TWMS J. App. and Eng. Math. V.14, N.3, 2024, pp. 1242-1253

REVISED INTERVAL ASM METHOD FOR MULTI-OBJECTIVE INTERVAL TRANSPORTATION PROBLEMS

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Abstract. In this paper, a revised version of interval ASM method is proposed for solving multi objective interval transportation problems without converting them to their crisp equivalent forms. Multi-objective transportation problems (MOITP) involving interval parameters are considered. By assigning suitable weights to each objective function, the multiple objective interval transportation problem is reduced to a single objective interval transportation problem (SOITP). Initial basic feasible solution is obtained by applying the proposed method with the help of new interval arithmetic and ranking functions and the optimality is tested using the interval version of MODI method. A numerical example is provided to illustrate the efficiency of the proposed method.

Keywords: Interval Numbers, New interval arithmetic, Ranking, Interval initial feasible solution, ASM method.

AMS Subject Classification: 90C08, 90C70, 90B06, 90C29, 90C90.

1. Introduction

Transportation problem (TP) is a special category of linear programming problem. The main objective of a transportation problem involves origins and destinations, for example, factories where products are produced, and a demanded quantity of these manufactured products are supplied to a particular number of destinations in such a way that the total transportation cost is minimum. When some or all the decision parameters (say cost, supply, demand etc) of the transportation problem are interval numbers, then it is called an interval transportation problem. Multi-objective interval transportation model involves searching for the best transportation set-up that meets the decision maker's preferences by considering the conflicting objectives/criteria such as transportation cost, transportation time, environmental and social issues.The purpose of this study is to investigate the

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§ Manuscript received: September 08, 2022; accepted: November 23, 2022.

best compromise solution of multi-objective interval transportation problem under interval version of revised ASM method.

Several methods exist in the literature to solve these kinds of problems. The interval and fuzzy extensions of traditional shipping issues have been developed by Chanas et al. [3]. Das et al. [5] introduced a multi-objective shipping issue with interval price, supply, as well as end factors. Solving linear indoctrination with interval coefficients has been discussed by Chinneck and Ramadan [4]. New arithmetic procedures for interval numbers have been proposed by Ganesan and Veeramani [6]. The properties of interval matrices have been discussed by Ganesan [7]. Hitchcock [9] spoke about the distribution of a product from multiple suppliers to a variety of locations. Pandian and Natrajan [14] projected a latest technique designed for solving completely interval integer shipping issues that finds the best answer. Sudhakar and Navaneetha Kumar [22] suggested a latest method for solving integer interval shipping problems. Roy and Mahapatra [17] created a multi-objective interval-valued shipping probabilistic problem using log-normal distributions. A duality speculation for interval linear indoctrination trouble was suggested by Ramesh and Ganesan [15]. Abdul Quddoos et al. [2] discussed an improved adaptation of the ASM-method for addressing transportation issues. Grey position management speculation based on grey statistics was used by Jignasha Patel and Jayesh Dhodiya [11] to solve a multi-objective interval shipping problem. Shraddha Mishra [19] investigated several approaches to resolving transportation issues. Akilbasha and Natarajan [1] presented an original correct method for resolving completely interval numeral transportation troubles. Sophia Porchelvi and Anitha [20] conducted a qualified research of the best solution for the trouble of interval shipping and interval transhipment. Ramesh et al. [16] suggested an unique method for solving the multi-objective interval transportation issue. Muzaffa Makhmudov and Chang seong Ko [13] projected a fuzzy set theory-based conciliation variance resolution technique to the uncertain transportation problem. In the field of sustainable development, Gurupada Maity et al. [8] suggested a time modification multi-objective interval-valued shipping trouble. Sudha and Ganesan [21] presented a time-saving alternative strategy for solving an interval numeral transportation problem. Indira and Jayalakshmi [10] suggested a entirely interval numeral transportation problem and used the Row-Column Minima technique to get the best interval elucidation. In this paper précised contributions is solved multi-objective interval transportation problems as well as can use n- number of objective interval transportation problem the proposed method to get the minimum time and maximum profit. The advantage for solving the interval transportation problem without affecting interval nature of the transportation problem we have obtained the interval optimum solution.

