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FIBONACCI RANGE LABELING ON DIRECT PRODUCT OF PATH AND CYCLES GRAPHS

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Abstract. The primary concept of direct product constitute from the idea of product graphs establish from Weichsel [13], where the direct product of two graphs is connected if and only if both are connected and are not bipartite. From Imrich and Klavzar [6], the direct product $G\times H$ of graphs G and H is the graph with the vertex set $V(G) \times V(H)$ and for which vertices (x, y) and (x', y') being adjacent in $G\times H \iff xx' \in E(H)$ and $yy' \in E(G)$. Here, we characterize for direct product of graphs and prove on certain class of direct product of path and cycles graphs with Fibonacci range labeling.

Keywords: Direct product, Fibonacci range labeling, Fibonacci range graph, golden ratio.

AMS Subject Classification: (2020) Primary 05C78.

1. Introduction

In 1962, the Kronecker product of graphs proposed by Weichsel [13], establish that the direct product of two graphs G and H is connected if and only if both G and H are connected and are not bipartite. Imrich and Klavzar [6], gave three fundamental results on product graphs: the Cartesian product, the direct product and the strong product. Certain names on the direct product are used by different authors such as cardinal product, tensor product, Kronecker product, cross product, categorical product, conjuction etc. In particular, explicit formulae is obtain on direct product of graphs in terms of graph labeling and several other papers appeared from the works of Jha et al. [7], Schwarz and Troxell [11], Jha et al. [8] and for more survey on product graphs and labeling, see Chang and Kuo [4], Liu and Yeh [10], Jha [9] and others. Our aim in this paper, is to obtain similar categorical result for the direct product of path and cycles graph from the Fibonacci range labeling with the objective of determining a common ratio between the connected vertex set and edge set obtained from the product of two graphs. In the next section we prove

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our main theorems, proposition and remarks for any arbitrary labeling between any two vertices α and β. We also present examples on Fibonacci range labeling which constitute from product of two graphs viz., path and cycles graph.

2. Direct product of Path and Cycles graph

Recall from Weichsel [13], the direct product of two graphs G and H is connected if and only if both G and H are connected and are not bipartite. Imrich and Klavzar [6] defined the direct product $G{\times}H$ of graphs G and H is the graph with the vertex set $V(G) \times V(H)$ and for which vertices (x, y) and (x', y') being adjacent in $G{\times}H \iff xx'{\in} E(H)$ and $yy'{\in}E(G)$. Formally, we define a graph $G = (V, E)$ is said to be a Fibonacci range labeling if we label the vertices $x \in V$ with distinct labels $f(x) \to \{f_2, f_3, f_4, ..., f_{p+1}\}$ such that, when the edge $e = (\alpha, \beta)$ is labeled with $f^*(e = \alpha\beta) = \lceil \frac{f(\alpha)^2+f(\beta)^2}{f(\alpha)+f(\beta)} \rceil$ or $f^*(e = \alpha\beta) = \lfloor \frac{f(\alpha)^2+f(\beta)^2}{f(\alpha)+f(\beta)} \rfloor$, then the resulting edge gets unique label. Also, the ratio of each edge to the subsequent edge is in the form of the golden ratio given by $R_t(E_{i,i+1}) = \frac{E_{i+1}}{E_i} \approx \psi$ (for larger i), where $R_t(E_{i,i+1})$ is the ratio of the resulting induced edges $(e_1, e_2), (e_2, e_3)$, ..., (e_{n-1}, e_n) and $\psi = 1.618$ known as the golden ratio. If a graph G exhibit a Fibonacci range labeling then it is defined to be a Fibonacci range graph. Here, we consider all graphs to be simple, finite and undirected with no loops. In this section, we shows some result on the vertex edge connectivity for $P_n \times K_2$, $P_n \times K_3$, $P_n \times C_3$, $P_n \times C_4$, $C_n \times K_1$ in terms of the resultant graph obtained from the two product graph.

