

A SUITABLE WAY OF NORMALIZING NEW SI TO MAKE c AND h UNITIES

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ABSTRACT. Yarman’s Approach, which serves as the basis of YARK gravitation theory (as abbreviated from “Yarman-Arik-Kholmetskii”), together with its recently developed extension QTG (Quantal Theory of Gravity), motivated us to question the suitability of Natural Units commonly used in Quantum Field Theory (QFT) and other areas of physics. That and the consensus of the *General Conference on Weights and Measures* (CGPM) towards the establishment of “New SI” inspired us to explore an appropriate way of normalizing the metric system in order to make the utmost theoretical speed limit of light c and the Planck Constant h unities, as well as universal constants, respectively. Our metrological approach herein reveals that the correction factor \mathcal{L} introduced to the retired definition of vacuum permeability μ_0 — as extracted from an indiscriminate *Fine-Structure Constant* α value — does not suffice to align the computed α with the latest experimental measurements of α . One may therefore require a rectified value for the elementary charge e along with the need to restore its uncertainty digits. All this is especially relevant within the context of the 20 May 2019 international decision to fix the Planck Constant to a definite value while letting the kilogram vary instead. One thus remarkably ends up with the necessity to either restore the uncertainty parts of the elementary charge in contrast to the SI redefinition, or to recalculate the correction factor \mathcal{L} that latterly appears in vacuum permeability, or both. Another far-reaching option is the idea of restituting the uncertainties for the Planck Constant and/or lightspeed too when SI is normalized and then re-normalized without disturbing the meaningfulness of the related physical dimensions.

Keywords: Yarman’s Approach, SI and Metrology, Natural Units, Planck Constant, Fine-Structure Constant, Elementary Charge

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1. INTRODUCTION

Doubts about the internal consistency of Natural Units (SI formalism adopted from the historical *Heaviside-Lorentz* approach where $\epsilon_0 = \mu_0 = \hbar = c = 1$) drove us to embark upon our meticulous method below for making respectively the speed of light c and the Planck Constant h unities, as well as universal constants, in the proper way.

The normalization and re-normalization (*i.e.*, ergonomization) of weights & measures we propose herein, including common units of the electromagnetic domain, point to a

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different value for the elementary charge e compared to that given by the “New SI” (Système International [d’Unités]: International System of Units) or under Natural Units.

In particular, contrary to the 20 May 2019 international redefinition of units by the concerted efforts of metrology workgroups worldwide that resulted in the “New SI” [cf. 1], the need to either restore the uncertainty digits of the value of the elementary charge e , or to recalculate the correction factor k which arises from a respecified vacuum permeability μ_0 , or both, becomes apparent. Another far-reaching option is the idea of restituting the uncertainties for the Planck Constant h and/or lightspeed c as well. The latter is especially implied by the well-known $c^2 = 1/\mu_0\varepsilon_0$ rule. Furthermore, it is shown that setting $\varepsilon_0 = \mu_0 = 1$ along with $h = c = 1$ turns out to be improper.

Our metrological approach preserves the proposed corresponding physical dimensions at every step of the way and does not disturb their empirical meaningfulness.

Otherwise, the correction factor $k=1.000\ 000\ 000\ 55(15)$ for μ_0 with its allowable uncertainty digits (as extracted from $|(2h\alpha_{\text{exp}})/(e^2c)| / 4\pi \times 10^{-7}$ or $|\mu_0| / 4\pi \times 10^{-7} = 1.0000000055(15)$ [2]) — based on an indiscriminately chosen *Fine-Structure Constant* α value [cf. 3] — does not by itself suffice to bring together the two sides of the world of base units at the 10^{-10} level. This is already evidenced by the controversial up-to-date measurements of α cited in this manuscript. In short, the deviation is too large and unacceptable as it stands, which confirms the fear that CGPM’s (*Conférence Générale des Poids et Mesures*) aforesaid 2019 decision constitutes a grave error.

Such a discrepancy of more than one order of magnitude between the relative uncertainty for α as computed from the adopted values of “New SI” and that of its latest experimental results [4, 5] was noticed by our team at the onset and has long-awaited being addressed; whereby we believe that our resolution herein — while not poised to affect the laity’s daily reliance on the metric system and while not necessitating new instrumentation or measurement techniques — is vital for the scientific community, whose dependence on a correct metrological standard is of the utmost significance.

2. MAKING c AND h UNITIES IN THE PROPER WAY

Yarman’s Approach, which was later extended to YARK theory of gravitation (with YARK standing for “Yarman-Arik-Kholmetskii” the way abbreviated from the principal authors’ surnames), and recently to its more developed form QTG (Quantal Theory of Gravity) [6], holds that lightspeed (c) — either in differential or integral form — and Planck Constant (h) are Lorentz Invariant universal physical constants outside or under gravity (unlike is the case with General Relativity), or indeed any other type of force interaction whatever the frame of reference (cf. [7]).

Based on this outlook (regarding which, one may refer to [6, 7] for an exposition of how the said framework underlies our main reasoning towards this contribution), it would stand to reason that they should both be defined as “1” in their respective dimensions. To this end, however, Natural Units was found problematic as shall be seen.

