

EXTREMUM SEEKING CONTROL OF UNCERTAIN SYSTEMS

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ABSTRACT. Extremum seeking is used in control problems where the reference trajectory or reference set point of the system is not known but it is searched in real time in order to maximize or minimize a performance function representing the optimal behaviour of the system. In this paper, extremum seeking algorithm is applied to the systems with parametric uncertainties.

Keywords: extremum seeking, uncertain systems, sliding mode control, adaptive control.

AMS Subject Classification: 93C40

1. INTRODUCTION

Extremum seeking control is used for searching a maximum or minimum point of a performance function representing the desired behaviour of the system. The algorithm fits for the problems where the performance function of the system is completely or partially not known or may change in time. The system may be nonlinear and may have structured or unstructured uncertainties and disturbances. Some application areas are maximization of the braking and traction forces in automotive applications, wind mill adjustment to maximize the power generation, spark timing and cam timing control in internal combustion engines, and optimization of bioreactors.

In the literature there are mainly four types of extremum seeking schemes: perturbation based, sliding mode based, numerical optimization based, and gradient based extremum seeking algorithms. In the perturbation based extremum seeking algorithms studied in [1] - [3], a perturbation is added to the search signal. By observing the effect of the perturbation on the performance function measurement, it is determined whether to increase or decrease the search signal to reach its optimum value and hence maximize (or minimize) the performance function. It is assumed that the shape, i.e. the gradient of the performance function is unknown. In the seminal paper of Krstic and Wang [1], stability proof for this type of extremum seeking control is given with the tools of averaging and singular perturbations.

In the sliding mode based extremum seeking approach studied in [4] - [8], similar to the previous scheme, the gradient of the performance function is considered unknown. A sliding surface is defined where on that surface the performance function is forced to follow an increasing (or decreasing) function. Since the shape of the performance function is not known, this is a control problem with uncertain direction of control vector. Henceforth, the search signals are selected as discontinuous periodic switchings.

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The numerical optimization based extremum seeking schemes in [9], and [10] uses iterative methods such as steepest descent. Numerical optimization algorithm chooses the next state and a state regulator manages the system to follow the new state.

In the gradient based extremum seeking algorithms of [11], and [12], in contrast to the above schemes, an explicit structure of the objective function is required.

In this paper, sliding mode based extremum seeking algorithm is developed for systems with parametric uncertainties. The control scheme consists of two phases. In the first phase, a sliding mode regulator brings the system to an equilibrium point characterized with an adaptation parameter. In the second phase, the value of this adaptation parameter is changed via the sliding mode-based extremum seeking scheme to operate the system in the maximum point of its performance function where the gradient, i.e. the shape of the performance function is unknown.

The paper is organized as follows. In Section 2, the problem formulation is given. Section 3 explains sliding mode regulator for the parametric uncertain system. Section 4 introduces extremum seeking algorithm. Section 5 presents an exemplary control system. The paper concludes with Section 6.

2. PROBLEM FORMULATION

Consider the following nonlinear system with parameter uncertainties

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(\mathbf{x}, \mathbf{p}) + b(\mathbf{x}, \mathbf{p})u\end{aligned}\quad (1)$$

Here, $\mathbf{x} = [x_1 \ x_2] \in R^2$ is the state, $y = x_1$ is the system output, $u \in R$ is the control, and $\mathbf{p} \in R^q$ is the uncertain parameter vector, where q is the number of uncertain parameters.

Assumption 1. The functions $f(\mathbf{x}, \mathbf{p})$ and $b(\mathbf{x}, \mathbf{p})$ can be written as

$$f = \bar{f} + \Delta f; |\Delta f| \leq F(\mathbf{x}) \quad (2)$$

$$0 < b = \bar{b} + \Delta b; |\Delta b| \leq B(\mathbf{x}) \quad (3)$$

where \bar{f} and \bar{b} represent the nominal parts, Δf and Δb represent unknown terms, which are bounded with some known functions $F(\mathbf{x})$ and $B(\mathbf{x})$.

The objective is to control the uncertain system (1) with assumptions (2) and (3) at the extremum point of a performance function, where the gradient, i.e. the shape of the performance function is unknown. Only the magnitude of the performance function can be measured during the control phase.