Proposition 2.1. *Consider α, β and l be in Z^+ with $\alpha < \beta$ then*

(i) $\alpha < \frac{\alpha^2+\beta^2}{\alpha+\beta} < \beta$

(ii) $l < \frac{l^2+(l+2)^2}{l+(l+2)} < l + 2$, *where* $l > 2$

(iii) $l < \frac{l^2+(l+3)^2}{l+(l+3)} < l + 3$

(iv) $l < \frac{l^2+(l+5)^2}{l+(l+5)} < l + 5$

(v) $l - 1 < \frac{1^2+l^2}{1+l} < l$

Remark 2.1. *The Fibonacci numbers are $\{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...,\}$ here, $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, ..., but all the vertices labeled should be distinct, so we consider the label from f_2 only.*

Lemma 2.1. *For any direct product $G_{a,b} \times H_{c,d} \equiv G_{\alpha,\beta}$*

Proof. For any two graph $G(V, E)$ and $H(V_*E_*)$ the direct product of $G \times H$ is defined for vertex as $V(G \times H) = V \times V_*$ and edge $E(G \times H) = \{\{(v_x\ w_x), (v_y\ w_y)\} : \{v_xv_y\}{\in} E$ and $\{w_xw_y{\in} E_*\}\}$. Then for any direct product $G_{a,b} \times H_{c,d} = H_{c,d} \times G_{a,b} \equiv G_{\alpha,\beta}$ such that $\frac{f(a)^2+f(b)^2}{f(a)+f(b)} \times \frac{f(c)^2+f(d)^2}{f(c)+f(d)} = \frac{f(c)^2+f(d)^2}{f(c)+f(d)} \times \frac{f(a)^2+f(b)^2}{f(a)+f(b)} \equiv \frac{f(\alpha)^2+f(\beta)^2}{f(\alpha)+f(\beta)}$.

Theorem 2.1. *For $n \geq 3$, the product graph $P_n \times K_2$ is a Fibonacci range labeling.*

Proof. Let P_n be the path with vertices $v_1, v_2, v_3, ..., v_n$ and u_i, w_i be the vertices of K_2 which are attached to the vertices of P_n. The order of the graph G is $p = 3n$ and size is $q = 3n - 1$. Define a function $f : V(G) \to \{f_2, f_3, f_4, ..., f_{p+1}\}$ defined by
$f^*(e = \alpha\beta) = \lceil \frac{f(\alpha)^2+f(\beta)^2}{f(\alpha)+f(\beta)} \rceil$ or $f^*(e = \alpha\beta) = \lfloor \frac{f(\alpha)^2+f(\beta)^2}{f(\alpha)+f(\beta)} \rfloor$

(i) $f(v_1) = 1$

(ii) $f(v_2) = 2$

(iii) $f(v_i) = f_{i+1}$, where $3 \leq i \leq n$

(iv) $f(u_i) = f_{k+i} + f_{k+(i-1)}$, where $1 \leq i \leq n$

(v) $f(w_i) = f_{2k+i} + f_{2k+(i-1)}$, where $k = 1, 2, 3, ...$, the k-copies

then we get the edge label as

(i) $f(v_1v_2) = 1$

(ii) $f(v_2v_3) = 3$

(iii) $f(v_iv_{i+1}) = f(v_i) + f_{i-1}$, for $3 \leq i \leq (n-1)$

(iv) $f(v_iu_i) = f(u_i) - i$, for $1 \leq i \leq 3$

(v) $f(v_iu_i) = f(u_i) - \{f_{i+1} - 1\}$, for $4 \leq i \leq n$

(vi) $f(v_iw_i) = f(w_i) - i$, for $1 \leq i \leq 3$

(vii) $f(v_iw_i) = f(w_i) - f_{i+1}$, for $4 \leq i \leq n$

From the above computations, the edge gets distinct label. Therefore, by Proposition 2.1, (i) (ii) and (v) all the edge label are unique and distinct. Hence, $P_n \times K_2$ for $n \geq 3$ is a Fibonacci range labeling. We show an illustration given in Fig. 1 for the graph $P_3 \times K_2$, $P_4 \times K_2$ and the ratio of its induced edge is in the form of the golden ratio $\psi = 1.618$

