} \end{array} \right\}; \quad \overline{\mathcal{G}_q(e_2)} = \left\{ \begin{array}{c} \frac{x_1}{\langle(0.2,0.3,0.6),(0.4,0.5,0.6)\rangle} \\ \frac{x_2}{\langle(0.3,0.1,0.6),(0.5,0.2,0.4)\rangle} \\ \frac{x_3}{\langle(0.5,0.4,0.7),(0.7,0.4,0.3)\rangle} \end{array} \right\};$$

$$\overline{\mathcal{G}_q(e_3)} = \left\{ \begin{array}{c} \frac{x_1}{\langle(0.6,0.3,0.5),(0.4,0.3,0.7)\rangle} \\ \frac{x_2}{\langle(0.5,0.3,0.4),(0.6,0.5,0.4)\rangle} \\ \frac{x_3}{\langle(0.4,0.2,0.7),(0.5,0.4,0.6)\rangle} \end{array} \right\}$$

Thus, PSFS set is obtained and is represented by matrix form of $\overline{\mathcal{F}_p} \sqcup \overline{\mathcal{G}_q}$:

$$\begin{bmatrix} \langle(0.5,0.3,0.5),(0.4,0.3,0.7)\rangle & \langle(0.5,0.1,0.4),(0.6,0.3,0.4)\rangle & \langle(0.7,0.4,0.3),(0.8,0.2,0.3)\rangle \\ \langle(0.6,0.1,0.6),(0.4,0.3,0.6)\rangle & \langle(0.5,0.1,0.6),(0.6,0.1,0.4)\rangle & \langle(0.7,0.3,0.4),(0.7,0.4,0.3)\rangle \\ \langle(0.6,0.2,0.5),(0.4,0.2,0.7)\rangle & \langle(0.5,0.3,0.4),(0.6,0.3,0.4)\rangle & \langle(0.7,0.2,0.3),(0.5,0.4,0.5)\rangle \end{bmatrix}$$

Thus, PSFS set is obtained and is represented by matrix form of $\overline{\mathcal{F}_p} \sqcap \overline{\mathcal{G}_q}$:

$$\begin{bmatrix} \langle(0.4,0.3,0.6),(0.2,0.3,0.7)\rangle & \langle(0.3,0.1,0.8),(0.3,0.3,0.5)\rangle & \langle(0.6,0.4,0.3),(0.4,0.2,0.8)\rangle \\ \langle(0.2,0.1,0.7),(0.4,0.3,0.6)\rangle & \langle(0.3,0.1,0.6),(0.5,0.1,0.5)\rangle & \langle(0.5,0.3,0.7),(0.5,0.4,0.3)\rangle \\ \langle(0.3,0.2,0.7),(0.3,0.2,0.8)\rangle & \langle(0.5,0.3,0.6),(0.6,0.3,0.4)\rangle & \langle(0.4,0.2,0.7),(0.4,0.4,0.6)\rangle \end{bmatrix}$$

Definition 3.6. *A PSFS set* $\overline{\emptyset_\theta(e)(x)} = \left\langle \overline{\emptyset(e)(x)}, \overline{\theta(e)(x)} \right\rangle$ *is said to be a possibility null spherical fuzzy soft set* $\overline{\emptyset_\theta} : E \to \overline{S\mathcal{F}(X)} \times \overline{S\mathcal{F}(X)}$, *where* $\overline{\emptyset(e)(x)} = (0,0,1)$ *and* $\overline{\theta(e)(x)} = (0,0,1),\ \forall\, x \in X$.

Definition 3.7. *A PSFS set* $\overline{\Omega_\Lambda(e)(x)} = \left\langle \overline{\Omega(e)(x)}, \overline{\Lambda(e)(x)} \right\rangle$ *is said to be a possibility absolute spherical fuzzy soft set* $\overline{\Omega_\Lambda} : E \to \overline{S\mathcal{F}(X)} \times \overline{S\mathcal{F}(X)}$, *where* $\overline{\Omega(e)(x)} = (1,0,0)$, $\overline{\Lambda(e)(x)} = (1,0,0),\ \forall\, x \in X$.

Theorem 3.1. *Let* $\overline{\mathcal{F}_p}$ *be a PSFS set on* (X, E). *Then the following properties are holds:*

(1) $\overline{\mathcal{F}_p} = \overline{\mathcal{F}_p} \sqcup \overline{\mathcal{F}_p}$, $\overline{\mathcal{F}_p} = \overline{\mathcal{F}_p} \sqcap \overline{\mathcal{F}_p}$
(2) $\overline{\mathcal{F}_p} \sqsubseteq \overline{\mathcal{F}_p} \sqcup \overline{\mathcal{F}_p}$, $\overline{\mathcal{F}_p} \sqsubseteq \overline{\mathcal{F}_p} \sqcap \overline{\mathcal{F}_p}$
(3) $\overline{\mathcal{F}_p} \sqcup \overline{\emptyset_\theta} = \overline{\mathcal{F}_p}$, $\overline{\mathcal{F}_p} \sqcap \overline{\emptyset_\theta} = \overline{\emptyset_\theta}$
(4) $\overline{\mathcal{F}_p} \sqcup \overline{\Omega_\Lambda} = \overline{\Omega_\Lambda}$, $\overline{\mathcal{F}_p} \sqcap \overline{\Omega_\Lambda} = \overline{\mathcal{F}_p}$.

Remark 3.1. *Let* $\overline{\mathcal{F}_p}$ *be a PSFS set on* (X, E). *If* $\overline{\mathcal{F}_p} \neq \overline{\Omega_\Lambda}$ *or* $\overline{\mathcal{F}_p} \neq \overline{\emptyset_\theta}$, *then* $\overline{\mathcal{F}_p} \sqcup \overline{\mathcal{F}_p^c} \neq \overline{\Omega_\Lambda}$ *and* $\overline{\mathcal{F}_p} \sqcap \overline{\mathcal{F}_p^c} \neq \overline{\emptyset_\theta}$.

Theorem 3.2. *Let* $\overline{\mathcal{F}_p}$ *and* $\overline{\mathcal{G}_q}$ *are any two PSFS sets over* (X, E). *Then the commutative and De Morgan's laws are holds:*

(1) $\overline{\mathcal{F}_p} \sqcup \overline{\mathcal{G}_q} = \overline{\mathcal{G}_q} \sqcup \overline{\mathcal{F}_p}$
(2) $\overline{\mathcal{F}_p} \sqcap \overline{\mathcal{G}_q} = \overline{\mathcal{G}_q} \sqcap \overline{\mathcal{F}_p}$
(3) $\left(\overline{\mathcal{F}_p} \sqcup \overline{\mathcal{G}_q}\right)^c = \overline{\mathcal{F}_p^c} \sqcap \overline{\mathcal{G}_q^c}$
(4) $\left(\overline{\mathcal{F}_p} \sqcap \overline{\mathcal{G}_q}\right)^c = \overline{\mathcal{F}_p^c} \sqcup \overline{\mathcal{G}_q^c}$.