To make c unity, while initially preserving the *second* as the arbitrary unit period of time (as defined through the lapse of 9 192 631 770 ticks of radiation from any Cesium-133 atomic clock operating in an unperturbed ground-state hyperfine transition mode) — and keeping in mind that the *meter* is taken as 1 out of 299,792,458 portions of the rectilinear

distance travelled by light in empty space throughout the aforementioned duration — we can straightforwardly multiply the *meter* by 299,792,458, which gives us a new unit of length (*NLU*):

$$c = \frac{1 \text{ meter} \times 299,792,458}{1 \text{ second}}, \quad (2.1a)$$

$$c = \frac{1 \text{ NLU}}{1 \text{ second}}, \quad (2.1b)$$

$$1 \text{ NLU} = 299,792,458 \text{ meters}. \quad (2.1c)$$

Notice immediately that the chosen unit period of time (*i.e.*, the *second*) is exact and unalterable by definition, and the only recourse left for the possibility of any absolute error in the measurement of c (which, as we remarked previously, is implied through $c^2 = 1/\mu_0\epsilon_0$ owing to the barely co-cancelling correction factors in the said formula) is by virtue of the introduction of a miniscule indeterminacy to the rectilinear spatial distance — *e.g.*, such as 299,792,45(7.99xxxx) *meters* or 0.999 999 99 99(xxx) *NLU*; with each digit designated as “x” denoting any number from 0 to 9, and the sections in parantheses denoting uncertainty in measurement.

It is especially important to point to the fact that the said uncertainties on c occur to be in satisfactory concordance with the relative uncertainty on its measurement, which is about 2×10^{-11} [8].

Given that Planck’s Constant has the dimensions of $\text{m}^2 \times \text{kg} / \text{sec}$, and keeping in mind the latest decision by the *General Conference on Weights and Measures* to fix it to a certain value [cf. 1], it is possible to substitute *NLU* in place of the *meter* to find out the new unit of mass (*NWU*) that will make h unity:

$$h = \frac{6.62607015 \times 10^{-34} \text{ kg} \times \text{m}^2}{\text{s}}, \quad (2.2a)$$

$$h = \frac{6.62607015 \times 10^{-34}}{299,792,458^2} \text{ kg} \cdot 1 \frac{\text{NLU}^2}{\text{s}}, \quad (2.2b)$$

$$1 \text{ NWU} = \frac{6.62607015 \times 10^{-34}}{299,792,458^2} \text{ kg}, \quad (2.2c)$$

$$1 \text{ NWU} = 7.37249732381271 \times 10^{-51} \text{ kg}, \quad (2.2d)$$

$$h = 1 \frac{\text{NWU} \times \text{NLU}^2}{\text{s}}. \quad (2.2e)$$

The letter W was chosen in the acronym NWU to stress the historically inseparable physical connection between mass and weight. Otherwise, NWU is a unit of mass free from gravitational acceleration that is the equivalent of about $7.3725\dots \times 10^{-51}$ kg .

In Eq.(2.2e), regardless of the co-moving frame’s perspective of the frame of the distant observer, or vice versa, “ NLU per second” will not vary according to Yarman’s Approach or YARK/QTG, but “ $NWU \times NLU$ ” product components will vary each. Because Planck’s Constant is now firmly secured onto a given value, they must vary conformally and in opposite directions. To rephrase, while the proper reference frame subjected to a force will not notice any physical change in its mass and/or length, a comparison between the local observer’s measurement of the said quantities and the distant observer’s shall entail a difference inasmuch as their paired result staying invariant and thus, along with the Lorentz Invariance of “ NLU per second”, ensuring the universal constancy of h .

A plausible revisit of the idea of restituting the uncertainty digits to the Planck Constant from the indeterminacies that manifestly occur in length and/or mass (*e.g.*, in NLU and/or NWU) is independent of this factuality.

Thus, we have not touched the *second* as the adopted basic unit period of time at this stage, but simply modified the traditional mass and distance units to unicitize c and h . After this set of operations, it is possible to linearly transform the units, including the *second*, without destroying the unicity of c and h . But since the *second* is quite common, applicable in other areas which we shall momentarily tread, and possible to determine with great precision when taking as reference atomic or stellar objects (such as Pulsars), let us hence keep using it.

3. DISCREPANCY BETWEEN THE RELATIVE UNCERTAINTIES OF COMPUTED α AND MEASURED α

At this point, we can advance further into the electromagnetic realm through the established MKS (meter-kilogram-second) definition of the *Fine-Structure Constant*:

$$\alpha = \frac{1}{4\pi\epsilon_0} \frac{e^2}{\hbar c} , \tag{3.1a}$$

$$\alpha = \frac{e^2}{2\epsilon_0 hc} = \frac{e^2 c \mu_0}{2h} , \tag{3.1b}$$

where, in accordance with *Ampère’s Force Law* and *Maxwell’s Equations*, we must adhere to the classical relationship

$$c^2 = \frac{1}{\mu_0 \epsilon_0} \tag{3.2a}$$

as presenting us with the *de facto* definition of the “permittivity of free space” ϵ_0 , since the “magnetic permeability of vacuum” μ_0 was historically fixed at:

$$\mu_0 = 4\pi \times 10^{-7} \frac{\text{Henry}}{\text{meter}} . \tag{3.2b}$$

Bear in mind that the long-established understanding shared by CODATA 2014 (cf. TABLE I of [9]) was such that μ_0 and ε_0 were taken to be exact. This changed with the CODATA 2018 recommended values after the emergence of a ubiquitous correction factor (which we herein dub \hat{k}) [2]. Likewise, the CGPM redefinition that culminated in the “New SI” [cf. 10] tries to carry the relative uncertainty from an indiscriminately chosen α to the vacuum permeability μ_0 — and from thereon to ε_0 via Eq.(3.2a) — whilst foregoing the uncertainties in e , h and c altogether.