In this paper we propose an interval version of revised ASM method under a new interval arithmetic to solve the problem of multi-objective interval transportation without converting into crisp form. The structure of this article is given as follows: Section 2 deals with the basic definitions. Section 3 deals with the problem of multi-objective interval transportation and the related results. Section 4 introduces revised interval ASM method under generalized interval arithmetic. Section 5 provides an example to illustrate the theory developed in this paper. Section 6 concludes this paper.

2. Preliminaries

The purpose of this segment is to provide some observations, ideas and results which are useful in our further consideration.

2.1. **Interval numbers.** Let $\tilde{a} = [a_1, a_2] = \{x \in R : a_1 \leq x \text{ and } \mathrm{a_1, a_2 \in R}\}$ be an interval on the real line R. If $\tilde{a} = a_1 = a_2 = a$, then $\tilde{a} = [a, a] = a$ is a real number (or a degenerate interval). We shall make use of the terms interval and interval number interchangeably. The mid-point and width (or half-width) of an interval number $\tilde{a} = [a_1, a_2]$ are defined as $m(\tilde{a}) = \left(\dfrac{a_1 + a_2}{2}\right)$ and $w(\tilde{a}) = \left(\dfrac{a_2 - a_1}{2}\right)$ The interval number $\tilde{a}$ can also be expressed in terms of its midpoint and width as $\tilde{a} = [a_1, a_2] = \langle m(\tilde{a}), w(\tilde{a})\rangle$ We use IR to denote the set of all interval numbers definedon the real line R.

2.2. **Ranking of Interval Numbers.** Sengupta and Pal [17] suggested an easy and powerful index to compare any two intervals on IR through the satisfaction of decision-makers.

Definition 2.1. *Let $\preceq$ be an extended order relation between the interval numbers $\tilde{a} = [a_1, a_2], \tilde{b} = [b_1, b_2]$ in IR then for $m(\tilde{a}) < m(\tilde{b})$ we construct a premise $(\tilde{a} \preceq \tilde{b})$ which implies that $\tilde{a}$ is inferior to $\tilde{b}$ (or $\tilde{b}$ is superior to $\tilde{a}$).*

An acceptability function $A_{\preceq} : IR \times IR \to [0, \infty)$ is defined as:

$$A_{\preceq}(\tilde{a}, \tilde{b}) = A(\tilde{a} \preceq \tilde{b}) = \frac{m(\tilde{b}) - m(\tilde{a})}{w(\tilde{b}) + w(\tilde{a})}, \text{ where } w(\tilde{b}) + w(\tilde{a}) \neq 0.$$

$A_{\preceq}$ may be interpreted as the grade of acceptability of the first interval number to be inferior to the second interval number. For any two interval numbers $\tilde{a}$ and $\tilde{b}$ in IR either $A(\tilde{a} \preceq \tilde{b}) \geq 0$ (or) $A(\tilde{b} \succeq \tilde{a}) \succeq 0$ (or) $A(\tilde{a} \preceq \tilde{b}) = 0$ (or) $A(\tilde{b} \succeq \tilde{a}) = 0$ (or) $A(\tilde{a} \preceq \tilde{b}) + A(\tilde{b} \preceq \tilde{a}) = 0$. If $A(\tilde{a} \preceq \tilde{b}) = 0$ and $A(\tilde{b} \preceq \tilde{a}) = 0$, then we say that the interval numbers $\tilde{a}$ and $\tilde{b}$ are equivalent (non-inferior to each other) and we denote it by $\tilde{a} \approx \tilde{b}$. Also if $A(\tilde{a} \preceq \tilde{b}) \geq 0$, then $\tilde{a} \preceq \tilde{b}$ and if $A(\tilde{b} \preceq \tilde{a}) \geq 0$, then $\tilde{b} \preceq \tilde{a}$.

2.3. **A New Interval Arithmetic.** Ming Ma et al. [10] suggested a new fuzzy arithmetic focused on the index of locations and the index function of fuzziness. For the ordinary arithmetic the position index number is taken, while in the lattice L the fuzziness index functions are assumed to obey the lattice law which is the least upper bound and the greatest lower bound. That is for $a, b \in L$ we define $a \vee b = \max\{a, b\}$ and $a \wedge b = \min\{a, b\}$.