Proof. The proof follows from Definition 3.4 and 3.5. $\square$

Theorem 3.3. *Let $\overbracket{\mathcal{F}_p}$, $\overbracket{\mathcal{G}_q}$ and $\overbracket{\mathcal{H}_r}$ are three PSFS sets over (X, E). Then the associative laws and distributive laws are holds:*

(1) $\overbracket{\mathcal{F}_p} \sqcup (\overbracket{\mathcal{G}_q} \sqcup \overbracket{\mathcal{H}_r}) = (\overbracket{\mathcal{F}_p} \sqcup \overbracket{\mathcal{G}_q}) \sqcup \overbracket{\mathcal{H}_r}$.
(2) $\overbracket{\mathcal{F}_p} \sqcap (\overbracket{\mathcal{G}_q} \sqcap \overbracket{\mathcal{H}_r}) = (\overbracket{\mathcal{F}_p} \sqcap \overbracket{\mathcal{G}_q}) \sqcap \overbracket{\mathcal{H}_r}$.
(3) $\overbracket{\mathcal{F}_p} \sqcup (\overbracket{\mathcal{G}_q} \sqcap \overbracket{\mathcal{H}_r}) = (\overbracket{\mathcal{F}_p} \sqcup \overbracket{\mathcal{G}_q}) \sqcap (\overbracket{\mathcal{F}_p} \sqcup \overbracket{\mathcal{H}_r})$.
(4) $\overbracket{\mathcal{F}_p} \sqcap (\overbracket{\mathcal{G}_q} \sqcup \overbracket{\mathcal{H}_r}) = (\overbracket{\mathcal{F}_p} \sqcap \overbracket{\mathcal{G}_q}) \sqcup (\overbracket{\mathcal{F}_p} \sqcap \overbracket{\mathcal{H}_r})$.
(5) $(\overbracket{\mathcal{F}_p} \sqcup \overbracket{\mathcal{G}_q}) \sqcap \overbracket{\mathcal{F}_p} = \overbracket{\mathcal{F}_p}$.
(6) $(\overbracket{\mathcal{F}_p} \sqcap \overbracket{\mathcal{G}_q}) \sqcup \overbracket{\mathcal{F}_p} = \overbracket{\mathcal{F}_p}$.

Proof. The proof follows from Definition 3.4 and 3.5. □

Definition 3.8. *Let $(\overbracket{\mathcal{F}_p}, A)$ and $(\overbracket{\mathcal{G}_q}, B)$ be two PSFS sets on (X, E). Then the operation "$(\overbracket{\mathcal{F}_p}, A)$ AND $(\overbracket{\mathcal{G}_q}, B)$" is denoted by $(\overbracket{\mathcal{F}_p}, A) \wedge (\overbracket{\mathcal{G}_q}, B)$ and is defined by $(\overbracket{\mathcal{F}_p}, A) \wedge (\overbracket{\mathcal{G}_q}, B) = (\overbracket{\mathcal{H}_r}, A \times B)$, where $\overbracket{\mathcal{H}_r(\alpha, \beta)} = \Big\langle \overbracket{\mathcal{H}_\nabla(\alpha, \beta)(x)}, \overbracket{r(\alpha, \beta)(x)} \Big\rangle$ such that $\overbracket{\mathcal{H}(\alpha, \beta)} = \overbracket{\mathcal{F}(\alpha)} \sqcap \overbracket{\mathcal{G}(\beta)}$ and $\overbracket{r(\alpha, \beta)} = \overbracket{p(\alpha)} \sqcap \overbracket{q(\beta)}$, for all $(\alpha, \beta) \in A \times B$.*

Definition 3.9. *Let $(\overbracket{\mathcal{F}_p}, A)$ and $(\overbracket{\mathcal{G}_q}, B)$ be two PSFS sets on (X, E). Then the operation "$(\overbracket{\mathcal{F}_p}, A)$ OR $(\overbracket{\mathcal{G}_q}, B)$" is denoted by $(\overbracket{\mathcal{F}_p}, A) \vee (\overbracket{\mathcal{G}_q}, B)$ and is defined by $(\overbracket{\mathcal{F}_p}, A) \vee (\overbracket{\mathcal{G}_q}, B) = (\overbracket{\mathcal{H}_r}, A \times B)$, where $\overbracket{\mathcal{H}_r(\alpha, \beta)} = \Big\langle \overbracket{\mathcal{H}_\nabla(\alpha, \beta)(x)}, \overbracket{r(\alpha, \beta)(x)} \Big\rangle$ such that $\overbracket{\mathcal{H}(\alpha, \beta)} = \overbracket{\mathcal{F}(\alpha)} \sqcup \overbracket{\mathcal{G}(\beta)}$ and $\overbracket{r(\alpha, \beta)} = \overbracket{p(\alpha)} \sqcup \overbracket{q(\beta)}$, for all $(\alpha, \beta) \in A \times B$.*

Theorem 3.4. *Let $(\overbracket{\mathcal{F}_p}, A)$ and $(\overbracket{\mathcal{G}_q}, B)$ be two PSFS sets on (X, E), then*

(1) $\Big((\overbracket{\mathcal{F}_p}, A) \wedge (\overbracket{\mathcal{G}_q}, B)\Big)^c = \Big(\overbracket{\mathcal{F}_p}, A\Big)^c \vee \Big(\overbracket{\mathcal{G}_q}, B\Big)^c$
(2) $\Big((\overbracket{\mathcal{F}_p}, A) \vee (\overbracket{\mathcal{G}_q}, B)\Big)^c = \Big(\overbracket{\mathcal{F}_p}, A\Big)^c \wedge \Big(\overbracket{\mathcal{G}_q}, B\Big)^c$.

Proof. (i) Suppose that $(\overbracket{\mathcal{F}_p}, A) \wedge (\overbracket{\mathcal{G}_q}, B) = (\overbracket{\mathcal{H}_r}, A \times B)$ and $\Big((\overbracket{\mathcal{F}_p}, A) \wedge (\overbracket{\mathcal{G}_q}, B)\Big)^c = (\overbracket{\mathcal{H}_r^c}, A \times B)$. Now $\overbracket{\mathcal{H}_r^c(\alpha, \beta)} = \Big\langle \overbracket{\mathcal{H}^c(\alpha, \beta)(x)}, \overbracket{r^c(\alpha, \beta)(x)} \Big\rangle$. By Theorem 3.2 and Definition 3.8, $\overbracket{\mathcal{H}^c(\alpha, \beta)} = \Big(\overbracket{\mathcal{F}(\alpha)} \sqcap \overbracket{\mathcal{G}(\beta)}\Big)^c = \overbracket{\mathcal{F}^c(\alpha)} \sqcup \overbracket{\mathcal{G}^c(\beta)}$ and $\overbracket{r^c(\alpha, \beta)} = \Big(\overbracket{p(\alpha)} \sqcap \overbracket{q(\beta)}\Big)^c = \overbracket{p^c(\alpha)} \sqcup \overbracket{q^c(\beta)}$. Also, $\Big(\overbracket{\mathcal{F}_p}, A\Big)^c \vee \Big(\overbracket{\mathcal{G}_q}, B\Big)^c = (\overbracket{\Lambda_o}, A \times B)$, where $\overbracket{\Lambda_o(\alpha, \beta)} = \Big\langle \overbracket{\Lambda(\alpha, \beta)(x)}, \overbracket{o(\alpha, \beta)(x)} \Big\rangle$ such that $\overbracket{\Lambda(\alpha, \beta)} = \overbracket{\mathcal{F}^c(\alpha)} \sqcup \overbracket{\mathcal{G}^c(\beta)}$, $\overbracket{o(\alpha, \beta)} = \overbracket{p^c(\alpha)} \sqcup \overbracket{q^c(\beta)}$ for all $(\alpha, \beta) \in A \times B$. Thus, $\overbracket{\mathcal{H}_r^c} = \overbracket{\Lambda_o}$. Hence $\Big((\overbracket{\mathcal{F}_p}, A) \wedge (\overbracket{\mathcal{G}_q}, B)\Big)^c = \Big(\overbracket{\mathcal{F}_p}, A\Big)^c \vee \Big(\overbracket{\mathcal{G}_q}, B\Big)^c$.
(ii) Suppose $(\overbracket{\mathcal{F}_p}, A) \vee (\overbracket{\mathcal{G}_q}, B) = (\overbracket{\mathcal{H}_r}, A \times B)$ and $\Big((\overbracket{\mathcal{F}_p}, A) \vee (\overbracket{\mathcal{G}_q}, B)\Big)^c = (\overbracket{\mathcal{H}_r^c}, A \times B)$. Now, $\overbracket{\mathcal{H}_r^c(\alpha, \beta)} = \Big\langle \overbracket{\mathcal{H}^c(\alpha, \beta)(x)}, \overbracket{r^c(\alpha, \beta)(x)} \Big\rangle$. By Theorem 3.2 and Definition 3.9, $\overbracket{\mathcal{H}^c(\alpha, \beta)} = \Big(\overbracket{\mathcal{F}(\alpha)} \sqcup \overbracket{\mathcal{G}(\beta)}\Big)^c = \overbracket{\mathcal{F}^c(\alpha)} \sqcap \overbracket{\mathcal{G}^c(\beta)}$ and $\overbracket{r^c(\alpha, \beta)} = \Big(\overbracket{p(\alpha)} \sqcup \overbracket{q(\beta)}\Big)^c = \overbracket{p^c(\alpha)} \sqcap \overbracket{q^c(\beta)}$. Also $\Big(\overbracket{\mathcal{F}_p}, A\Big)^c \wedge \Big(\overbracket{\mathcal{G}_q}, B\Big)^c = (\overbracket{\Lambda_o}, A \times B)$, where $\overbracket{\Lambda_o(\alpha, \beta)} = \Big\langle \overbracket{\Lambda(\alpha, \beta)(x)}, \overbracket{o(\alpha, \beta)(x)} \Big\rangle$ such that