$$R_t(e_ie_{i+1}) = \begin{cases} 3 & for\ i = 1 \\ 1.33 & for\ i = 2 \\ 1.750 & for\ i = 3 \\ 1.571 & for\ i = 4 \\ 1.636 & for\ i = 5 \\ 1.611 & for\ i = 6 \\ 1.620 & for\ i = 7 \\ 1.617 & for\ i = 8 \\ 1.618 & for\ i = 9 \\ for\ higher\ order\ of\ n & \\ \cdots & \\ 1.618 & for\ i = n \end{cases} \tag{1}$$

$$R_t(e_j^1e_{j+1}^1) = \begin{cases} 1.583 & for\ j = 1 \\ 1.631 & for\ j = 2 \\ 1.645 & for\ j = 3 \\ 1.608 & for\ j = 4 \\ for\ higher\ order\ of\ n & \\ \cdots & \\ 1.618 & for\ j = n \end{cases} \tag{2}$$

$$R_t(e_k^2 e_{k+1}^2) = \begin{cases} 1.611 & for\ k = 1 \\ 1.620 & for\ k = 2 \\ 1.617 & for\ k = 3 \\ 1.618 & for\ k = 4 \\ for\ higher\ order\ of\ n & \\ \dots & \\ 1.618 & for\ k = n \end{cases} \tag{3}$$

For larger (i, j, k), $R_t(e_i e_{i+1})$, $R_t(e_j^1 e_{j+1}^1)$, $R_t(e_k^2\ e_{k+1}^2)$ approaches to the value of 1.618 i.e., $\approx \psi$, where $\psi = 1.618$ is the value of the golden ratio. Hence, for $P_n \times K_2$ the ratio of its edge label converges to the golden ratio when higher order are considered for n.

Example 2.1. *A Fibonacci range graph of $P_3 \times K_2$, $P_4 \times K_2$ is illustrated in view of the following graph.*

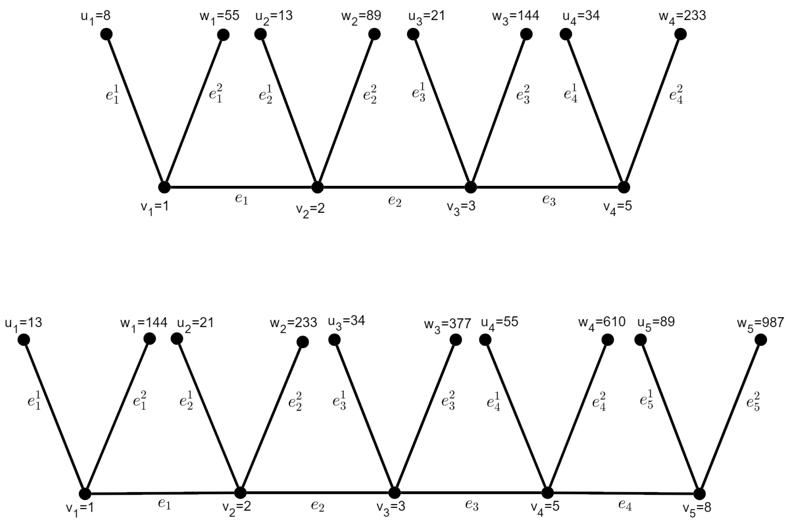

FIGURE 1. Fibonacci range graph of $P_3 \times K_2$, $P_4 \times K_2$

The values of the ratio fluctuate and differ as the order of n increases and it converges to ψ for higher order of n.

Theorem 2.2. *For $n \geq 3$, the product graph $P_n \times K_3$ is a Fibonacci range labeling.*

Proof. This proof follows from Theorem 2.1, by replacing K_2 with K_3 with the added vertex label

(i) $f(z_i) = f_{3k+i} + f_{3k+(i-1)}$, where $k = 1, 2, 3, \ldots$, the k-copies

then it will generate the edge label as

(i) $f(v_i z_i) = f(z_i) - i$, for $1 \leq i \leq 3$

(ii) $f(v_i z_i) = f(z_i) - f_{i+1}$, for $4 \leq i \leq n$

And the rest follows the same from Theorem 2.1, the following labeling is illustrated in Fig. 2 where the product graph $P_3 \times K_3$, $P_4 \times K_3$ is shown with the corresponding edge label required to appear towards the golden ratio ψ.