Assumption 2. The performance function of the system, which is

$$z = J(y) \quad (4)$$

is smooth and has a unique maximum. For example the performance function can be written as a quadratic function of

$$J(y) = J^* + \beta [y(t) - y^*]^2 \quad (5)$$

where J^* denotes the extremum value, y^* is the optimum operation point of the system, and β is a constant. The controller does not know the values of J^* , y^* , and β . Only the magnitude of J should be at hand while the system output changes. The controller will seek the optimum operation point, i.e. y^* that maximizes the performance function.

3. SLIDING MODE REGULATION VIA ADAPTIVE PARAMETER

In [8], the idea of introducing a free parameter to the closed loop through the control law and adapting this parameter to seek the optimum operating point with respect to a performance function is given. Here, this method is applied to the system (1). A sliding surface (6) is defined where on that surface the equilibrium point of the system is characterized by an adaptation parameter θ

$$s_1 = a(x_1 - \theta) + x_2 \quad (6)$$

By taking the time derivative of (6), one can obtain

$$\dot{s}_1 = a(x_2 - \dot{\theta}) + f + bu \quad (7)$$

The control input is selected as

$$u = \frac{1}{b} \left[-\bar{f} - a(x_2 - \dot{\theta}) - U(x) \operatorname{sgn}(s_1) \right] \quad (8)$$

Putting (8) in (7)

$$\dot{s}_1 = a(x_2 - \dot{\theta}) + \bar{f} + \Delta f - \frac{b}{\bar{b}} \bar{f} - \frac{b}{\bar{b}} a(x_2 - \dot{\theta}) - \frac{b}{\bar{b}} U \operatorname{sgn}(s_1) \quad (9)$$

Grouping same factors in (9), one can get

$$\dot{s}_1 = \left(1 - \frac{b}{\bar{b}} \right) \left[a(x_2 - \dot{\theta}) + \bar{f} \right] + \Delta f - \frac{b}{\bar{b}} U \operatorname{sgn}(s_1) \quad (10)$$

From (10) one can write

$$s_1 \dot{s}_1 \leq |s_1| \left| 1 - \frac{b}{\bar{b}} \right| \left| a(x_2 - \dot{\theta}) + \bar{f} \right| + |s_1| \left| F - \frac{b}{\bar{b}} U \right| |s_1| \quad (11)$$

Since the inequalities of

$$\frac{b}{\bar{b}} \geq \frac{\bar{b} - B}{\bar{b}} \quad (12)$$

$$\frac{|\bar{b} - b|}{\bar{b}} = \frac{\Delta b}{\bar{b}} \leq \frac{B}{\bar{b}} \quad (13)$$

are true, one can write following inequality from (11)

$$s_1 \dot{s}_1 \leq |s_1| \frac{B}{\bar{b}} \left| a(x_2 - \dot{\theta}) + \bar{f} \right| + |s_1| \left| F - \frac{\bar{b} - B}{\bar{b}} U \right| |s_1| \quad (14)$$

In order to make right hand side of (14) negative definite, U is selected as

$$U = \frac{\bar{b}}{\bar{b} - B} \left[U_0 + \frac{B}{\bar{b}} \left| a(x_2 - \dot{\theta}) + \bar{f} \right| + F \right] \quad (15)$$

where $U_0 > 0$. So (14) becomes

$$s_1 \dot{s}_1 \leq -\sigma_0 |s_1| \quad (16)$$

where σ_0 is a positive number. Since (16) is the finite time stability condition, after a finite time interval, s_1 will equal to zero. Then, (6) becomes

$$a(x_1 - \theta) + x_2 = 0 \quad (17)$$

By choosing a big enough to let (17) have a fast dynamics, x_1 will track θ . Putting (15) to the (8), the control input is calculated from

$$u = \frac{1}{b} \left[-\bar{f} - a(x_2 - \dot{\theta}) \right] - \frac{1}{\bar{b} - B} \left[U_0 + \frac{B}{\bar{b}} \left| a(x_2 - \dot{\theta}) + \bar{f} \right| + F \right] \operatorname{sgn}(s_1) \quad (18)$$