$\overbracket{\Lambda(\alpha,\beta)} = \overbracket{\mathcal{F}^c(\alpha)} \sqcap \overbracket{\mathcal{G}^c(\beta)}$ and $\overbracket{o(\alpha,\beta)} = \overbracket{p^c(\alpha)} \sqcap \overbracket{q^c(\beta)}$ for all $(\alpha,\beta) \in A \times B$. Thus, $\overbracket{\mathcal{H}_r^c} = \overbracket{\Lambda_o}$. Hence $\left((\overbracket{\mathcal{F}_p}, A) \vee (\overbracket{\mathcal{G}_q}, B)\right)^c = \left(\overbracket{\mathcal{F}_p}, A\right)^c \wedge \left(\overbracket{\mathcal{G}_q}, B\right)^c$. □

4. Similarity measure between two PSFS sets

In this section, finding similarity measure between PSFS sets is given below.

Definition 4.1. *Let X be a non-empty set of the universe and E be a set of parameter. Suppose that $\overbracket{\mathcal{F}_p}$ and $\overbracket{\mathcal{G}_q}$ are two PSFS sets on (X,E). The similarity measure between two PSFS sets $\overbracket{\mathcal{F}_p}$ and $\overbracket{\mathcal{G}_q}$ is defined as $Sim(\overbracket{\mathcal{F}_p}, \overbracket{\mathcal{G}_q}) = \varphi(\overbracket{\mathcal{F}}, \overbracket{\mathcal{G}}) \cdot \psi(\overbracket{p}, \overbracket{q})$ such that*

$$\varphi(\overbracket{\mathcal{F}}, \overbracket{\mathcal{G}}) = \frac{T_1\left(\overbracket{\mathcal{F}(e)(x)}, \overbracket{\mathcal{G}(e)(x)}\right) + T_2\left(\overbracket{\mathcal{F}(e)(x)}, \overbracket{\mathcal{G}(e)(x)}\right) + S\left(\overbracket{\mathcal{F}(e)(x)}, \overbracket{\mathcal{G}(e)(x)}\right)}{3} \text{ and}$$

$$\psi(\overbracket{p}, \overbracket{q}) = 1 - \frac{\sum |(\alpha_{1i}+\alpha_{2i}) - (\beta_{1i}+\beta_{2i})|}{\sum |(\alpha_{1i}+\alpha_{2i}) + (\beta_{1i}+\beta_{2i})|},$$

where $T_1\left(\overbracket{\mathcal{F}(e)(x)}, \overbracket{\mathcal{G}(e)(x)}\right) = \frac{\sum_{i=1}^n (\nu_{\mathcal{F}(e_i)}(x) \cdot \nu_{\mathcal{G}(e_i)}(x))}{\sum_{i=1}^n \left(1 - \sqrt{(1-\nu^2_{\mathcal{F}(e_i)}(x)) \cdot (1-\nu^2_{\mathcal{G}(e_i)}(x))}\right)}$,

$T_2\left(\overbracket{\mathcal{F}(e)(x)}, \overbracket{\mathcal{G}(e)(x)}\right) = \frac{\sum_{i=1}^n (\omega^2_{\mathcal{F}(e_i)}(x) \cdot \omega^2_{\mathcal{G}(e_i)}(x))}{\sum_{i=1}^n \left(1 - \sqrt{(1-\omega^4_{\mathcal{F}(e_i)}(x)) \cdot (1-\omega^4_{\mathcal{G}(e_i)}(x))}\right)}$,

$S\left(\overbracket{\mathcal{F}(e)(x)}, \overbracket{\mathcal{G}(e)(x)}\right) = \sqrt{1 - \frac{\sum_{i=1}^n |\zeta^2_{\mathcal{F}(e_i)}(x) - \zeta^2_{\mathcal{G}(e_i)}(x)|}{\sum_{i=1}^n 1 + ((\zeta^2_{\mathcal{F}(e_i)}(x)) \cdot (\zeta^2_{\mathcal{G}(e_i)}(x)))}}$ *and*

$\alpha_{1i} = \frac{\nu^2_{p(e_i)}(x)}{\nu^2_{p(e_i)}(x) + \zeta^2_{p(e_i)}(x)}, \quad \alpha_{2i} = \frac{\nu^2_{p(e_i)}(x)}{\nu^2_{p(e_i)}(x) + \omega^2_{p(e_i)}(x)}$

$\beta_{1i} = \frac{\nu^2_{q(e_i)}(x)}{\nu^2_{q(e_i)}(x) + \zeta^2_{q(e_i)}(x)}, \quad \beta_{2i} = \frac{\nu^2_{q(e_i)}(x)}{\nu^2_{q(e_i)}(x) + \omega^2_{q(e_i)}(x)}$, *where* $1 \leq i \leq n$.

Theorem 4.1. *Let $\overbracket{\mathcal{F}_p}$, $\overbracket{\mathcal{G}_q}$ and $\overbracket{\mathcal{H}_r}$ be the any three PSFS sets over (X,E). Then the following statements are holds:*

1. $Sim(\overbracket{\mathcal{F}_p}, \overbracket{\mathcal{G}_q}) = Sim(\overbracket{\mathcal{G}_q}, \overbracket{\mathcal{F}_p})$
2. $0 \leq Sim(\overbracket{\mathcal{F}_p}, \overbracket{\mathcal{G}_q}) \leq 1$
3. $\overbracket{\mathcal{F}_p} = \overbracket{\mathcal{G}_q} \implies Sim(\overbracket{\mathcal{F}_p}, \overbracket{\mathcal{G}_q}) = 1$
4. $\overbracket{\mathcal{F}_p} \sqsubseteq \overbracket{\mathcal{G}_q} \sqsubseteq \overbracket{\mathcal{H}_r} \implies Sim(\overbracket{\mathcal{F}_p}, \overbracket{\mathcal{H}_r}) \leq Sim(\overbracket{\mathcal{G}_q}, \overbracket{\mathcal{H}_r})$
5. $\overbracket{\mathcal{F}_p} \sqcap \overbracket{\mathcal{G}_q} = \{\phi\} \Leftrightarrow Sim(\overbracket{\mathcal{F}_p}, \overbracket{\mathcal{G}_q}) = 0$.

Proof. The proof (i), (ii) and (v) are trivial. (iii) Suppose that $\overbracket{\mathcal{F}_p} = \overbracket{\mathcal{G}_q}$ implies that $\nu_{\mathcal{F}(e)}(x) = \nu_{\mathcal{G}(e)}(x)$, $\omega_{\mathcal{F}(e)}(x) = \omega_{\mathcal{G}(e)}(x)$, $\zeta_{\mathcal{F}(e)}(x) = \zeta_{\mathcal{G}(e)}(x)$, $\nu_{p(e)}(x) = \nu_{q(e)}(x)$, $\omega_{p(e)}(x) = \omega_{q(e)}(x)$ and $\zeta_{p(e)}(x) = \zeta_{q(e)}(x)$.