Example 2.2. *A Fibonacci range graph of* $P_3 \times K_3$, $P_4 \times K_3$ *is illustrated in view of the following graph.*

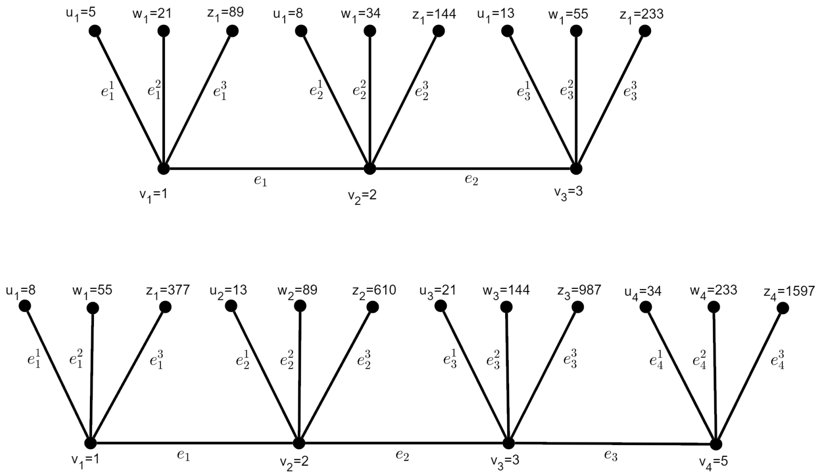

FIGURE 2. Fibonacci range graph of $P_3 \times K_3$, $P_4 \times K_3$

The values of the ratio fluctuate and differ as the order of n increases and it converges to ψ *for higher order of n.*

Corollary 2.1. *Without loss of generality, from Theorem 2.1 and Theorem 2.2, for any direct product of two graph graph* $P_n \times K_m$ *is a Fibonacci range labeling for any values of* $n \geq 3$, $m \geq 2$ *and by lemma 2.1, it follows.*

Theorem 2.3. *For* $n \geq 3$, *the product graph* $P_n \times C_3$ *is a Fibonacci range labeling.*

Proof. Let $P_n \times C_3$ be the graph with vertices v_1, v_2, v_3 ,..., v_n and u_1, u_2, u_3 ,..., u_m. The order of the graph is $p = 2n - 1$ and size $q = 3n - 3$ edges. Define a function $f : V(G) \to \{f_2, f_3, f_4, ..., f_{p+1}\}$ defined by

$f^*(e = \alpha\beta) = \lceil \frac{f(\alpha)^2 + f(\beta)^2}{f(\alpha)+f(\beta)} \rceil$ or $f^*(e = \alpha\beta) = \lfloor \frac{f(\alpha)^2 + f(\beta)^2}{f(\alpha)+f(\beta)} \rfloor$

(*i*) $f(v_i) = f_{i+1}$, where $1 \leq i \leq n$

(*ii*) $f(u_i) = f_{k+(i+1)} + f_{k+i}$, where $k = 1, 2, 3, ...,$ the k-copies

then we get the edge gets label as

(*i*) $f(v_1v_2) = 1$

(*ii*) $f(v_2v_3) = 3$

(*iii*) $f(v_iv_{i+1}) = f(v_i) + f_{i-1}$, for $3 \leq i \leq (n-1)$

(*iv*) $f(v_iu_i) = f(u_i) - f_{i+1}$, for $1 \leq i \leq (n-3)$ and $1 \leq i \leq (m-2)$

(*v*) $f(v_{i+1}u_i) = f(u_i) - f_{i+2}$, for $1 \leq i \leq (n-3)$ and $1 \leq i \leq (m-2)$