Whereas the CODATA 2018 correction factor for μ_0 was given in Tiesinga et al.’s TABLE XXXI as 1.000 000 000 55(15) [2], it was considered, at the time of the adoption of the Resolution of the 26th Reunion of the CGPM [1], equal to the relative standard uncertainty of the recommended value of the *Fine-Structure Constant* — *i.e.*, 2.3×10^{-10} ; implying the coefficient 1.000 000 000 82(20) — while there was still anticipation for its better determination through experimentation in the future.

Either way, it will be shown that the said correction factor \hat{k} is at least one order of magnitude off from correctly aligning up-to-date empirical measurements of α with the α computed based on the given constants as internationally redefined in 2019, and this will constitute one *raison d’être* of the present metrological exercise.

To demonstrate this fact, let us take the finite differential of α (to be calculated from the 2019 values of the constants) the way provided on the LHS of Eq.(3.1b). After adopting the constraints $\Delta e = 0$, $\Delta h = 0$, $\Delta c = 0$ as implied by the 20 May 2019 international consensus of CGPM (signifying that the uncertainties on the Planck Constant, lightspeed and elementary charge are zero) [1], we arrive at:

$$\Delta\alpha_{\text{calc}} = \frac{2e \Delta e (2\varepsilon_0 hc) - e^2 2\Delta\varepsilon_0 hc}{4\varepsilon_0^2 h^2 c^2}, \quad \left| \quad \text{given } \Delta e = \Delta h = \Delta c = 0; \quad (3.3a)$$

$$\Delta\alpha_{\text{calc}} = \frac{2e \Delta e (2\varepsilon_0 \cancel{h} \cancel{c}) - e^2 2\Delta\varepsilon_0 \cancel{h} \cancel{c}}{4\varepsilon_0^2 \cancel{h}^2 \cancel{c}^2}, \quad (3.3b)$$

$$\Delta\alpha_{\text{calc}} = \frac{\cancel{2e \Delta e (2\varepsilon_0)} - e^2 2\Delta\varepsilon_0}{4 2\varepsilon_0^2 hc}, \quad (3.3c)$$

$$\Delta\alpha_{\text{calc}} = -\frac{e^2 \Delta\varepsilon_0}{2\varepsilon_0^2 hc}. \quad (3.3d)$$

As a corollary to this exercise, we can crosscheck whether the relative accuracy 8.1×10^{-11} on the latest experimental value of the *Fine-Structure Constant* — *i.e.*, α_{exp} [5] — is identical to Eq.(3.3d), that is, α_{calc} as computed from the latest fixed values of the relevant constants:

$$\frac{\Delta\alpha_{\text{exp}}}{\alpha_{\text{exp}}} \stackrel{?}{=} -\frac{e^2 \Delta\varepsilon_0}{2\varepsilon_0^2 hc \alpha_{\text{exp}}}, \quad (3.4a)$$

where, by convention, the finite difference on the formerly fixed ε_0 is

$$\Delta\varepsilon_0 = (\varepsilon_0/\mathcal{k}) - \varepsilon_0, \tag{3.4b}$$

$$\Delta\varepsilon_0 = \varepsilon_0 (1/\mathcal{k} - 1); \tag{3.4c}$$

with \mathcal{k} being the aforementioned correction factor 1.000 000 000 55(15).

Seeing as e is fixed at 1.602176634E-19 *Coulombs*, c is fixed at 299,792,458 (m/s) and h is fixed at 6.62607015E-34 *Js* — along with the values $\alpha_{\text{calc}} = 1/137.035999083(14)$ and $\alpha_{\text{exp}} = 1/137.035999206(11)$ [5] — Eq.(3.4a) numerically yields:

$$(8.1 \times 10^{-11}) \stackrel{?}{=} \frac{(1.6 \times 10^{-19})^2 \cdot \left[\frac{10^7}{4\pi |c_{\text{MKS}}|^2} \cdot \left(\frac{1}{1.0000000005515} - 1 \right) \right]}{2 \cdot \left(\frac{10^7}{4\pi |c_{\text{MKS}}|^2} \right)^2 \cdot (6.6 \times 10^{-34}) \cdot 299792458 \cdot \left(\frac{1}{137.03599920611} \right)}, \tag{3.5a}$$

$$(8.1 \times 10^{-11}) \stackrel{?}{=} \frac{-(1.6 \times 10^{-19})^2 \cdot \left[\frac{10^7}{4\pi |c_{\text{MKS}}|^2} \cdot \frac{(1-1.0000000005515)}{1.0000000005515} \right]}{2 \cdot \left(\frac{10^7}{4\pi |c_{\text{MKS}}|^2} \right)^2 \cdot (6.6 \times 10^{-34}) \cdot |c_{\text{MKS}}| \cdot 0.007297352562781}, \tag{3.5b}$$

$$(8.1 \times 10^{-11}) \stackrel{?}{=} \frac{-(2.6 \times 10^{-38-11}) \cdot -0.0000000005515 \cdot 2\pi \cdot 299792458}{10^7 \cdot (6.6 \times 10^{-34}) \cdot 0.007297352562781 \cdot 1.0000000005515}, \tag{3.5c}$$

$$(8.1 \times 10^{-11}) \neq (5.5 \times 10^{-10}), \tag{3.5d}$$

with the ratio of the LHS of Eq.(3.5d) to its RHS equalling 15 percent. This verily amounts to more than an order of magnitude discrepancy between the relative inaccuracy of α_{exp} and that of α_{calc} just as we remarked previously.