Now, $T_1\left(\overbracket{\mathcal{F}(e)(x)}, \overbracket{\mathcal{G}(e)(x)}\right) = \frac{\sum_{i=1}^n \nu^2_{\mathcal{F}(e_i)}(x)}{\sum_{i=1}^n (1-1+\nu^2_{\mathcal{F}(e_i)}(x))} = \frac{\sum_{i=1}^n \nu^2_{\mathcal{F}(e_i)}(x)}{\sum_{i=1}^n \nu^2_{\mathcal{F}(e_i)}(x)} = 1$

and $T_2\left(\overbracket{\mathcal{F}(e)(x)}, \overbracket{\mathcal{G}(e)(x)}\right) = \frac{\sum_{i=1}^n \omega^4_{\mathcal{F}(e_i)}(x)}{\sum_{i=1}^n (1-1+\omega^4_{\mathcal{F}(e_i)}(x))} = \frac{\sum_{i=1}^n \omega^4_{\mathcal{F}(e_i)}(x)}{\sum_{i=1}^n \omega^4_{\mathcal{F}(e_i)}(x)} = 1$

and $S\left(\overbracket{\mathcal{F}(e)(x)}, \overbracket{\mathcal{G}(e)(x)}\right) = \sqrt{(1-0)} = 1$. Thus, $\varphi(\overbracket{\mathcal{F}}, \overbracket{\mathcal{G}}) = \frac{1+1+1}{3} = 1$ and $\psi(\overbracket{p}, \overbracket{q}) = 1$.

Hence $Sim(\overbracket{\mathcal{F}_p}, \overbracket{\mathcal{G}_q}) = 1$.

(iv) Given that

$$\left\{\begin{array}{l} \overbrace{\mathcal{F}_p} \sqsubseteq \overbrace{\mathcal{G}_q} \implies \nu_{\mathcal{F}(e)}(x) \leq \nu_{\mathcal{G}(e)}(x), \quad \omega_{\mathcal{F}(e)}(x) \leq \omega_{\mathcal{G}(e)}(x), \quad \zeta_{\mathcal{F}(e)}(x) \geq \zeta_{\mathcal{G}(e)}(x) \\ \nu_{p(e)}(x) \leq \nu_{q(e)}(x), \quad \omega_{p(e)}(x) \leq \omega_{q(e)}(x), \quad \zeta_{p(e)}(x) \geq \zeta_{q(e)}(x) \\ \overbrace{\mathcal{F}_p} \sqsubseteq \overbrace{\mathcal{H}_r} \implies \nu_{\mathcal{F}(e)}(x) \leq \nu_{\mathcal{H}(e)}(x), \quad \omega_{\mathcal{F}(e)}(x) \leq \omega_{\mathcal{H}(e)}(x), \quad \zeta_{\mathcal{F}(e)}(x) \geq \zeta_{\mathcal{H}(e)}(x) \\ \nu_{p(e)}(x) \leq \nu_{r(e)}(x), \quad \omega_{p(e)}(x) \leq \omega_{r(e)}(x), \quad \zeta_{p(e)}(x) \geq \zeta_{r(e)}(x) \\ \overbrace{\mathcal{G}_q} \sqsubseteq \overbrace{\mathcal{H}_r} \implies \nu_{\mathcal{G}(e)}(x) \leq \nu_{\mathcal{H}(e)}(x), \quad \omega_{\mathcal{G}(e)}(x) \leq \omega_{\mathcal{H}(e)}(x), \quad \zeta_{\mathcal{G}(e)}(x) \geq \zeta_{\mathcal{H}(e)}(x) \\ \nu_{q(e)}(x) \leq \nu_{r(e)}(x), \quad \omega_{q(e)}(x) \leq \omega_{r(e)}(x), \quad \zeta_{q(e)}(x) \geq \zeta_{r(e)}(x) \end{array}\right\} \tag{1}$$

Clearly, $\nu_{\mathcal{F}(e)}(x) \cdot \nu_{\mathcal{H}(e)}(x) \leq \nu_{\mathcal{G}(e)}(x) \cdot \nu_{\mathcal{H}(e)}(x)$ implies that

$$\sum_{i=1}^{n}(\nu_{\mathcal{F}(e_i)}(x) \cdot \nu_{\mathcal{H}(e_i)}(x)) \leq \sum_{i=1}^{n}(\nu_{\mathcal{G}(e_i)}(x) \cdot \nu_{\mathcal{H}(e_i)}(x)) \tag{2}$$

Clearly, $\nu^2_{\mathcal{F}(e)}(x) \leq \nu^2_{\mathcal{G}(e)}(x)$ implies that $-\nu^2_{\mathcal{F}(e)}(x) \geq -\nu^2_{\mathcal{G}(e)}(x)$ and

$$(1-(\nu^2_{\mathcal{F}(e)}(x))) \cdot (1-(\nu^2_{\mathcal{H}(e)}(x))) \geq (1-(\nu^2_{\mathcal{G}(e)}(x))) \cdot (1-(\nu^2_{\mathcal{H}(e)}(x))) \text{ and}$$

$$\sqrt{(1-(\nu^2_{\mathcal{F}(e)}(x))) \cdot (1-(\nu^2_{\mathcal{H}(e)}(x)))} \geq \sqrt{(1-(\nu^2_{\mathcal{G}(e)}(x))) \cdot (1-(\nu^2_{\mathcal{H}(e)}(x)))} \text{ and}$$

$$1-\sqrt{(1-(\nu^2_{\mathcal{F}(e)}(x))) \cdot (1-(\nu^2_{\mathcal{H}(e)}(x)))} \leq 1-\sqrt{(1-(\nu^2_{\mathcal{G}(e)}(x))) \cdot (1-(\nu^2_{\mathcal{H}(e)}(x)))} \text{ and}$$

$$\sum_{i=1}^{n} 1-\sqrt{(1-(\nu^2_{\mathcal{F}(e_i)}(x))) \cdot (1-(\nu^2_{\mathcal{H}(e_i)}(x)))} \leq \sum_{i=1}^{n} 1-\sqrt{(1-(\nu^2_{\mathcal{G}(e_i)}(x))) \cdot (1-(\nu^2_{\mathcal{H}(e_i)}(x)))} \tag{3}$$

Equation (2) is divided by (3),

$$\frac{\sum_{i=1}^{n}(\nu_{\mathcal{F}(e_i)}(x) \cdot \nu_{\mathcal{H}(e_i)}(x))}{\sum_{i=1}^{n} 1-\sqrt{(1-(\nu^2_{\mathcal{F}(e_i)}(x))) \cdot (1-(\nu^2_{\mathcal{H}(e_i)}(x)))}} \leq \frac{\sum_{i=1}^{n}(\nu_{\mathcal{G}(e_i)}(x) \cdot \nu_{\mathcal{H}(e_i)}(x))}{\sum_{i=1}^{n} 1-\sqrt{(1-(\nu^2_{\mathcal{G}(e_i)}(x))) \cdot (1-(\nu^2_{\mathcal{H}(e_i)}(x)))}} \tag{4}$$

Clearly, $\omega^2_{\mathcal{F}(e)}(x) \cdot \omega^2_{\mathcal{H}(e)}(x) \leq \omega^2_{\mathcal{G}(e)}(x) \cdot \omega^2_{\mathcal{H}(e)}(x)$ implies that

$$\sum_{i=1}^{n}(\omega^2_{\mathcal{F}(e_i)}(x) \cdot \omega^2_{\mathcal{H}(e_i)}(x)) \leq \sum_{i=1}^{n}(\omega^2_{\mathcal{G}(e_i)}(x) \cdot \omega^2_{\mathcal{H}(e_i)}(x)) \tag{5}$$