4. EXTREMUM SEEKING ADAPTATION RULE

As introduced in Section 2, the performance function can be a quadratic function as shown in (5). The sliding mode regulator given in Section 3 ensures that the state x_1 will track the adaptation parameter θ . Hence, (5) can be written as

$$J(\theta) = J^* + \beta [\theta(t) - x_1^*] \quad (19)$$

So, the problem turns into adapting the θ value to seek the optimum point of the performance function, e.g. (19). A second sliding surface is selected as

$$s_2 = J(\theta) - g(t) \quad (20)$$

where $g(t)$ is a time increasing function with the slope of ρ . Taking the time derivative of (20), one can obtain

$$\dot{s}_2 = \frac{dJ}{d\theta} \dot{\theta} - \rho \quad (21)$$

The adaptation rule for the parameter θ is selected as

$$\dot{\theta} = K \operatorname{sgn} \left[\sin \left(\frac{\pi s_2}{\alpha} \right) \right] \quad (22)$$

where K and α are positive constants. By putting (22) into (21) one can get

$$\dot{s}_2 = \frac{dJ}{d\theta} K \operatorname{sgn} \left[\sin \left(\frac{\pi s_2}{\alpha} \right) \right] - \rho \quad (23)$$

Theorem 4.1. *In (23) as long as the condition*

$$\left| \frac{dJ}{d\theta} \right| > \frac{\rho}{K} \quad (24)$$

holds, then, after a finite time interval, the time derivative of s_2 will be equal to zero.

Proof. Assuming that the initial value of the sliding surface variable s_2 is between the interval of

$$\alpha < s_2(0) < 2\alpha \quad (25)$$

then, the following mathematical expressions can be written on this interval

$$\operatorname{sgn} \left[\sin \left(\frac{\pi s_2}{\alpha} \right) \right] = -\operatorname{sgn}(s_2 - \alpha) = \operatorname{sgn}(s_2 - 2\alpha) \quad (26)$$

Expressions of (26) can be justified from Figure 1. It is seen that when the value of s_2 is between $\alpha < s_2 < 2\alpha$, then the function $\operatorname{sgn} \left[\sin \left(\frac{\pi s_2}{\alpha} \right) \right]$ is equal to -1 and the values of the functions $-\operatorname{sgn}(s_2 - \alpha)$ and $\operatorname{sgn}(s_2 - 2\alpha)$ are also equal to -1 on this interval. Now, according to (24), consider the case that the current operation point satisfies

$$\frac{dJ}{d\theta} < -\frac{\rho}{K} \quad (27)$$

From (23) and (26), one can write

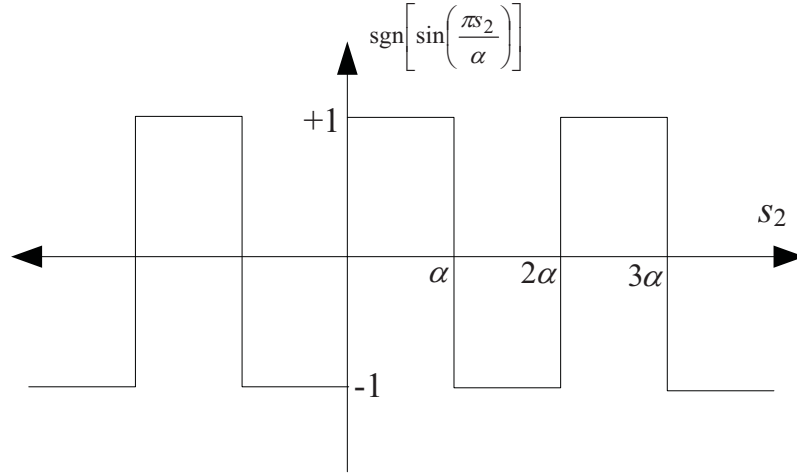
$$\dot{s}_2 = \frac{dJ}{d\theta} K \operatorname{sgn}(s_2 - 2\alpha) - \rho \quad (28)$$

By defining a new variable γ as

$$\gamma = s_2 - 2\alpha \quad (29)$$

then, since $\dot{\gamma} = \dot{s}_2$, (28) can be written as

$$\dot{\gamma} = \frac{dJ}{d\theta} K \operatorname{sgn}(\gamma) - \rho \quad (30)$$

FIGURE 1. Function of $\text{sgn} \left[\sin \left(\frac{\pi s_2}{\alpha} \right) \right]$.