As it so happens, a correction factor \mathcal{k} that seems to be haphazardly produced from what appears to be the chosen $\alpha = 1/137.035999046(27)$ [3] is indeed badly misaligned with the latest state-of-the-art laboratory assessment of the *Fine-Structure Constant* [5].

One way to remedy the non-conformance of concern is to modify the correction factor \mathcal{k} to the value 1.000 000 000 54(897) for $\alpha_{\text{CODATA2018}} = 1/137.035999083(49)$ [2]. Another is to pull it down to 0.999 999 999 88(59) for $\alpha_{\text{QED-based}} = 1/137.035999174(35)$ [4]. Yet another is to pull it even further down to 0.999 999 999 65(41) for $\alpha_{\text{exp}} = 1/137.035999206(11)$ [5]. The list can go on and so forth.

However, such modifications soon grow huge and are sure to adversely affect laboratory measurements where permittivity and permeability are substantial factors to consider in particularly solid state physics. We will come back to this point in the end.

4. NORMALIZING SI IN THE ELECTROMAGNETIC DOMAIN

Now we face a challenge, since we have at hand the task of normalizing *Henry*, *Farad*, and consequently *Ohm* and *Siemens* units. Fortunately, we have the possibility of quickly finding out the desired “*new Ohm*” through a novel upcoming reformulation of the *Fine-Structure Constant*, given that

$$\varepsilon_0 = \frac{10^7}{4\pi |c_{\text{MKS}}|^2} \left(\frac{\text{C}^2}{\text{N} \times \text{m}^2} \right) = \frac{10^7}{4\pi |c_{\text{MKS}}|^2} \left(\frac{\text{Farad}}{\text{m}} \right), \quad (4.1)$$

which has the dimensions of *Coulomb* squared per *Newton* times *Area* (i.e., *Farad per meter*); where the modulus of the utmost theoretical speed limit of light in empty space in MKS units is taken (i.e., just the number 299,792,458 without any units, denoted as $|c_{\text{MKS}}|^2$).

Our reformulation of the Fine-Structure Constant will be as follows: By replacing the abovegiven permittivity term in the denominator of Eq.(3.1a) with the RHS of Eq.(4.1), we can right away get to

$$\alpha = \frac{e^2}{hc} \times \left(\frac{4\pi |c_{\text{MKS}}|^2}{2 \times 10^7} \frac{\text{meter}}{\text{Farad}} \right) = \frac{e^2 2\pi}{h} \times \left(\frac{299,792,458}{10^7} \frac{\text{second}}{\text{Farad}} \right). \quad (4.2a)$$

Because 1 *second* over 1 *Farad* equals 1 *Ohm*, and since $299,792,458/10^7$ makes 29.9792458, it is then possible to arrive at

$$\alpha = \frac{e^2}{\hbar} \Omega^{-1} \times 29.9792458 \Omega; \quad (4.2b)$$

at which point, we have taken into account the dimension of the ratio e^2/\hbar as being Ω^{-1} (*Siemens*). Thus, through the cancellation of the unit of resistance (in *Ohm*) with its reciprocal (in *Siemens*), this relationship alternatively furnishes the expected dimensionless Fine-Structure Constant α in the form of a specific electrical resistance times elementary charge squared divided by the reduced Planck Constant.

However, due to the 2019 redefinition of the SI base units where the correction factor k is carried over to the vacuum permeability μ_0 in Eq.(3.2b) before the 10^{-7} term, the amount of Ω in Eq.(4.2b) becomes precisely 29.97924581(653355), with the last six digits denoting uncertainty in measurement.

Therefore — and keeping in mind that the elementary charge is now exactly $1.602176634\text{E} - 19$ *Coulombs* and the Planck Constant exactly $6.62607015\text{E} - 34$ *Js* — the 1 “*new Ohm*” (*NOhm*) which will yield α_{calc} as the designated $1/137.035999083(14)$ will be larger than *Ohm* by as much as $1.00000000055(15) \times 29.9792458$ times the 2π coming from the reduced Planck Constant:

$$1\text{N}\Omega = [29.97924581(653355) \times 2\pi] \Omega = 188.36515683(476849) \Omega. \quad (4.2c)$$

But here already we are at a crossroads, because, as we mentioned above, the latest recommended or measured value of α is at odds with α_{calc} arrived at by relying on the 2019 redefinitions despite the claimed CODATA 2018 accuracy of 1.5×10^{-10} relative standard uncertainty [2]. Recall that the situation is especially worse with the QED-based 2012 measurement [4] that gave $1/137.035999174(35)$ as well as with the findings by Guellati-Khélifa et al. [5] that gave $1/137.035999206(11)$; for which it is said: “*Surprisingly, her new measurement differs from Müller’s 2018 result [cf. 3] in the tenth digit, a bigger discrepancy than the margin of error of either measurement.*” [11].

Since no amount of adjusting the uncertainty parts of 29.97924581(653355) will rectify the conspicuous one order of magnitude mismatch at hand — and since both the adopted *Coulumb*² in the numerator of Eq.(4.1) and the $\text{kg} \times \text{m}^2/\text{s}$ times m/s terms (having the dimensions of the Planck Constant times the speed of light) out of $\text{N} \times \text{m}^2$ in the denominator of Eq.(4.1) are absolute — the only sensible way out to ascertain a suitable α without changing anything else at this juncture is to re-evaluate the elementary charge value, which we will come back to soon.