(*vi*) $f(v_{n-2}u_{m-1}) = f(u_{m-1}) - \{f_m - 1\}$

(vii) $f(v_{n-1}u_{m-1}) = f(u_{m-1}) - \{f_{m+1} - 1\}$

$(viii)$ $f(v_{n-1}u_m) = f(u_m) - \{f_{m+1} - 1\}$

(ix) $f(v_n u_m) = f(u_m) - \{f_{m+2} - 2\}$

From the above computations, the edge gets distinct label. Therefore, by Proposition 2.1, (i) (ii) and (v) all the edge label are unique and distinct. Hence, $P_n \times C_3$ for $n \geq 3$ is a Fibonacci range labeling. We show an illustration given in Fig. 3 for the graph $P_5 \times C_3$, $P_6 \times C_3$ and the ratio of its induced edge is in the form of the golden ratio $\psi = 1.618$

$$R_t(e_i e_{i+1}) = \begin{cases} 3 & for\ i = 1 \\ 1.33 & for\ i = 2 \\ 1.750 & for\ i = 3 \\ 1.571 & for\ i = 4 \\ 1.636 & for\ i = 5 \\ 1.611 & for\ i = 6 \\ 1.620 & for\ i = 7 \\ 1.617 & for\ i = 8 \\ 1.618 & for\ i = 9 \\ 1.618 & for\ i = 10 \\ for\ higher\ order\ of\ n & \\ \ldots & \\ 1.618 & for\ i = n \end{cases} \tag{4}$$

$$R_t(e_j^1 e_{j+1}^1) = \begin{cases} 1.600 & for\ j = 1 \\ 1.625 & for\ j = 2 \\ 1.634 & for\ j = 3 \\ 1.612 & for\ j = 4 \\ for\ higher\ order\ of\ n & \\ \ldots & \\ 1.618 & for\ j = n \end{cases} \tag{5}$$

$$R_t(e_k^2 e_{k+1}^2) = \begin{cases} 1.631 & for\ k = 1 \\ 1.613 & for\ k = 2 \\ 1.640 & for\ k = 3 \\ 1.621 & for\ k = 4 \\ for\ higher\ order\ of\ n & \\ \ldots & \\ 1.618 & for\ k = n \end{cases} \tag{6}$$

For larger (i, j, k), $R_t(e_i e_{i+1})$, $R_t(e_j^1 e_{j+1}^1)$, $R_t(e_k^2\ e_{k+1}^2)$ approaches to the value of 1.618 i.e., $\approx \psi$, where $\psi = 1.618$ is the value of the golden ratio. Hence, for $P_n \times C_3$ the ratio of its edge label converges to the golden ratio when higher order are considered for n.

Example 2.3. *A Fibonacci range graph of $P_5 \times C_3$, $P_6 \times C_3$ is illustrated in view of the following graph.*

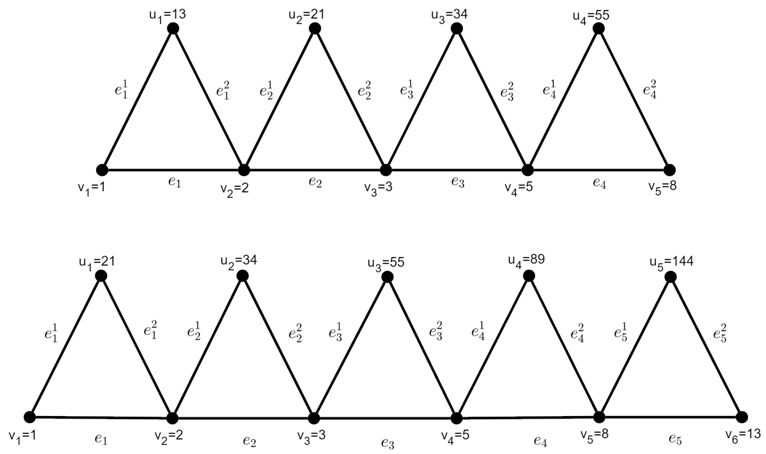

FIGURE 3. Fibonacci range graph of $P_5 \times C_3$, $P_6 \times C_3$

The values of the ratio fluctuate and differ as the order of n increases and it converges to ψ for higher order of n.