By multiplying (30) with γ , the following equality can be written

$$\gamma \dot{\gamma} = \frac{dJ}{d\theta} K |\gamma| - \rho \gamma \quad (31)$$

Since (27) is considered to be true, one can write (31) as

$$\gamma \dot{\gamma} = - \left| \frac{dJ}{d\theta} \right| K |\gamma| - \rho \gamma \quad (32)$$

From (32), following inequality can be written

$$\gamma \dot{\gamma} \leq - \left| \frac{dJ}{d\theta} \right| K |\gamma| + \rho |\gamma| = - |\gamma| \left[\left| \frac{dJ}{d\theta} \right| K - \rho \right] < 0 \quad (33)$$

(33) is finite time convergence condition, which means that after a finite time interval γ will equal to zero. From (29), after a finite time interval, when $\gamma = 0$, then

$$s_2 = 2\alpha \quad (34)$$

which means that after a finite time interval s_2 value will approach to a constant value. Above analysis has been conducted for the case of (27). Now, the analysis is conducted for the case of

$$\frac{dJ}{d\theta} > \frac{\rho}{K} \quad (35)$$

From (23) and (26), one can write

$$\dot{s}_2 = - \frac{dJ}{d\theta} K \text{sgn}(s_2 - \alpha) - \rho \quad (36)$$

This time, when γ is defined as

$$\gamma = s_2 - \alpha \quad (37)$$

then, since $\dot{\gamma} = \dot{s}_2$, (36) can be written as

$$\dot{\gamma} = - \frac{dJ}{d\theta} K \text{sgn}(\gamma) - \rho \quad (38)$$

By multiplying (38) with γ , the following equality can be written

$$\gamma\dot{\gamma} = -\frac{dJ}{d\theta}K|\gamma| - \rho\gamma \quad (39)$$

Since (35) is considered to be true, then one can write (39) as follows

$$\gamma\dot{\gamma} = -\left|\frac{dJ}{d\theta}\right|K|\gamma| - \rho\gamma \quad (40)$$

From (40), the following inequality can be written

$$\gamma\dot{\gamma} \leq -\left|\frac{dJ}{d\theta}\right|K|\gamma| + \rho|\gamma| = -|\gamma|\left[\left|\frac{dJ}{d\theta}\right|K - \rho\right] < 0 \quad (41)$$

Again, since (41) is the finite time stability condition, after a finite time interval, γ will equal to zero. From (37), after a finite time interval

$$s_2 = \alpha \quad (42)$$

will be true. So, it has been shown that when the initial value of s_2 is between $\alpha < s_2 < 2\alpha$, then, as long as the condition (24) holds, after a finite time interval, s_2 value will approach to α or 2α . The above analysis can be repeated not for only $\alpha < s_2 < 2\alpha$ but for any initial value of s_2 . For any $s_2(0)$, after a finite time interval, s_2 will approach to a constant value, which results that $\dot{s}_2 = 0$. \square

According to the sliding surface given in (20), after a finite time interval, when $\dot{s}_2 = 0$, it is true that

$$\dot{J} = \rho \quad (43)$$

which means that the value of the performance function will increase with the slope of ρ . In sliding mode, the equivalent value of $\dot{\theta}$ can be calculated as

$$\dot{s}_2 = \frac{dJ}{d\theta}\dot{\theta} - \rho = 0 \quad (44)$$

$$(\dot{\theta})_{eq} = \frac{\rho}{\frac{dJ}{d\theta}} \quad (45)$$

So, when the current value of the adaptation parameter is less than its optimum value, for example point P_5 in Figure 2, then the sign of $\frac{dJ}{d\theta}$ is positive and according to the (45), θ value will increase and approach towards its optimum value θ^* . On the contrary, when the current value of the adaptation parameter is bigger than its optimum value, the sign of $\frac{dJ}{d\theta}$ is negative and according to the (45), θ value will decrease and approach towards the optimum value.