Seeing as it has already been mentioned that 1 *second* over 1 *Farad* equals 1 *Ohm*, the “new Farad” (*Narad*) will be:

$$[29.97924581(653355) \times 2\pi] \Omega = \frac{1 \text{ second} \times [29.97924581(653355) \times 2\pi]}{\text{Farad}}, \quad (4.3a)$$

$$1 \text{ Narad} = 1/188.36515683(476849) \text{ Farad} = 0.005308837455(94831) \text{ Farad}. \quad (4.3b)$$

With this accomplished, we now have the means to find out the “new Henry” (*Nenry*) we want, given that the traditional unit of inductance 1 *Henry* is equal to 1 *second* squared over 1 *Farad*:

$$1 \text{ Henry} = \frac{1 \text{ s}^2}{1 \text{ Farad}} = \frac{1 \text{ s}^2}{[29.97924581(653355) \times 2\pi] \text{ Narad}}, \quad (4.4a)$$

$$1 \text{ Nenry} = 188.36515683(476849) \text{ Henry}. \quad (4.4b)$$

Returning to the relationship (3.2a), we can substitute these new units for a cross-check via also dismissing a priori the aforesaid unwieldy 1.000 000 000 55(15) multipliers, for they eventually cancel each other out from μ_0 's numerator and ϵ_0 's denominator in the ratio below (so long as an unacceptable extra correction factor to attain α without re-touching e is avoided):

$$c^2 = \frac{1}{\left((4\pi \times 10^{-7}) \frac{\text{Nenry} \times 299,792,458}{(29.9792458 \times 2\pi) \text{ NLU}} \right) \times \epsilon_0}, \quad (4.5a)$$

$$c^2 = \frac{1}{\left(\frac{2}{1} \times \frac{\text{Nenry}}{\text{NLU}} \right) \times \left(\frac{10^7}{4\pi |c_{\text{MKS}}|^2} \cdot \frac{(29.9792458 \times 2\pi) \times \text{Narad} \times 299,792,458}{\text{NLU}} \right)}, \quad (4.5b)$$

$$c^2 = \frac{1}{\left(\frac{2}{1} \times \frac{\text{Nenry}}{\text{NLU}} \right) \times \left(\frac{1}{2} \times \frac{\text{Narad}}{\text{NLU}} \right)}, \quad (4.5c)$$

$$\frac{1 \text{ NLU}^2}{1 \text{ second}^2} = \frac{1 \text{ NLU}^2}{1 \text{ Nenry} \times 1 \text{ Narad}}. \quad (4.5d)$$

This furnishes *en bloc* the magnetic permeability of vacuum in our new units as:

$$\mu_0 = 2 \times \frac{\text{Nenry}}{\text{NLU}}; \quad (4.6)$$

and subsequently, we now have the vacuum permittivity *en bloc* in our new units as:

$$\epsilon_0 = \left(\frac{1}{2}\right) \times \frac{\text{Narad}}{\text{NLU}}, \quad (4.7)$$

along with the related uncertainties at the 10^{-11} level.

What is interesting is that setting $\epsilon_0 = \mu_0 = 1$ turns out to be improper, and uncertainties in *Nenry* (inductance) and *Narad* (capacitance) must also thence vary in an interrelated fashion by exactly the same amount, but in opposite directions, to ensure the universality and constancy of the speed of light (as long as the aforesaid unacceptable and unphysical extra correction factor meant for ϵ_0 so as not to re-touch the 2019 redefinition of e is eschewed).

Yet, one must not forget that *Henrys per meter* is classically the equivalent of *Newtons per Ampere squared* — from the retired definition of *Ampere* as the constant electrical current that generates 2×10^{-7} *Newtons* attractive force per 1 *meter* length of wire between two endlessly extended parallel wires of negligible cross-section placed 1 *meters* apart [12]. Note how the force of pull generated from two identical parallel steady electrical currents I_1 and I_2 intensifies, say, thrice after a reduction of the distance D thrice as per:

$$\text{Magnetic force intensity per unit length} = \frac{\mu_0 \times I_1 \times I_2}{2\pi \times D}. \quad (4.8)$$

Be that as it may, we shall introduce our new force and new current units in accordance with NOT the latest internationally fixed elementary charge value (which was hinted above to be in need of correction — because, otherwise, the necessary alteration to the dimensionless correction factor k becomes quite considerable), but according to an updated form of the old standard definition.

To do so, we start by calculating 1 *Newton* in our new mass and distance units:

$$1 \text{ Newton} = 1 \left(\frac{\text{kg} \times \text{m}}{\text{s}^2} \right) = \left(\frac{10^{51} \text{NWU} \times \text{NLU}}{7.37249732381271 \times \text{s}^2 \times 299,792,458} \right), \quad (4.9a)$$

$$1 \text{ Newton} = \left(\frac{10^{51} \text{NWU} \times \text{NLU}}{0.221021909430423 \times 10^{10} \text{s}^2} \right), \quad (4.9b)$$

$$1 \text{ Newton} = 4.52443833544382 \times 10^{41} \left(\frac{\text{NWU} \times \text{NLU}}{\text{s}^2} \right). \quad (4.9c)$$