For any two intervals $\tilde{a} = [a_1, a_2], \tilde{b} = [b_1, b_2] \in IR$ and for $* \in \{+, -, \cdot, \div\}$ the arithmetic operations on $\tilde{a}$ and $\tilde{b}$ are defined as:

$$\begin{aligned}\tilde{a} * \tilde{b} &= [a_1, a_2] * [b_1, b_2] = \langle m(\tilde{a}), w(\tilde{a})\rangle * \langle m(\tilde{b}), w(\tilde{b})\rangle \\ &= \langle m(\tilde{a}) * m(\tilde{b}), \max\{w(\tilde{a}), w(\tilde{b})\}\rangle\end{aligned}$$

We must be relevant the interval arithmetic operations of addition, subtraction, multiplication, and division in scrupulous here. Applying the Ming Ma's new arithmetic operations the fuzzy number in its parametric form is similar to expressing the interval number in midpoint width form. We can use only maximum or only the minimum of width for all the four arithmetic operations. Also maximum width can be used for addition and multiplication and minimum width can be used for subtraction and division.

3. Main results

Consider a fully interval transportation problem with m sources and n destinations involving interval numbers. Let $\tilde{a}_i \preceq \tilde{0}$ be the availability at source i and $\tilde{b}_j (\tilde{b}_j \preceq \tilde{0})$ be the requirement at destination j. Let $\tilde{c}_{ij} (\tilde{c}_{ij} \preceq \tilde{0})$ be the unit interval transportation cost from source i to destination j. Let $\tilde{x}_{ij}$ denote the number of units to be transported from source i to destination j. Now the problem is to find a feasible way of transporting the available amount at each source to satisfy the demand at each destination so that the total interval transportation cost is minimized.

3.1. **General interval transportation problem.** The mathematical model of an interval transportation problem is as follows

$$\text{Minimize } \tilde{Z} \approx \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij}$$

$$\text{subject to } \sum_{j=1}^{n} \tilde{x}_{i}j \approx \tilde{a}_i, i = 1, 2, 3, \cdots, m$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j, j = 1, 2, 3, \cdots, n \tag{1}$$

$$\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j, \text{ where } 1, 2, 3, \cdots, m; j = 1, 2, 3, \cdots n$$

and $\tilde{x}_{ij} \succeq \tilde{0}$ for all i and j.
and $\tilde{a}_i, \tilde{b}_j, \tilde{c}_{ij}, \tilde{x}_{ij}$ in IR where $\tilde{c}_{ij}$ is the interval unit transportation cost from i^{th} source to the j^{th} destination.

3.2. **General form of interval transportation problems with multiple objectives.** The mathematical formulation of interval transportation problems with multiple objectives when all the cost coefficient, supply and demand are interval numbers is given by:

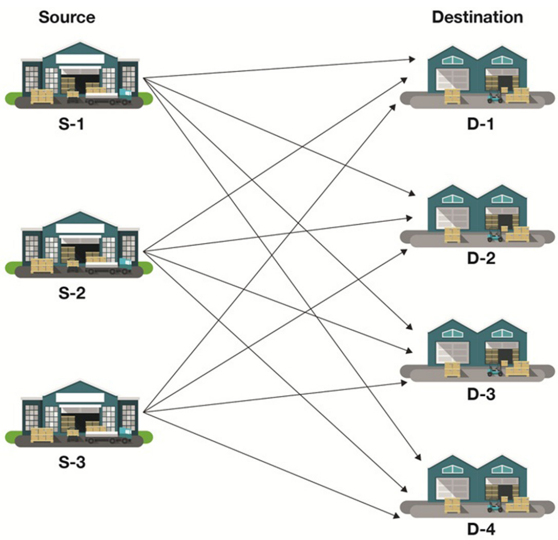

FIGURE 1. Multi objective transportation problem

$$
\begin{aligned}
&\text{Minimize } \tilde{Z}^M \approx \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}^k \tilde{x}_{ij} \text{ where} k = 1, 2, \cdots, K \\
&\text{subject to } \sum_{j=1}^{n} \tilde{x}_i j \approx \tilde{a}_i, i = 1, 2, 3, \cdots, m \\
&\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j, j = 1, 2, 3, \cdots, n \\
&\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j, \text{ where } 1, 2, 3, \cdots, m; j = 1, 2, 3, \cdots n
\end{aligned} \tag{2}
$$

and $\tilde{x}_{ij} \succeq \tilde{0}$ for all i and j.
Where $\tilde{Z}^k \approx \{\tilde{Z}^1, \tilde{Z}^2, \cdots \tilde{Z}^K\}$ is a vector of K objective functions and the superscript on both $\tilde{Z}^k$ and c_{ij} are used to identify the number of objective functions $(k = 1, 2, \cdots, K)$.