Theorem 2.4. *For $n \geq 3$, the product graph $P_n \times C_4$ is a Fibonacci range labeling.*

Proof. This proof follows from Theorem 2.3, by replacing the order of C_3 cycle with C_4 cycle with the change in vertex label

(*i*) $f(w_i) = f_{k+i} + f_{k+(i-1)}$

(*ii*) $f(u_i) = f_{2k+i} + f_{2k+(i-1)}$, where $k = 1, 2, 3, ...$, the k-copies

then it will generate the edege label as

(*i*) $f(u_i w_i) = f(u_i) - \{f(w_i) - 2\}$
(*ii*) $f(u_{i+1} w_i) = f(u_{i+1}) - \{f(w_i) - 2\}$

And the rest follows the same from Theorem 2.3, the following labeling is illustrated in Fig. 4 where the product graph $P_4 \times C_4$, $P_5 \times C_4$ is shown with the corresponding edge label required to appear towards the golden ratio ψ.

Example 2.4. *A Fibonacci range graph of $P_4 \times C_4$, $P_5 \times C_4$ is illustrated in of the following graph.*

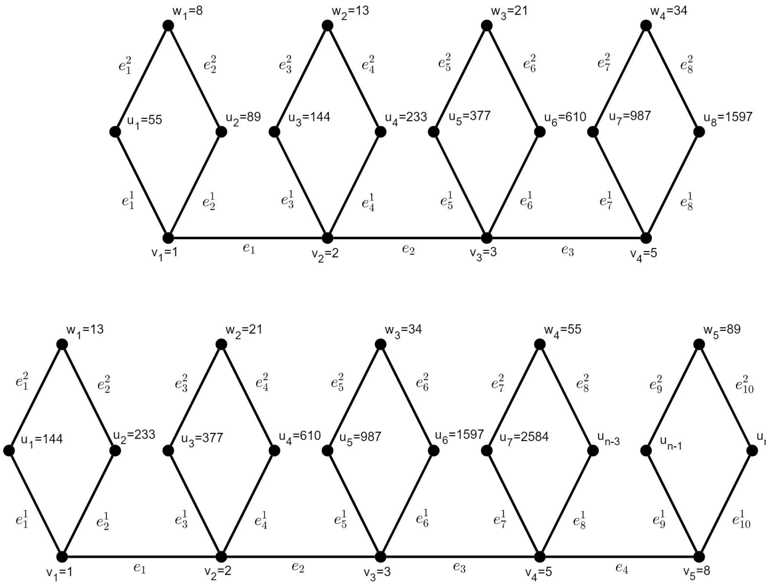

FIGURE 4. Fibonacci range graph of $P_4 \times C_4$, $P_5 \times C_4$

The values of the ratio fluctuate and differ as the order of n increases and it converges to ψ for higher order of n.

Corollary 2.2. *Without loss of generality, from Theorem 2.3 and Theorem 2.4, for any direct product of two graph $P_n \times C_m$ is a Fibonacci range labeling for any values of $n \geq 3$, $m \geq 3$ and by lemma 2.1, it follows.*

Theorem 2.5. *For $n \geq 3$, the product graph $C_n \times K_1$ is a Fibonacci range labeling.*

Proof. Let C_n be the cycles with vertices $v_1, v_2, v_3, ...v_n v_1$ and u_1, u_2, u_3, ..., u_n be the pendent vertex adjacent to v_i. Define a function $f : V(G) \to \{f_2, f_3, f_4, ...f_{n+1}\}$ defined by

$f^*(e = \alpha\beta) = \lceil \frac{f(\alpha)^2+f(\beta)^2}{f(\alpha)+f(\beta)} \rceil$ or $f^*(e = \alpha\beta) = \lfloor \frac{f(\alpha)^2+f(\beta)^2}{f(\alpha)+f(\beta)} \rfloor$