Let's call $1 / 4.52443833544382\text{E}+41$ *Newton* force hereinafter "1 *Lewton*". After this, we can refer to the dimensional equivalence of the LHS and RHS of equation (4.1) to calculate the "New Coloumb" (*Loulomb*):

$$\left(\frac{C^2}{N \times m^2} \right) = \left(\frac{\text{Farad}}{m} \right), \quad (4.10a)$$

$$\frac{C^2 \times 299,792,458^2}{4.524\dots \times 10^{41} \left(\frac{NWU \times NLU}{s^2} \right) \times NLU^2} = \frac{\text{Narad} \times 299,792,458}{[29.97924581(653355) \times 2\pi]^{-1} \times NLU}, \quad (4.10b)$$

$$\frac{C^2}{4.524\dots \times 10^{41} NWU \times NLU^3} = \frac{[29.97924581(653355) \times 2\pi] \text{Narad}}{299,792,458 \times NLU \times s^2}, \quad (4.10c)$$

$$C^2 = \frac{4.524\dots \times 10^{41} NWU \times NLU^2 \times \text{Narad}}{1.591549430(04121) \times 10^6 \times s^2}, \quad (4.10d)$$

$$C^2 = 2.842788448(81787) \times 10^{35} \left(\frac{NWU \times NLU^2 \times \text{Narad}}{s^2} \right), \quad (4.10e)$$

$$C = \sqrt{2.842788448(81787) \times 10^{35} \text{Loulomb}} = 5.331780611(40729) \times 10^{17} \text{Loulomb}, \quad (4.10f)$$

$$\text{Loulomb} = \frac{C}{5.331780611(40729) \times 10^{17}}, \quad (4.10g)$$

$$\text{Loulomb} = 1.875546037(77303) \times 10^{-18} C; \quad (4.10h)$$

revealing incidentally that the uncertainty in NWU must, at the outset of the equality (4.10b), be of the order of 10^{-9} .

Immediately, one will notice that the ratio of 1 *Loulomb* in *Coulombs* to the novelly adopted SI elementary charge value — *i.e.*, 11.7062376143(293) — whose uncertainty digits have been arbitrarily and inappropriately cut off by 2019 so as to result in 1.602176634E – 19 *Coulombs*, is verily the inverse of the squareroot of the Fine-Structure Constant ($1/\sqrt{\alpha}$); which is otherwise the so-called “Planck charge”. In fact, one can, based on the precise QED-based value of α [4], re-calculate the elementary charge from Eq.(4.10h) to find 1.602176633(4668) $\times 10^{-19}$ *Coulombs*, or else, based on the most recent and accurate α_{exp} from [5], do the same and find 1.602176633(28115) $\times 10^{-19}$ *Coulombs*, or else still, based on the CODATA 2018 recommended value [2], do the same and arrive at 1.602176633(99797) $\times 10^{-19}$ *Coulombs*; but with the uncertainty parts restored either way, and all of the procedure eventually depending on how definitive the MKS quantity of the Planck Constant is initially taken as.

This is precisely where our far-reaching idea to reconstitute the formerly existing miniscule indeterminacy in h too comes under scrutiny. Not only does the uncertainty in NLU , but also the latently revealed uncertainty in NWU entails it. Since we already came to consign the restoration of the uncertainty parts to e just above in order to avoid an unduly large modification in the values of μ_0 and ϵ_0 , we can also readily consider that the Planck Constant, along with the speed of light in vacuum (with an implied indeterminacy in the rectilinear spatial distance), can be given back their uncertainty digits.

In what follows, because $1\text{ C} = 1\text{ A} \cdot 1\text{ s}$, we are able to straightforwardly establish the *New Ampere (Lampere)*:

$$5.331780611(40729) \times 10^{17} \text{ Loulomb} = 1 (\text{A} \cdot \text{s}), \quad (4.11a)$$

$$\text{Loulomb} = \left(\frac{\text{A}}{5.331780611(40729) \times 10^{17}} \right) \cdot \text{s}, \quad (4.11b)$$

$$\text{Lampere} = 1.875546037(77303) \times 10^{-18} \text{ A}, \quad (4.11c)$$

$$1 \text{ Loulomb} = 1 (\text{Lampere} \cdot \text{s}). \quad (4.11d)$$

We now reached the appropriate milestone to re-adjust the scales, since we are dealing with unreasonably huge or too miniscule numbers — where, in particular, *NLU* itself is a very long distance unit and is quite inapplicable in a real experiment of *Ampère's Force Law* as well as other laboratory setups. An acceptable recalibration would require the placement of the aforementioned parallel wires at about 30 *cm* from each other, so that every 30 *cm* long segment of the cable is exerted force upon by the full length of the other. This is possible if we take 10^{-9} *NLU* as the designated unit length for both the distance and the current-carrying cable segment. In such a case, the resulting magnetic force will be the same as when taking 1 *meter* as the unit length for both the separation between the wires and the individual wire segment.

Let us thereby, with the substitution of our “normalized SI” refinements, backtrack to *Ampère's Force Law* as had been shown in Eq.(4.8) to find the commensurate LHS and RHS terms in Eq.(4.12) below for the wires in question under vacuum conditions, when given 2×10^{-7} *Newtons* or $9.04887667... \times 10^{34}$ *Lewtons* force — times the neoteric correction factor of 1.00000000055(15) — per 10^{-9} *NLU* (*i.e.*, 0.299792458 *meters* or “1/3.33564095198152nd of a *meter*”) length of cable over the same 10^{-9} *NLU* distance between the cables:

$$\frac{F \times 1.00000000055(15)}{\ell_{10^{-9} \text{ NLU}}} = \frac{\mu_0 I_1 I_2}{2\pi D_{10^{-9} \text{ NLU}}}, \quad (4.12a)$$