3.3. **Weighted sum.** By assigning suitable weights $w_k > 0$ such that $\sum\limits_{k=1}^{K} w_k = 1$ to the K objective functions, the multiple objective interval transportation problem (2) becomes a single objective interval transportation problem

$$
\begin{aligned}
&\text{Minimize } \tilde{\mathbf{Z}} \approx \sum_{k=1}^{K} w_k \tilde{Z}^k \text{ where } \sum_{k=1}^{K} w_k = 1 \text{ and } w_k > 0. \\
&\text{subject to } \sum_{j=1}^{n} \tilde{x}_i j \approx \tilde{a}_i, i = 1, 2, 3, \cdots, m \\
&\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j, j = 1, 2, 3, \cdots, n \\
&\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j, \text{ where } 1, 2, 3, \cdots, m; j = 1, 2, 3, \cdots n
\end{aligned} \tag{3}
$$

and $\tilde{x}_{ij} \succeq \tilde{0}$ for all i and j.

Definition 3.1. *(Non-dominated solution). A feasible vector $\tilde{\mathbf{x}}^* \in X$ (X is the feasible region) yields a non-dominated solution of (2) if and only if, there is no other feasible vector $\tilde{\mathbf{x}}^*$ such that $\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} \tilde{c}_{ij}^k \tilde{x}_{ij} \preceq \sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} \tilde{c}_{ij}^k \tilde{x}_{ij}^*$, for all k and $\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} \tilde{c}_{ij}^k \tilde{x}_{ij}^* \prec \sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} \tilde{c}_{ij}^k \tilde{x}_{ij}^*$, for some $k, k = 1, 2, \cdots K$.*

Definition 3.2. *(Efficient solution). A point $\tilde{\mathbf{x}}^* \in X$ efficient iff there does not exist another $\tilde{\mathbf{x}} \in X$ such that $\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} \tilde{c}_{ij}^k \tilde{x}_{ij} \preceq \sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} \tilde{c}_{ij}^k \tilde{x}_{ij}^*$ all k and $\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} \tilde{c}_{ij}^k \tilde{x}_{ij}^* \not\approx \sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} \tilde{c}_{ij}^k \tilde{x}_{ij}^*$, for some k.*

Definition 3.3. *(Compromise solution). A feasible vector $\tilde{\mathbf{x}}^* \in X$ is called a compromise solution of (2) if and only if $\tilde{\mathbf{x}}^* \in IR^n$ and $\tilde{\mathbf{Z}}(\tilde{\mathbf{x}}^*) \wedge_{\tilde{\mathbf{x}} \in X} \tilde{\mathbf{Z}}(\tilde{\mathbf{x}})$, where $\wedge$ stands for minimum and X is the set of efficient solutions.*

Theorem 3.1. *If $\tilde{\mathbf{x}}^* \in X$ is an optimum solution the single objective interval transportation problem (3), then it is also a compromised (Pareto optimal) solution to the multi-objective interval transportation problem (2).*