Clearly, $\omega^4_{\mathcal{F}(e)}(x) \leq \omega^4_{\mathcal{G}(e)}(x)$ implies that $-\omega^4_{\mathcal{F}(e)}(x) \geq -\omega^4_{\mathcal{G}(e)}(x)$ and

$$(1-(\omega^4_{\mathcal{F}(e)}(x))) \cdot (1-(\omega^4_{\mathcal{H}(e)}(x))) \geq (1-(\omega^4_{\mathcal{G}(e)}(x))) \cdot (1-(\omega^4_{\mathcal{H}(e)}(x))) \text{ and}$$

$$\sqrt{(1-(\omega^4_{\mathcal{F}(e)}(x))) \cdot (1-(\omega^4_{\mathcal{H}(e)}(x)))} \geq \sqrt{(1-(\omega^4_{\mathcal{G}(e)}(x))) \cdot (1-(\omega^4_{\mathcal{H}(e)}(x)))} \text{ and}$$

$$1-\sqrt{(1-(\omega^4_{\mathcal{F}(e)}(x))) \cdot (1-(\omega^4_{\mathcal{H}(e)}(x)))} \leq 1-\sqrt{(1-(\omega^4_{\mathcal{G}(e)}(x))) \cdot (1-(\omega^4_{\mathcal{H}(e)}(x)))} \text{ and}$$

$$\sum_{i=1}^{n} 1-\sqrt{(1-(\omega^4_{\mathcal{F}(e_i)}(x))) \cdot (1-(\omega^4_{\mathcal{H}(e_i)}(x)))} \leq \sum_{i=1}^{n} 1-\sqrt{(1-(\omega^4_{\mathcal{G}(e_i)}(x))) \cdot (1-(\omega^4_{\mathcal{H}(e_i)}(x)))} \tag{6}$$

Equation (5) is divided by (6),

$$\frac{\sum_{i=1}^{n}(\omega^2_{\mathcal{F}(e_i)}(x) \cdot \omega^2_{\mathcal{H}(e_i)}(x))}{\sum_{i=1}^{n} 1-\sqrt{(1-(\omega^4_{\mathcal{F}(e_i)}(x))) \cdot (1-(\omega^4_{\mathcal{H}(e_i)}(x)))}} \leq \frac{\sum_{i=1}^{n}(\omega^2_{\mathcal{G}(e_i)}(x) \cdot \omega^2_{\mathcal{H}(e_i)}(x))}{\sum_{i=1}^{n} 1-\sqrt{(1-(\omega^4_{\mathcal{G}(e_i)}(x))) \cdot (1-(\omega^4_{\mathcal{H}(e_i)}(x)))}} \tag{7}$$

Clearly, $\zeta^2_{\mathcal{F}(e)}(x) \geq \zeta^2_{\mathcal{G}(e)}(x)$ and $\zeta^2_{\mathcal{F}(e)}(x) - \zeta^2_{\mathcal{H}(e)}(x) \geq \zeta^2_{\mathcal{G}(e)}(x) - \zeta^2_{\mathcal{H}(e)}(x)$.
Hence

$$\sum_{i=1}^{n}\left|\zeta^2_{\mathcal{F}(e_i)}(x) - \zeta^2_{\mathcal{H}(e_i)}(x)\right| \geq \sum_{i=1}^{n}\left|\zeta^2_{\mathcal{G}(e_i)}(x) - \zeta^2_{\mathcal{H}(e_i)}(x)\right| \tag{8}$$

Also, $\zeta^2_{\mathcal{F}(e)}(x)\cdot\zeta^2_{\mathcal{H}(e)}(x)\geq\zeta^2_{\mathcal{G}(e)}(x)\cdot\zeta^2_{\mathcal{H}(e)}(x)$ implies that

$$\sum_{i=1}^{n}1+(\zeta^2_{\mathcal{F}(e_i)}(x)\cdot\zeta^2_{\mathcal{H}(e_i)}(x))\geq\sum_{i=1}^{n}1+(\zeta^2_{\mathcal{G}(e_i)}(x)\cdot\zeta^2_{\mathcal{H}(e_i)}(x)) \tag{9}$$

Equation (8) is divided by (9), we get

$$\frac{\sum_{i=1}^{n}|\zeta^2_{\mathcal{F}(e_i)}(x)-\zeta^2_{\mathcal{H}(e_i)}(x)|}{\sum_{i=1}^{n}1+(\zeta^2_{\mathcal{F}(e_i)}(x)\cdot\zeta^2_{\mathcal{H}(e_i)}(x))}\geq\frac{\sum_{i=1}^{n}|\zeta^2_{\mathcal{G}(e_i)}(x)-\zeta^2_{\mathcal{H}(e_i)}(x)|}{\sum_{i=1}^{n}1+(\zeta^2_{\mathcal{G}(e_i)}(x)\cdot\zeta^2_{\mathcal{H}(e_i)}(x))} \text{ and}$$

$$1-\frac{\sum_{i=1}^{n}|\zeta^2_{\mathcal{F}(e_i)}(x)-\zeta^2_{\mathcal{H}(e_i)}(x)|}{\sum_{i=1}^{n}1+(\zeta^2_{\mathcal{F}(e_i)}(x)\cdot\zeta^2_{\mathcal{H}(e_i)}(x))}\leq 1-\frac{\sum_{i=1}^{n}|\zeta^2_{\mathcal{G}(e_i)}(x)-\zeta^2_{\mathcal{H}(e_i)}(x)|}{\sum_{i=1}^{n}1+(\zeta^2_{\mathcal{G}(e_i)}(x)\cdot\zeta^2_{\mathcal{H}(e_i)}(x))} \text{ and}$$

$$\sqrt{1-\frac{\sum_{i=1}^{n}|\zeta^2_{\mathcal{F}(e_i)}(x)-\zeta^2_{\mathcal{H}(e_i)}(x)|}{\sum_{i=1}^{n}1+(\zeta^2_{\mathcal{F}(e_i)}(x)\cdot\zeta^2_{\mathcal{H}(e_i)}(x))}}\leq\sqrt{1-\frac{\sum_{i=1}^{n}|\zeta^2_{\mathcal{G}(e_i)}(x)-\zeta^2_{\mathcal{H}(e_i)}(x)|}{\sum_{i=1}^{n}1+(\zeta^2_{\mathcal{G}(e_i)}(x)\cdot\zeta^2_{\mathcal{H}(e_i)}(x))}} \tag{10}$$

Adding Equations (4), (7), (10) and divided by 3,

$$\varphi(\overline{\mathcal{F}},\overline{\mathcal{H}})\leq\varphi(\overline{\mathcal{G}},\overline{\mathcal{H}}) \tag{11}$$

By Equation (1), Clearly $\alpha_{1i}\leq\beta_{1i}\leq\gamma_{1i}$ and $\alpha_{2i}\leq\beta_{2i}\leq\gamma_{2i}$, where
$\alpha_{1i}=\frac{\nu^2_{p(e_i)}(x)}{\nu^2_{p(e_i)}(x)+\zeta^2_{p(e_i)}(x)},\quad \alpha_{2i}=\frac{\nu^2_{p(e_i)}(x)}{\nu^2_{p(e_i)}(x)+\omega^2_{p(e_i)}(x)}$
$\beta_{1i}=\frac{\nu^2_{q(e_i)}(x)}{\nu^2_{q(e_i)}(x)+\zeta^2_{q(e_i)}(x)},\quad \beta_{2i}=\frac{\nu^2_{q(e_i)}(x)}{\nu^2_{q(e_i)}(x)+\omega^2_{q(e_i)}(x)}.$
$\gamma_{1i}=\frac{\nu^2_{r(e_i)}(x)}{\nu^2_{r(e_i)}(x)+\zeta^2_{r(e_i)}(x)},\quad \gamma_{2i}=\frac{\nu^2_{r(e_i)}(x)}{\nu^2_{r(e_i)}(x)+\omega^2_{r(e_i)}(x)}.$
Clearly, $(\alpha_{1i}+\alpha_{2i})\leq(\beta_{1i}+\beta_{2i})\leq(\gamma_{1i}+\gamma_{2i})$ and $(\alpha_{1i}+\alpha_{2i})-(\gamma_{1i}+\gamma_{2i})\leq(\beta_{1i}+\beta_{2i})$ $-(\gamma_{1i}+\gamma_{2i})$. Since $(\alpha_{1i}+\alpha_{2i}),(\beta_{1i}+\beta_{2i}),(\gamma_{1i}+\gamma_{2i})$ are numerical values.
Hence $\Big|(\beta_{1i}+\beta_{2i})-(\gamma_{1i}+\gamma_{2i})\Big|\leq\Big|(\alpha_{1i}+\alpha_{2i})-(\gamma_{1i}+\gamma_{2i})\Big|$ and

$$-\Big|(\alpha_{1i}+\alpha_{2i})-(\gamma_{1i}+\gamma_{2i})\Big|\leq-\Big|(\beta_{1i}+\beta_{2i})-(\gamma_{1i}+\gamma_{2i})\Big| \text{ and} \tag{12}$$

$$\Big|(\alpha_{1i}+\alpha_{2i})+(\gamma_{1i}+\gamma_{2i})\Big|\leq\Big|(\beta_{1i}+\beta_{2i})+(\gamma_{1i}+\gamma_{2i})\Big| \tag{13}$$