$$\frac{2 \times 10^{-7} \times 4.52444... \times 10^{41} \text{ Lewton}}{1.00000000055(15)^{-1} \times 10^{-9} \text{ NLU}} = \frac{2 \text{ NHenry} \times (5.33178... \times 10^{17})^2 \text{ Lampere}^2}{\text{NLU} \times 2\pi \times 10^{-9} \text{ NLU}}, \quad (4.12b)$$

$$\frac{9.048876665(89718) \times 10^{34} \text{ L}}{10^{-9} \text{ NLU}} = \frac{9.048876675(87811) \times 10^{34} \text{ NHenry} \times \text{Lampere}^2}{10^{-9} \text{ NLU}^2}, \quad (4.12c)$$

$$\frac{9.048876665(89718) \times 10^{34} \text{ Lewton}}{10^{-9} \text{ Lampere}^2} = \frac{9.048876675(87811) \times 10^{34} \text{ NHenry}}{10^{-9} \text{ NLU}}; \quad (4.12d)$$

where the numerical inaccuracy in calculation can be overlooked, and indeterminacies in *Lampere*² and *NLU* appearing under the fractions can still come into play.

Notice that the *Lampere* value in Eq.(4.12d) is to be reduced at present by as much as $10^{4.5}$, leading to astronomical quantities such as $(5.93099733586697E - 23 \text{ Amperes})^2$ in the denominator of the LHS. This presents us with the opportunity to carry over the 10^{34} orders of magnitude from the numerator of the RHS to the denominator of the LHS so as to re-normalize and rename our units:

$$\frac{9.048876665(89718) \times 10^{34} \text{ Lewton}}{10^{25} \text{ Lampere}^2} = \frac{9.048876675(87811) \text{ Nenry}}{\text{NanoNLU}}, \quad (4.13a)$$

$$\frac{90.48876665(89718) \text{ GigaYotta Lewton}}{100 \times 10^{25} \text{ Lampere}^2} = \frac{9.048876675(87811) \text{ Nenry}}{100 \times \text{NanoNLU}}, \quad (4.13b)$$

$$\frac{0.9048876665(89718) \text{ ExaYotta Lewton}}{\text{Diecto-teradeca Lampere}^2} = \frac{9.048876675(87811) \text{ CentiNenry}}{\text{NanoNLU}}, \quad (4.13c)$$

$$\frac{0.9048876665(89718) \text{ Xiyotton}}{\text{Dihtedempere}^2} = \frac{0.9048876675(87811) \text{ DeciNenry}}{\text{NanoNLU}}; \quad (4.13d)$$

where the numerical inaccuracy in calculation can once more be overlooked, and where the indeterminacies of the quantities in the denominators can yet again come into play.

Over here, *ExaYotta Lewton* or “*Xiyotton*” is 10^{42} *Lewtons* (amounting to practically 2.21022 *Newtons*), while *CentiNenry* is 0.01 and *DeciNenry* is 0.1 *Nenry* (*i.e.*, 18.8 H), with their divisor *NanoNLU* denoting 10^{-9} *NLU* (*i.e.*, ≈ 0.3 m), and *Diecto-teradeca Lampere* or “*Dihtedempere*” simply constitutes 10^{17} ($= 10^{[2 \times 2] + [12 + 1]}$) *Lampere* (*i.e.*, 0.188 A).

Thus, after so far instituting the novelly proposed SI normalization vis-à-vis the present contribution, the humane re-normalization (*i.e.*, ergonomization) of all units — including those for the prospective re-parametrizations of Joule, Watt, Volt, Weber, Tesla, etc. . . — can be straightforwardly achieved.

5. SUMMARY OF RESULTS

The normalization and subsequent re-normalization procedure we brought to attention for the base units under SI formalism has been explained at length in the previous sections. Our metrological approach can be readily extended to all accustomed units in gravitational physics and electromagnetism.

Crucially, we have shown that the system of Natural Units improperly tries to set $\epsilon_0 = \mu_0 = \hbar = c = 1$. One should instead have $\epsilon_0 = 1/2$ and $\mu_0 = 2$ (with their respective dimensions) following the appropriate normalization procedure shown herein when unicitizing c and h (again with their corresponding dimensions). This also means that the impedance of free space $Z_0 = \sqrt{(\mu_0/\epsilon_0)}$ becomes 2 *NOhms*.

More importantly, we point out how the correction factor \mathcal{K} may not be altered in excess for the sake of preserving, in particular, the established parameters of solid state physics that rely on precise dielectric constants [cf. 13, 14] and magnetic susceptibilities [cf. 15, 16]; especially given that vacuum fluctuations change “*completely the idea of vacuum ... allowing for non linear electrodynamic effects in vacuum*” and it “*therefore behaves as a non linear polarisable and magnetisable medium*” [17].

Consequently, the removal of the uncertainty parts from the elementary charge quantity is what could be said to adversely affect the calculated value of the *Fine-Structure Constant* to the detriment of state-of-the-art empirical findings — as no amount of manipulating the uncertainty digits of the updated correction factor 1.000 000 000 55(15) carried over to ϵ_0 suffices to match the one order of magnitude deviation from especially the latest laboratory result [5] and the QED-based older result [4] that is in conformity with it. Otherwise, the failure to correctly align α_{calc} — as computed from the fixed constants of the 2019 redefinition — with α_{exp} as given by [5] through \hbar would entail the introduction of an extra unphysical and devoid-of-meaning dimensionless coefficient to ensure the required modification to vacuum permittivity. Since this is quite unacceptable, the more logical recourse at this stage is to re-touch the elementary charge value as discussed.