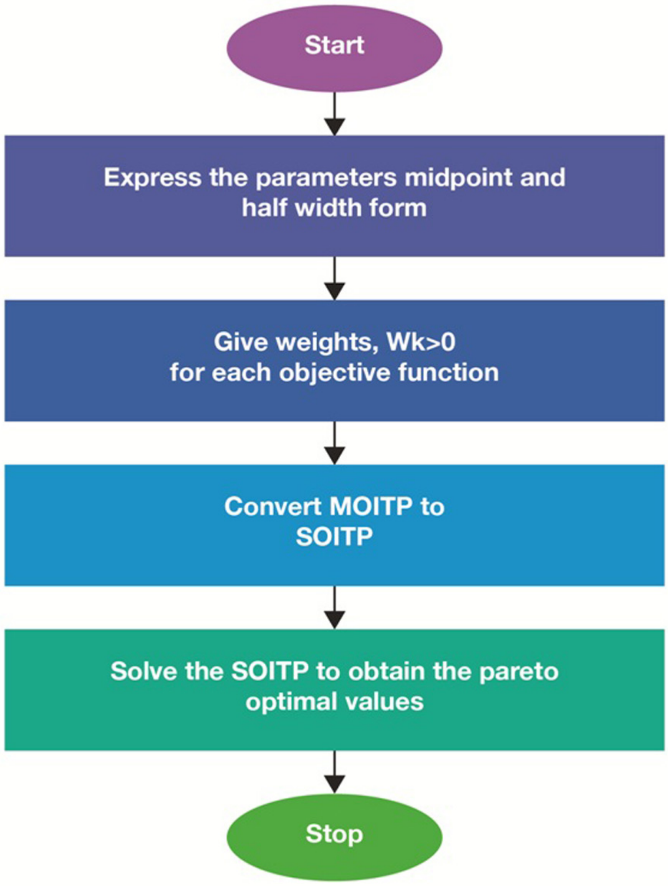

FIGURE 2. Flow chart: 1 Revised ASM method

4. Interval version of Revised ASM method

Step 1: Present the problem of multi-objective interval transportation in the transport table.

Step 2: Examine if the total supply is equal to the total demand, then it is a balanced problem, If not, add dummy row / column with zero cost and profit.

Step 3: Express the all interval parameters (supply, demand and unit transportation cost, profit etc) in the Multi-objective shipping problem in $\langle m(\tilde{a}), w(\tilde{(a)}\rangle$ form.

Step 4: Construct the single objective interval transportation problem (3) by giving suitable weights $w_k > 0$ such that $\sum_{k=1}^{K} w_k = 1$ to the K objective functions.

Step 5: In the resultant tableau, subtract the lowest ingredient of every row from every element of the relevant row, and then subtract the lowest ingredient of each column from every ingredient of the relevant column.

Step 6: Every row and column into the shortened tableau has at least one interval zero. choose the first interval zero and calculate the number of interval zeros in the row and column (except the selected one) as a subscript of the selected interval zero. In the transportation tableau, repeat this technique for all interval zeros.

Step 7: Choose the cell with the smallest value of subscript that has an interval zero and enter the highest amount possible in that cell. If there is a tie for any interval zeros in Step 6, choose the cell of that interval zero to break the tie so that the sum of all the components in the strip and column is the highest. Give everything you have to that cell.

Step 8: Cross out the row (or column) for which a certain source's supply is depleted for future consideration (or the demand for a given destination is satisfied). If both the column and row demand are completely satisfied at the same time, the column (or row, but not both) is simply eliminated, and the left behind row (or column) is assigned a zero supply (or demand) interval for future calculations.

Step 9: Do again the process until all of the demands have been met and all of the supplies have been depleted.

Step 10: Check the optimality of the IBFS obtained by means of the interval description of the MODI method. Stepping stone method is available to check the optimality of the initial basic feasible solution. But the stepping stone method, we have to draw as many closed paths as equal to the unoccupied cells for their evaluation. To the contrary, in MODI method, only closed path for the unoccupied cell with highest opportunity cost is drawn.

5. Mathematical Example

Consider a multi-objective interval transport issue discussed by Muzaffar Maakhmudov and Chang Seong Ko [13]

Table 1. Interval transportation time

	T1	T2	T3	T4	Supply
P1	[1, 2]	[1, 3]	[5, 9]	[4, 8]	8
P2	[1, 2]	[7, 10]	[2, 6]	[3, 5]	19
P3	[7, 9]	[7, 11]	[3, 5]	[5, 7]	17
Demand	11	3	14	16	

Table 2. Interval transportation profit

	T1	T2	T3	T4	Supply
P1	[3, 5]	[2, 6]	[2, 4]	[1, 5]	8
P2	[4, 6]	[7, 9]	[7, 10]	[9, 11]	19
P3	[4, 8]	[1,3]	[3, 6]	[1, 2]	17
Demand	11	3	14	16	

Since $\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j$, the given MOITP is balanced. In the Ming Ma's new arithmetic operations, we can use minimum for subtraction and division and maximum for addition and multiplication. If we use minimum instead of maximum the width of the final solution will be minimum.