Equation (12) is divided by (13), we get

$$-\frac{\Big|(\alpha_{1i}+\alpha_{2i})-(\gamma_{1i}+\gamma_{2i})\Big|}{\Big|(\alpha_{1i}+\alpha_{2i})+(\gamma_{1i}+\gamma_{2i})\Big|}\leq-\frac{\Big|(\beta_{1i}+\beta_{2i})-(\gamma_{1i}+\gamma_{2i})\Big|}{\Big|(\beta_{1i}+\beta_{2i})+(\gamma_{1i}+\gamma_{2i})\Big|} \text{ and}$$

$$1-\frac{\Big|(\alpha_{1i}+\alpha_{2i})-(\gamma_{1i}+\gamma_{2i})\Big|}{\Big|(\alpha_{1i}+\alpha_{2i})+(\gamma_{1i}+\gamma_{2i})\Big|}\leq 1-\frac{\Big|(\beta_{1i}+\beta_{2i})-(\gamma_{1i}+\gamma_{2i})\Big|}{\Big|(\beta_{1i}+\beta_{2i})+(\gamma_{1i}+\gamma_{2i})\Big|}.$$

Hence

$$\psi(\overline{p},\overline{r})\leq\psi(\overline{q},\overline{r}) \tag{14}$$

Multiply by Equations (11) and (14), $\varphi(\overline{\mathcal{F}},\overline{\mathcal{H}})\cdot\psi(\overline{p},\overline{r})\leq\varphi(\overline{\mathcal{G}},\overline{\mathcal{H}})\cdot\psi(\overline{q},\overline{r})$.
Hence $Sim(\overline{\mathcal{F}_p},\overline{\mathcal{H}_r})\leq Sim(\overline{\mathcal{G}_q},\overline{\mathcal{H}_r})$. This proves (iv). □

Example 4.1. *Calculate the similarity measure between the two PSFS sets namely* $\overline{\mathcal{F}_p}$ *and* $\overline{\mathcal{G}_q}$. *We choose the first sample of* $\overline{\mathcal{F}_p}$ *and* $\overline{\mathcal{G}_q}$, $E=\{e_1,e_2,e_3,e_4\}$ *can be defined as below:*

$\overbracket{\mathcal{F}_p(e)}$	e_1	e_2	e_3	e_4
$\overbracket{\mathcal{F}(e)}$	$\langle 0.65, 0.25, 0.45\rangle$	$\langle 0.55, 0.35, 0.45\rangle$	$\langle 0.75, 0.35, 0.25\rangle$	$\langle 0.35, 0.25, 0.55\rangle$
$\overbracket{p(e)}$	$\langle 0.45, 0.25, 0.45\rangle$	$\langle 0.35, 0.25, 0.55\rangle$	$\langle 0.65, 0.15, 0.25\rangle$	$\langle 0.35, 0.25, 0.45\rangle$

$\overbracket{\mathcal{G}_q(e)}$	e_1	e_2	e_3	e_4
$\overbracket{\mathcal{G}(e)}$	$\langle 0.55, 0.25, 0.35\rangle$	$\langle 0.45, 0.15, 0.55\rangle$	$\langle 0.65, 0.25, 0.45\rangle$	$\langle 0.25, 0.35, 0.55\rangle$
$\overbracket{q(e)}$	$\langle 0.35, 0.35, 0.55\rangle$	$\langle 0.35, 0.25, 0.45\rangle$	$\langle 0.65, 0.15, 0.55\rangle$	$\langle 0.55, 0.25, 0.45\rangle$

Using Definition 4.1 and routine calculation, we get

$$T_1\left(\overbracket{\mathcal{F}(e)(x)}, \overbracket{\mathcal{G}(e)(x)}\right) = \frac{0.3575+0.2475+0.4875+0.0875}{1.209851} = \frac{1.18}{1.209851} = 0.975327.$$

$$T_2\left(\overbracket{\mathcal{F}(e)(x)}, \overbracket{\mathcal{G}(e)(x)}\right) = \frac{0.00390625+0.00275625+0.00765625+0.00765625}{0.030633} = \frac{0.021975}{0.030633} = 0.717364.$$

$$S\left(\overbracket{\mathcal{F}(e)(x)}, \overbracket{\mathcal{G}(e)(x)}\right) = \sqrt{1-\frac{0.32}{4.190225}} = 0.961058.$$

$$\varphi(\overbracket{\mathcal{F}}, \overbracket{\mathcal{G}}) = \frac{0.975327+0.717364+0.961058}{3} = 0.884583.$$

$$\psi(\overbracket{p}, \overbracket{q}) = 1-\frac{1.2416706}{9.861500} = 0.874089.$$

$$Sim(\overbracket{\mathcal{F}_p}, \overbracket{\mathcal{G}_q}) = 0.884583 \times 0.874089 = 0.773204.$$

5. Similarity Measure for Parental Choice of Colleges

In the selection of college teaching education, the evaluation of teacher education is carried out according to various standards of experts. There are various studies, primarily conducted that have investigated the reasons why parents select a college. Which they think best suits their college student's needs and parental aspirations for their college student. We identify a factor regarded as parental decision making: Academic Factor - divided into five identified elements namely Campus environment, overall cost, academic quality, student/faculty relationship and career opportunities. Our goal is to select the optimal one out of a great number of alternatives based on the assessment of experts against the criteria.

5.1. **Algorithms based on the similarity measures for PSFS set Model.** An algorithm for decision making problems using PSFS set model is explained. The algorithm for the selection of the best choice is given as:

Step 1. Input PSFS set in tabular form.
Step 2. Form the set of choice parameters $A \subseteq E$.
Step 3. Compute the values of T_1, T_2 and S.
Step 4. Calculate the φ value by taking $\frac{T_1+T_2+S}{3}$.
Step 5. Determine the value $\psi(\overbracket{p}, \overbracket{q}) = 1-\frac{\sum|(\alpha_{1i}+\alpha_{2i})-(\beta_{1i}+\beta_{2i})|}{\sum|(\alpha_{1i}+\alpha_{2i})+(\beta_{1i}+\beta_{2i})|}$ and $1 \leq i \leq 5$.
Step 6. Compute the similarity measure $= \varphi \cdot \psi$.
Step 7. Determine maximum similarity $= \max\{similarity^i\}$ and $1 \leq i \leq 5$.
Step 8. To choose the best solution.
Step 9. End.

5.2. **Survey study.** A parent intends to choose the popular college education. Here we intends to choose five colleges are nominated. The score of the college education evaluated by the experts is represented by $E = \{e_1$: campus environment, e_2: overall cost, e_3 : academic quality, e_4 : student/faculty relationship, e_5: career opportunities$\}$.