A further idea along this line is reinstating the uncertainty decimal places to the Planck Constant h and lightspeed c as well. The latter is strongly implied by the well-known $c^2 = 1/\mu_0\epsilon_0$ rule (Eq.(3.2a)). In both cases, as long as we are dealing with the dimensions of mass and length that naturally feature statistical errors in measurement, the restoration of the uncertainty parts to the values of the said constants is inescapable.

To overcome these bottlenecks, the 2019 international SI resolution must be revisited in just the way alluded to, wherefore we have taken the initiative to restore the uncertainty digits of the elementary charge value, and proceeded to reinstate a normalized *Ampere* called *Lampere* contingent upon a normalized *Coulomb* christened *Loulomb*. In this manner, as per the Eqs.(4.12d, 4.13d), the indeterminacy in electrical current now correctly reflects on the indeterminacy in inductance, whose uncertainty was already inserted at an early stage of our normalization.

Accordingly, the rectified elementary charge value is, contrary to what is asserted by the proponents of the system of Natural Units, $\sqrt{\alpha}$ *Loulombs* — thus requiring the elimination of the $\sqrt{4\pi}$ term from the *Heaviside-Lorentz* formalism. By this reason also do we maintain that the elementary charge uncertainty parts must indeed be restored.

Interesting of note is the parallelism between Quantum Field Theory (QFT) and YARK (Yarman-Arik-Kholmetskii) theory — absent with respect to classical relativity theory that says nothing about the discretization which arises from the presence of the Planck Constant — after the fact that both can now be said to rely on a shared understanding of $\hbar = c = 1$ that is rectified thanks to the proper SI normalization carried out in this contribution. Thus, the YARK formalism begets $2\epsilon_0 = (1/2)\mu_0 = (1/2)Z_0 = \hbar = c = (1/\sqrt{\alpha})e = 1$, all with their respective normalized dimensions and uncertainties.

Further study on the re-parametrization of other commonplace units along the principles established here such as Joule, Watt, Volt, Weber, Tesla, etc... can be tackled later. A summary of our numerical exercise is provided in the comparative **Table 1**.



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TABLE 1. Old MKS (meter-kilogram-second) system and New Base Units: Proper normalization and ergonomic re-normalization of SI quantities

OLD UNIT	MODIFIER	NEW UNIT	NUMERICAL VALUE
second	1	1 second	exactly 1 s
meter	299,792,458	1 NLU	299,792,45(7.99....) m
kilogram	$(6.62607015 \times 10^{-34}) / 299,792,458^2$	1 NWU	7.372497323(81271)E – 51 kg
Ohm	$1.00000000055(15) \times 29.9792458 \times 2\pi$	1 NOhm	188.36515683(476849) Ohm
Siemens	reciprocal of above	1 Niemens	5.308837455(94831)E – 03 S
Farad	same as above	1 Narad	5.308837455(94831)E – 03 Fa
Henry	$1.00000000055(15) \times 29.9792458 \times 2\pi$	1 Nenry	188.36515683(476849) Henry
μ_0 (vacuum permeability)	redefined by new units	2 Nenry / NLU	$4\pi \times \ell \times 10^{-7}$ Henry/m
ϵ_0 (vacuum permittivity)	redefined by new units	Narad / 2 NLU	$10^7 \text{ Fa} / (4\pi \times \ell \times c_{\text{MKS}} ^2)$ m
Z_0 (vacuum impedance)	redefined by a new unit	2 NOhm	376.730313(669537) Ohm
Newton	$299,792,458 / (6.62607015 \times 10^{-34})$	1 Lewton	1/4.524438335(44382)E + 41 N
Coulomb	$\sqrt{(\{6.62607015 \times 10^{-34}\} / \{29.97924581(653355) \times 2\pi\})}$	1 Loulomb	1.875546037(77303)E – 18 C
Ampere	$\sqrt{(\{6.62607015 \times 10^{-34}\} / \{29.97924581(653355) \times 2\pi\})}$	1 Lampere	1.875546037(77303)E – 18 A
Elementary charge “e”	redefined by a new unit	$\sqrt{\alpha_{\text{exp}}}$ Loulomb	0.08542454309(37812) Lou. or 1.602176633(28115) $\times 10^{-19}$ C
FSC: α	$e^2 / 2\epsilon_0 hc$	dimensionless	1 / 137.035999...(..)
(RENORMALIZED NEW UNITS)			
Lewton	Exa-Yotta (or $\times 10^{42}$)	1 <i>Xiyotton</i>	10^{42} Lewton or <i>2.21022... N</i>
Xiyotton	$\times 0.1$	1 DeciXiyotton	10^{41} Lewton or <i>0.221022... N</i>
Lampere	Dihecto-Teradeca (or $\times 10^{17}$)	1 Dihtedempere	10^{17} Lampere or <i>0.187555... A</i>
Dihtedempere	$\times 10$	1 <i>ExaLampere</i>	10^{18} Lampere or <i>1.87555... A</i>
Nenry	$\times 0.01$	1 <i>CentiNenry</i>	10^{-2} Nenry or <i>1.88365... H</i>
Narad	$\times 100$	1 <i>HectoNarad</i>	10^2 Narad or <i>0.530884... Fa</i>
NLU	$\times 0.000000001$	1 <i>NanoNLU</i>	10^{-9} NLU or about <i>30 cm</i>
NWU	Hecto-Diyotta (or $\times 10^{50}$)	1 <i>Hectodiyotwu</i>	10^{50} NWU or <i>0.73725... kg</i>

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