In Figure 2, P_1 and P_4 characterizes the points where the condition (24) doesn't hold anymore. As long as the gradient is greater than the value in the right hand side of (24), the performance function will increase towards the points P_1 or P_4 . When the current operating point hits the point P_1 , it will be true that

$$\left|\frac{dJ}{d\theta}\right| < \frac{\rho}{K} \quad (46)$$

Then, from (23), it can be written that,

$$\dot{s}_2 = \frac{dJ}{d\theta}K \operatorname{sgn}\left[\sin\left(\frac{\pi s_2}{\alpha}\right)\right] - \rho \leq \left|\frac{dJ}{d\theta}\right|K - \rho < 0 \quad (47)$$

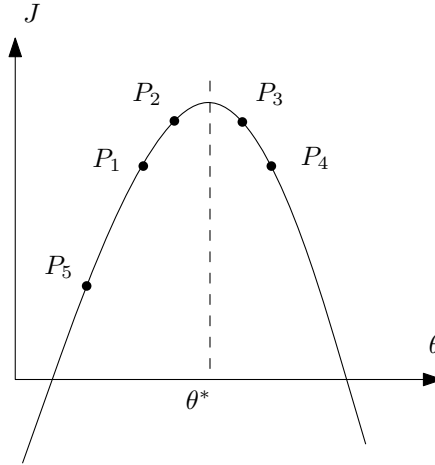


FIGURE 2. Sketch of the performance function.

Henceforth, s_2 will be not constant anymore as in (34) or (42) and it will start to decrease with the velocities of

$$\begin{aligned} \left(\frac{ds_2}{dt}\right)_{max} &= - \left| \frac{dJ}{d\theta} \right| K - \rho \\ \left(\frac{ds_2}{dt}\right)_{min} &= + \left| \frac{dJ}{d\theta} \right| K - \rho \end{aligned} \quad (48)$$

While s_2 decreases, according to (22), $\dot{\theta}$ will oscillate between $+K$ and $-K$. It can be shown that θ will still converge towards the optimum point with oscillation. According to (48) change of s_2 and \dot{s}_2 can be characterized as in Figure 3 and change of $\dot{\theta}$ according to (22) is shown in Figure 4. It is obvious from Figure 3 that s_2 passes the region I slower than the region II because of the smaller change rate. Consequently $\dot{\theta}$ takes the value of $+K$ more than $-K$ and hence θ will increase and approach to the point P_2 , i.e. continue to approach towards the optimum point θ^* .

If θ passes to the right side of the optimum value where the gradient is negative, change of s_2 and \dot{s}_2 will be similar as in Figure 5, where s_2 will pass the region II slower than the region I due to the smaller change rate. Consequently $\dot{\theta}$ takes the value of $-K$ more than $+K$ resulting that θ will decrease and approach to the optimum value. Finally, θ will oscillate in a small neighborhood of the extremum point as it is shown in Figure 2 with the points of P_2 and P_3 .

5. EXAMPLE SYSTEM

A mass-spring-damper with hardening spring is taken as the example system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{c}{m}x_2 - \frac{k}{m}(1+x_1^2)x_1 + \frac{1}{m}u \end{aligned} \quad (49)$$

where c , m , k are damping coefficient, mass and spring coefficient respectively. These are uncertain parameters, where their maximum and minimum values are known as

$$\begin{aligned} c_{min} &\leq c \leq c_{max} \\ k_{min} &\leq k \leq k_{max} \\ m_{min} &\leq m \leq m_{max} \end{aligned} \quad (50)$$

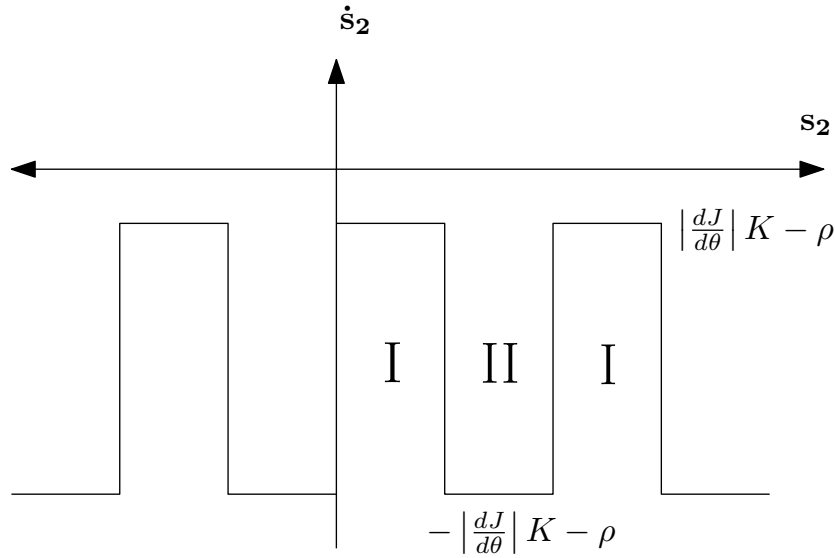


FIGURE 3. Change of s_2 and \dot{s}_2 around the neighborhood of the extremum point when the gradient is positive.