Table 1

PSFS set for the ideal college education property

$\overline{\mathcal{L}_{p(e)}}$	e_1	e_2	e_3	e_4	e_5
$\overline{\mathcal{L}(e)}$	$\langle 0.75, 0.3, 0.4\rangle$	$\langle 0.65, 0.35, 0.45\rangle$	$\langle 0.55, 0.45, 0.5\rangle$	$\langle 0.6, 0.35, 0.5\rangle$	$\langle 0.6, 0.45, 0.55\rangle$
$\overline{p(e)}$	$\langle 1, 0, 0\rangle$	$\langle 1, 0, 0\rangle$	$\langle 1, 0, 0\rangle$	$\langle 1, 0, 0\rangle$	$\langle 1, 0, 0\rangle$

Table 2

PSFS set for the first college education property

$\overline{\mathcal{A}_{p_1(e)}}$	e_1	e_2	e_3	e_4	e_5
$\overline{\mathcal{A}(e)}$	$\langle 0.55, 0.1, 0.45\rangle$	$\langle 0.4, 0.15, 0.5\rangle$	$\langle 0.3, 0.2, 0.55\rangle$	$\langle 0.35, 0.15, 0.6\rangle$	$\langle 0.5, 0.1, 0.6\rangle$
$\overline{p_1(e)}$	$\langle 0.45, 0.1, 0.55\rangle$	$\langle 0.5, 0.25, 0.4\rangle$	$\langle 0.6, 0.25, 0.45\rangle$	$\langle 0.65, 0.1, 0.35\rangle$	$\langle 0.65, 0.2, 0.45\rangle$

Table 3

PSFS set for the second college education property

$\overline{\mathcal{B}_{p_2(e)}}$	e_1	e_2	e_3	e_4	e_5
$\overline{\mathcal{B}(e)}$	$\langle 0.5, 0.1, 0.5\rangle$	$\langle 0.5, 0.15, 0.65\rangle$	$\langle 0.55, 0.2, 0.6\rangle$	$\langle 0.3, 0.15, 0.65\rangle$	$\langle 0.45, 0.1, 0.75\rangle$
$\overline{p_2(e)}$	$\langle 0.35, 0.2, 0.55\rangle$	$\langle 0.45, 0.35, 0.25\rangle$	$\langle 0.55, 0.15, 0.45\rangle$	$\langle 0.4, 0.35, 0.65\rangle$	$\langle 0.5, 0.2, 0.45\rangle$

Table 4

PSFS set for the third college education property

$\overline{\mathcal{C}_{p_3(e)}}$	e_1	e_2	e_3	e_4	e_5
$\overline{\mathcal{C}(e)}$	$\langle 0.6, 0.1, 0.45\rangle$	$\langle 0.45, 0.15, 0.55\rangle$	$\langle 0.3, 0.05, 0.75\rangle$	$\langle 0.35, 0.15, 0.65\rangle$	$\langle 0.25, 0.1, 0.7\rangle$
$\overline{p_3(e)}$	$\langle 0.65, 0.15, 0.4\rangle$	$\langle 0.15, 0.3, 0.6\rangle$	$\langle 0.35, 0.25, 0.15\rangle$	$\langle 0.5, 0.35, 0.75\rangle$	$\langle 0.5, 0.25, 0.65\rangle$

Table 5

PSFS set for the fourth college education property

$\overline{\mathcal{D}_{p_4(e)}}$	e_1	e_2	e_3	e_4	e_5
$\overline{\mathcal{D}(e)}$	$\langle 0.3, 0.1, 0.45\rangle$	$\langle 0.45, 0.05, 0.65\rangle$	$\langle 0.3, 0.2, 0.5\rangle$	$\langle 0.4, 0.15, 0.55\rangle$	$\langle 0.25, 0.1, 0.7\rangle$
$\overline{p_4(e)}$	$\langle 0.5, 0.2, 0.4\rangle$	$\langle 0.65, 0.35, 0.2\rangle$	$\langle 0.35, 0.5, 0.45\rangle$	$\langle 0.55, 0.2, 0.5\rangle$	$\langle 0.45, 0.2, 0.6\rangle$

Table 6

PSFS set for the fifth college education property

$\overline{\mathcal{E}_{p_5(e)}}$	e_1	e_2	e_3	e_4	e_5
$\overline{\mathcal{E}(e)}$	$\langle 0.6, 0.25, 0.45\rangle$	$\langle 0.45, 0.1, 0.6\rangle$	$\langle 0.3, 0.25, 0.65\rangle$	$\langle 0.45, 0.3, 0.55\rangle$	$\langle 0.5, 0.1, 0.6\rangle$
$\overline{p_5(e)}$	$\langle 0.65, 0.15, 0.5\rangle$	$\langle 0.55, 0.3, 0.25\rangle$	$\langle 0.45, 0.15, 0.75\rangle$	$\langle 0.55, 0.45, 0.4\rangle$	$\langle 0.45, 0.35, 0.55\rangle$

The SFNs values in Tables 2-6 are provided by the experts, depending on their assessment of the alternatives against the criteria under consideration. To find the college education property is closest to the ideal college education property, we should calculate the similarity measure of PSFS sets in Tables 2-6 with the one in Table 1 based on Definition 4.1. Calculating the similarity measure for the 1-5 colleges education property is given below.

Table 7

	T_1	T_2	S	φ	ψ	$Similarity$
$\overline{(\mathcal{L},\mathcal{A})}$	0.891096	0.266985	0.970638	0.709573	0.863467	0.612693
$\overline{(\mathcal{L},\mathcal{B})}$	0.911240	0.266985	0.919106	0.699110	0.766551	0.535904
$\overline{(\mathcal{L},\mathcal{C})}$	0.861358	0.146233	0.922766	0.643452	0.717221	0.461498
$\overline{(\mathcal{L},\mathcal{D})}$	0.773876	0.228346	0.952375	0.651533	0.787300	0.512952
$\overline{(\mathcal{L},\mathcal{E})}$	0.930259	0.474965	0.954239	0.786488	0.796558	0.626483

From the above results, we find that the **fifth college** education property is closest to the ideal college education property with the highest value of the similarity measure is $\mathbf{0.626483}$.

5.3. **Algorithms based on the similarity measures for SFS set Model.** An algorithm for decision making problems using SFS set model is explained. The algorithm for the selection of the best choice is given as:
Step 1. Input SFS set in tabular form.
Step 2. Form the set of choice parameters $A \subseteq E$.
Step 3. Compute the values of T_1, T_2 and S.
Step 4. Calculate the similarity measure $= \frac{T_1+T_2+S}{3}$.
Step 5. Determine maximum similarity $= Max\{similarity^i\}$ and $1 \leq i \leq 5$.
Step 6. To choose the best solution.
Step 7. End.
Calculating the similarity measure for the mentioned above 1-5 colleges education property using SFS set model as follows. We have

Table 8

	T_1	T_2	S	$Similarity$
$\overline{(\mathcal{L},\mathcal{A})}$	0.891096	0.266985	0.970638	0.709573
$\overline{(\mathcal{L},\mathcal{B})}$	0.911240	0.266985	0.919106	0.699110
$\overline{(\mathcal{L},\mathcal{C})}$	0.861358	0.146233	0.922766	0.643452
$\overline{(\mathcal{L},\mathcal{D})}$	0.773876	0.228346	0.952375	0.651533
$\overline{(\mathcal{L},\mathcal{E})}$	0.930259	0.474965	0.954239	0.786488

It is observed that the **first, second, third and fourth** colleges education property from the perspective of similarity measure are quite away from the ideal college education property. We find that the **fifth college** education property is closest to the ideal college education property with the highest value of the similarity measure is **0.786488**.

6. Comparative studies for PSFS set and SFS set

If the college education property unit chooses the threshold $\langle \mathbf{0.45}, \mathbf{0.1}, \mathbf{0.6} \rangle$, we should choose the **fifth college** education property as a potential college. On the contrary, when using SFS set approach without the generalization parameter, we can not distinguish which the colleges education property is the best one. So the possibility parameter has an important influence to the similarity measure of the college education property.

7. Conclusion and direction of future work

The main goal of this work is to present a possibility spherical fuzzy soft set to solve the phenomena related to decision making in which the sum of the squares of positive membership, neutral membership and negative membership is not exceed one. Finally, PSFS set approach is more scientific and reasonable than SFS set approach without the generalization parameter in the process of decision making. So in future, we should consider the possibility interval valued spherical fuzzy soft sets and bipolar fuzzy soft sets theory.

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References

[1] Alkhazaleh, S., Salleh, A. R. and Hassan, N., (2011), Possibility fuzzy soft set, Advances in Decision Sciences, 1-18.