(i) $f(v_i) = f_{i+1}$, where $1 \leq$ i $\leq$ n

(ii) $f(u_i) = f_{n+(i+1)}$, where n = number of vertices of the cycle C_n

then we get the edge gets label as

(i) $f(v_1 v_2) = 1$

(ii) $f(v_2 v_3) = 3$

(iii) $f(v_i v_{i+1}) = f(v_i) + f_{i-1}$, for $3 \leq i \leq (n-1)$

(iv) $f(v_n v_1) = f(v_n) - 1$

(v) $f(v_i u_i) = f(u_i) - f(v_i)$, for $1 \leq$ i ≤ 4

(vi) $f(u_i)$ - $f(v_i - 1)$, for $5 \leq$ i $\leq n$

From the above computations, the edge gets distinct label. Therefore, by Proposition 2.1, (i) (ii) and (v) all the edge label are unique and distinct. Hence, $C_n \times K_1$ for $n \geq 3$ is a

Fibonacci range labeling. We show an illustration given in Fig. 5 for the graph $C_8 \times K_1$ and the ratio of its induced edge is in the form of the golden ratio $\psi = 1.618$

$$R_t(e_ie_{i+1}) = \begin{cases} 3, & for\ i=1 \\ 1.33 & for\ i=2 \\ 1.750 & for\ i=3 \\ 1.571 & for\ i=4 \\ 1.636 & for\ i=5 \\ 1.611 & for\ i=6 \\ 1.620 & for\ i=7 \\ 1.617 & for\ i=8 \\ 1.618 & for\ i=9 \\ 1.618 & for\ i=10 \\ for\ higher\ order\ of\ n & \\ \dots & \\ 1.618 & for\ j=n \end{cases} \tag{7}$$

$$R_t(e^1_je^1_{j+1}) = \begin{cases} 1.611 & for\ j=1 \\ 1.620 & for\ j=2 \\ 1.617 & for\ j=3 \\ 1.622 & for\ j=4 \\ 1.617 & for\ j=5 \\ 1.617 & for\ j=6 \\ 1.617 & for\ j=7 \\ 1.617 & for\ j=8 \\ for\ higher\ order\ of\ n & \\ \dots & \\ 1.618 & for\ j=n \end{cases} \tag{8}$$

For larger (i, j), $R_t(e_ie_{i+1})$, $R_t(e^1_je^1_{j+1})$, approaches to the value of 1.618 i.e., $\approx \psi$, where $\psi = 1.618$ is the value of the golden ratio. Hence, for $C_n \times K_1$ the ratio of its edge label converges to the golden ratio when higher order are considered for n.

Example 2.5. *A Fibonacci range graph of $C_8 \times K_1$ is illustrated in view of the following graph.*

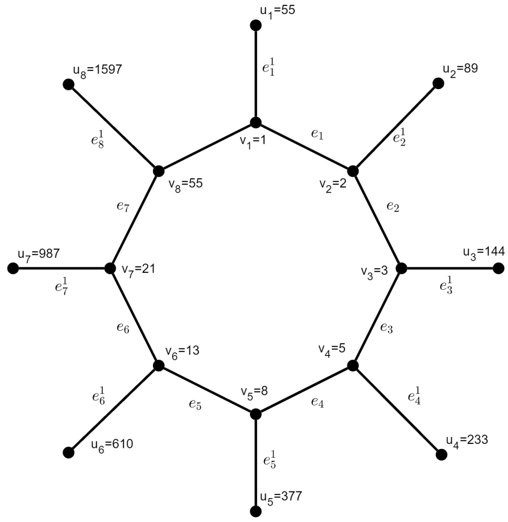

FIGURE 5. Fibonacci range graph of $C_8 \times K_1$

The values of the ratio fluctuate and differ as the order of n increases and it converges to ψ for higher order of n.

Corollary 2.3. *Similarily, the direct product of cycle $C_n \times K_1$ for $n \geq 3$ follows the same from Corollary 2.1 and Corollary 2.2.*

3. Conclusions

In this article, we demonstrate a brief result on the direct products of path and cycles graph by assigning a general framework based on Fibonacci range labeling. The objective to construct a distinct edge label is to achieve a common ratio (ψ) between the edge labeled. This simple approach to direct product of path and cycles with regard to graph labeling is indispensable on the complexity of product graphs.

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