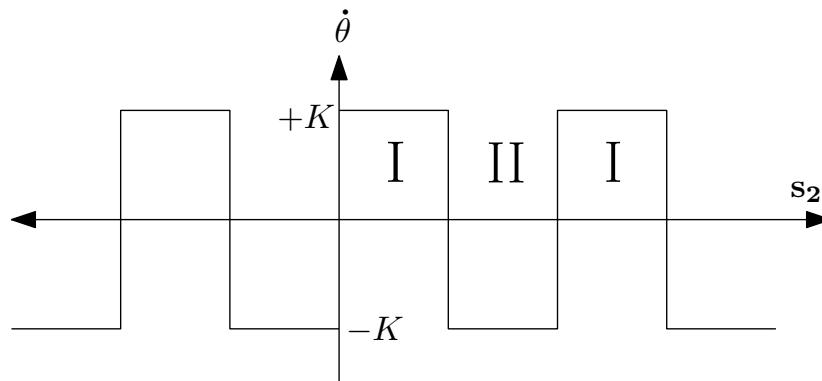


FIGURE 4. Change of $\dot{\theta}$.

The objective is to control the uncertain system (49) and (50) such that a performance function will be maximized. First, the system is brought to an equilibrium point characterized with an adaptation parameter. The sliding surface is selected as

$$s_1 = a(x_1 - \theta) + x_2 \tag{51}$$

Taking the time derivative of (51)

$$\dot{s}_1 = a(x_2 - \dot{\theta}) - \frac{c}{m}x_2 - \frac{k}{m}(1 + x_1^2)x_1 + \frac{1}{m}u \tag{52}$$

The control input is selected as

$$u = \bar{c}x_2 + \bar{k}(1 + x_1^2)x_1 - \bar{m} \left[a(x_2 - \dot{\theta}) + U(x)sgn(s_1) \right] \tag{53}$$

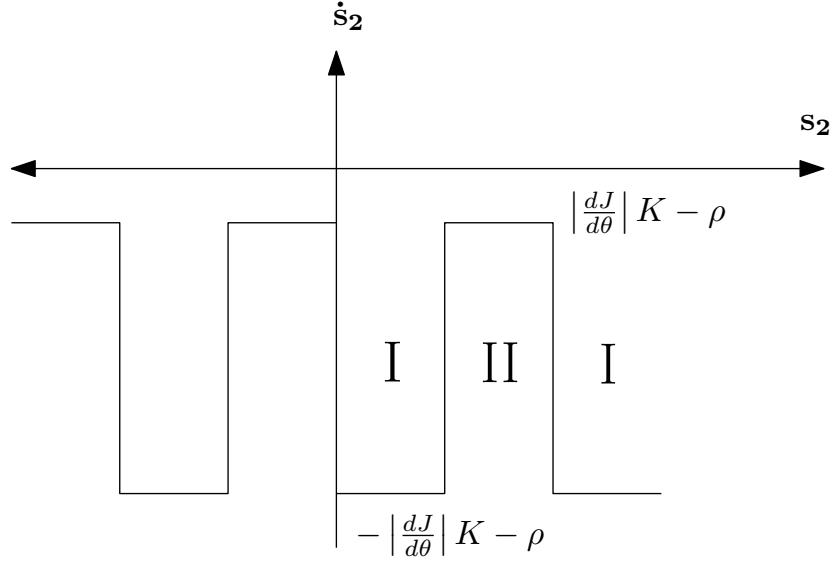


FIGURE 5. Change of s_2 and \dot{s}_2 around the neighborhood of the extremum point when the gradient is negative.

where \bar{c} , \bar{k} and \bar{m} are the average values of the c , k and m . Putting (53) into the (52)

$$\begin{aligned} \dot{s}_1 &= a(x_2 - \dot{\theta}) - \frac{c}{m}x_2 - \frac{k}{m}(1 + x_1^2)x_1 + \frac{\bar{c}}{m}x_2 \\ &\quad + \frac{\bar{k}}{m}(1 + x_1^2)x_1 - \frac{\bar{m}}{m}a(x_2 - \dot{\theta}) - \frac{\bar{m}}{m}U(x)\text{sgn}(s_1) \end{aligned} \quad (54)$$