[2] Alkhazaleh, S. and Salleh, A. R., (2012), Generalized interval valued fuzzy soft set, Journal of Applied Mathematics, 1-18.

[3] Ashraf, S., Abdullah, S., Aslam, M., Qiyasa, M. and Kutbi, M.A., (2019), Spherical fuzzy sets and its representation of spherical fuzzy t-norms and t-conorms, Journal of Intelligent and Fuzzy Systems, 36, 6089-6102.

[4] Atanassov, K., (1986), Intuitionistic fuzzy sets, Fuzzy sets and Systems, 20(1), 87-96.

[5] Bhattacharya, K. and De, S. K., (2020), Decision making under intuitionistic fuzzy metric distances, Annals of Optimization Theory and Practices, 3(2), 49-64.

[6] Bhattacharyee, N., Paramanik, R. and Mahato, S. K., (2020), Optimal redundancy allocation for the problem with chance constraints in fuzzy and intuitionistic fuzzy environments using soft computing technique, Annals of Optimization Theory and Practices, 3(2), 25-47.

[7] Bui Cong Cuong, (2014), Picture fuzzy sets, Journal of Computer Science and Cybernetics, 30(4), 409-420.

[8] Chen, TY., (2018), An interval valued Pythagorean fuzzy compromise approach with correlation based closeness indices for multiple criteria decision analysis of bridge construction methods, Complexity, 1-29.

[9] Fahmi, A., Amin, F. and Shah, S. B. H., (2020), Geometric operators based on linguistic interval valued intuitionistic neutrosophic fuzzy number and their application in decision making, Annals of Optimization Theory and Practices, 3(1), 47-71.

[10] Jana, C., Senapati, T. and Pal, M., (2019), Pythagorean fuzzy Dombi aggregation operators and its applications in multiple attribute decision making, International Journal of Intelligent Systems, 34(9), 2019-2038.

[11] Jana, C. and Pal, M., (2018), Application of bipolar intuitionistic fuzzy soft sets in decision making problem, International Journal of Fuzzy System Applications, 7(3), 32-55.

[12] Jana, C. and Pal, M., (2021), Multi criteria decision making process based on some single valued neutrosophic Dombi power aggregation operators, Soft Computing, 25(7), 5055-5072.

[13] Jana, C. and Pal, M., (2019), A Robust single valued neutrosophic soft aggregation operators in multi criteria decision making, Symmetry, 11, 110, 1-19.

[14] Jana, C., Pal, M. and Wang, J., (2019), A robust aggregation operator for multi criteria decision making method with bipolar fuzzy soft environment, Iranian Journal of Fuzzy Systems, 16(6), 1-16.

[15] Jana, C., Muhiuddin, G, and Pal, M., (2021), Multi criteria decision making approach based on SVTrN Dombi aggregation functions, Artificial Intelligence Review, 54(4), 3685-3723.

[16] Karaaslan, F., (2017), Possibility neutrosophic soft sets and PNS-decision making method, Applied Soft Computing 54, 403-414.

[17] Kirisci, M., (2020), Medical decision making with respect to the fuzzy soft sets, Journal of Interdisciplinary Mathematics, 1-10.

[18] Liang, D. and Xu, Z., (2017), The new extension of TOPSIS method for multiple criteria decision making with hesitant Pythagorean fuzzy sets, Applied Soft Computing, 60, 167-179.

[19] Mahmood, T., Kifayat, U., Khan, Q. and Naeem, J., (2018), An approach toward decision making and medical diagnosis problems using the concept of spherical fuzzy sets, Neural Computing and Applications, 2(2), 1-13.

[20] Maji, P. K., Biswas, R. and Roy, A. R., (2001), Fuzzy soft set, Journal of Fuzzy Mathematics, 9(3), 589-602.

[21] Maji, P. K., Biswas, R. and Roy, A. R., (2001), On intuitionistic fuzzy soft set, Journal of Fuzzy Mathematics, 9(3), 677-692.

[22] Molodtsov, D., (1999), Soft set theory first results, Computers and Mathematics with Applications, 37, 19-31.

[23] Palanikumar, M. and Arulmozhi, K., (2021), Possibility Pythagorean bipolar fuzzy soft sets and its application, Open Journal of Discrete Applied Mathematics, 4(2), 17-29.

[24] Peng, X. D., Yang, Y. and Song, J. P., (2015), Pythagorean fuzzy soft set and its application, Computer Engineering, 41(7), 224-229.

[25] Peng, X. and Yang, Y., (2016), Fundamental properties of interval valued Pythagorean fuzzy aggregation operators, International Journal of Intelligent Systems, 31(5), 444-487.

[26] Perez Dominguez, L., Rodriguez Picon, LA., Alvarado Iniesta, A., Cruz, DL. and Xu, Z., (2018), MOORA under Pythagorean fuzzy sets for multiple criteria decision making, Complexity, 1-10.

[27] Rahman, K., Abdullah, S., Shakeel, M., Khan, MSA. and Ullah, M., (2017), Interval valued Pythagorean fuzzy geometric aggregation operators and their application to group decision making problem, Cogent Mathematics, 4, 1-19.

[28] Rahman, K., Ali, A., Abdullah, S. and Amin, F., (2018), Approaches to multi attribute group decision making based on induced interval valued Pythagorean fuzzy Einstein aggregation operator, New Mathematics and Natural Computation, 14(3), 343-361.

[29] Taghaodi, R., (2019), A novel solution approach for solving intuitionistic fuzzy transportation problem of type-2, Annals of Optimization Theory and Practices, 2(2), 11-24.

[30] Ullah, K., Mahmood, T., Ali, Z. and Jan, N., (2019), On some distance measures of complex Pythagorean fuzzy sets and their applications in pattern recognition, Complex and Intelligent Systems, 1-13.

[31] Xiao, Z., Chen, W. J. and Li, L. L., (2013), A method based on interval valued fuzzy soft set for multi attribute group decision making problems under uncertain environment, Knowledge and Information Systems, 34, 653-669.

[32] Yager, R. R. and Abbasov, A. M., (2014), Pythagorean membership grades, complex numbers, and decision making, International Journal of Intelligent Systems, 28, 436-452.

[33] Yager, R. R., (2014), Pythagorean membership grades in multi criteria decision making, IEEE Transactions Fuzzy Systems, 22(4), 958-965.

[34] Yang, Y., Liang, C., Shiwei Ji and Liu, T., (2015), Adjustable soft discernibility matrix based on picture fuzzy soft sets and its applications in decision making, Journal of Intelligent and Fuzzy Systems, 29, 1711-1722.

[35] Zadeh, L. A., (1965), Fuzzy sets, Information and control, 8(3), 338-353.

[36] Zhang, X., (2016), Multi criteria Pythagorean fuzzy decision analysis a hierarchical qualiflex approach with the closeness index based ranking, Information Sciences, 330, 104-124.

[37] Zulqarnain, R. M., Xin, X. L., Saqlain, M. and Khan, W. A., (2021), TOPSIS method based on the correlation coefficient of interval valued intuitionistic fuzzy soft sets and aggregation operators with their application in decision making, Journal of Mathematics, 1-16.

[38] Zulqarnain, R. M., Xin, X.L., Siddique, I., Asghar Khan, W. and Yousif, M. A., (2021), TOPSIS method based on correlation coefficient under Pythagorean fuzzy soft environment and its application towards green supply chain management, Sustainability, 13(4), 16-42.

[39] Zulqarnain, R. M., Xin, X. L., Garg, H. and Khan, W. A., (2021), Aggregation operators of Pythagorean fuzzy soft sets with their application for green supplier chain management, Journal of Intelligent and Fuzzy Systems, 40(3), 5545-5563.

[40] Zulqarnain, R. M., Xin, X. L. and Saeed, M., (2020), Extension of TOPSIS method under intuitionistic fuzzy hypersoft environment based on correlation coefficient and aggregation operators to solve decision making problem, AIMS Mathematics, 6(3), 2732-2755.

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