By collecting the same factors, one can get

$$\dot{s}_1 = a(x_2 - \dot{\theta}) \left[1 - \frac{\bar{m}}{m} \right] + \frac{x_2}{m}(\bar{c} - c) + \frac{(1 + x_1^2)x_1}{m}(\bar{k} - k) - \frac{\bar{m}}{m}U(x)\text{sgn}(s_1) \quad (55)$$

From (55) the following inequality can be written

$$s_1\dot{s}_1 \leq |s_1|a|x_2 - \dot{\theta}| \frac{|m - \bar{m}|}{m} + |s_1| \frac{|x_2|}{m} |\bar{c} - c| + |s_1| \frac{(1 + x_1^2)|x_1|}{m} |\bar{k} - k| - \frac{\bar{m}}{m}U(x)|s_1| \quad (56)$$

In order to make right hand side of (56) negative definite, $U(x)$ can be selected as

$$U(x) \geq \frac{1}{\bar{m}} \left[a|x_2 - \dot{\theta}| |m - \bar{m}| + |x_2| |\bar{c} - c| \right] + \frac{1}{\bar{m}} \left[(1 + x_1^2)|x_1| |\bar{k} - k| + U_0 \right] \quad (57)$$

where $U_0 > 0$. Since in (57) the values of m , c , and k are not known, one can choose for $U(x)$ as

$$\begin{aligned} U(x) &= \frac{1}{\bar{m}} \left[a|x_2 - \dot{\theta}| (m_{max} - \bar{m}) + |x_2| (c_{max} - \bar{c}) \right] \\ &\quad + \frac{1}{\bar{m}} \left[(1 + x_1^2)|x_1| (k_{max} - \bar{k}) + U_0 \right] \end{aligned} \quad (58)$$

By selecting $U(x)$ as in (58), one assures that after a finite time interval, $s_1 = 0$ and x_1 will track θ . Putting (58) into (53), the control input can be calculated from

$$\begin{aligned} u &= \bar{c}x_2 + \bar{k}(1 + x_1^2)x_1 - \bar{m}a(x_2 - \dot{\theta}) \\ &\quad - \left[a|x_2 - \dot{\theta}| (m_{max} - \bar{m}) + |x_2| (c_{max} - \bar{c}) \right. \\ &\quad \left. + (1 + x_1^2)|x_1| (k_{max} - \bar{k}) + U_0 \right] \text{sgn}(s_1) \end{aligned} \quad (59)$$

The parameter θ will be adapted as given in Section 4. The adaptation rule is

$$\dot{\theta} = K \operatorname{sgn} \left[\sin \left(\frac{\pi s_2}{\alpha} \right) \right] \quad (60)$$

where s_2 is calculated from

$$s_2 = J(\theta) - g(t) \quad (61)$$

So, the final expression of the control input will be as follows

$$\begin{aligned} u = & \bar{c}x_2 + \bar{k}(1 + x_1^2)x_1 - \bar{m}a \left(x_2 - K \operatorname{sgn} \left[\sin \left(\frac{\pi s_2}{\alpha} \right) \right] \right) \\ & - \left[a \left| x_2 - K \operatorname{sgn} \left[\sin \left(\frac{\pi s_2}{\alpha} \right) \right] \right| (m_{max} - \bar{m}) + |x_2| (c_{max} - \bar{c}) \right. \\ & \left. + (1 + x_1^2)|x_1| (k_{max} - \bar{k}) + U_0 \right] \operatorname{sgn}(s_1) \end{aligned} \quad (62)$$

6. CONCLUSION

In this paper, sliding mode based extremum seeking algorithm is applied to the systems with parametric uncertainties. The control scheme consists of two phases. In the first phase, a sliding mode regulator brings the system to an equilibrium point characterized with an adaptation parameter. In the second phase, the adaptation parameter is changed via the extremum seeking scheme to operate the system in the extremum point of a performance function. The controller will operate the uncertain system in a-priori unknown optimum set point where the performance function is maximized.

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