We have expressed all of the interval parameters in terms of their midpoints and widths form, we have

Since Income $= -$ cost $(\max z = -\min(-z))$. By multiplying by -1, the income table is transformed into a price table.

TABLE 3. Interval transportation time in mid-point and width form

	T1	T2	T3	T4	Supply
P1	$\langle 1.5, 0.5\rangle$	$\langle 2, 1\rangle$	$\langle 7, 2\rangle$	$\langle 6, 2\rangle$	$\langle 8, 0\rangle$
P2	$\langle 1.5, 0.5\rangle$	$\langle 8.5, 1.5\rangle$	$\langle 4, 2\rangle$	$\langle 4, 1\rangle$	$\langle 19, 0\rangle$
P3	$\langle 8, 1\rangle$	$\langle 9, 2\rangle$	$\langle 4, 1\rangle$	$\langle 6, 1\rangle$	$\langle 17, 0\rangle$
Demand	$\langle 11, 0\rangle$	$\langle 3, 0\rangle$	$\langle 14, 0\rangle$	$\langle 16, 0\rangle$	$\langle 44, 0\rangle$

TABLE 4. Interval transportation profit in $\langle m(\tilde{a}), w(\tilde{a})\rangle$ form

	T1	T2	T3	T4	Supply
P1	$\langle -4, 1\rangle$	$\langle -4, 2\rangle$	$\langle -3, 1\rangle$	$\langle -3, 2\rangle$	$\langle 8, 0\rangle$
P2	$\langle -5, 1\rangle$	$\langle -8, 1\rangle$	$\langle -8.5, 1.5\rangle$	$\langle -10, 1\rangle$	$\langle 19, 0\rangle$
P3	$\langle -6, 2\rangle$	$\langle -2, 1\rangle$	$\langle -4.5, 1.5\rangle$	$\langle -1.5, 1\rangle$	$\langle 17, 0\rangle$
Demand	$\langle 11, 0\rangle$	$\langle 3, 0\rangle$	$\langle 14, 0\rangle$	$\langle 16, 0\rangle$	$\langle 44, 0\rangle$

By assigning suitable weights $w_1 = p, w_2 = (1-p)$, correspondingly to the charge and time interval parameters and combining them, SOITP becomes

TABLE 5. Single objective interval transportation problem

	T1	T2	T3	T4	Supply
P1	$\langle 1.5 - 5.5p, 1\rangle$	$\langle 2 - 6p, 2\rangle$	$\langle 7 - 10p, 2\rangle$	$\langle 6 - 9p, 2\rangle$	$\langle 8, 0\rangle$
P2	$\langle 1.5 - 6.5p, 1\rangle$	$\langle 8.5 - 16.5p, 1.5\rangle$	$\langle 4 - 12.5p, 2\rangle$	$\langle 4 - 14p, 1\rangle$	$\langle 19, 0\rangle$
P3	$\langle 8 - 14p, 2\rangle$	$\langle 9 - 11p, 2\rangle$	$\langle 4 - 8.5p, 1.5\rangle$	$\langle 6 - 7.5p, 1\rangle$	$\langle 17, 0\rangle$
Demand	$\langle 11, 0\rangle$	$\langle 3, 0\rangle$	$\langle 14, 0\rangle$	$\langle 16, 0\rangle$	$\langle 44, 0\rangle$

By applying the proposed algorithm, the IBFS is obtained as

TABLE 6. IBFS

	T1	T2	T3	T4	Supply
P1	$\langle 1.5 - 5.5p, 1\rangle$	$\langle 2 - 6p, 2\rangle$ $\langle 3, 0\rangle$	$\langle 7 - 10p, 2\rangle$	$\langle 6 - 9p, 2\rangle$ $\langle 5, 0\rangle$	$\langle 8, 0\rangle$
P2	$\langle 1.5 - 6.5p, 1\rangle$ $\langle 11, 0\rangle$	$\langle 8.5 - 16.5p, 1.5\rangle$	$\langle 4 - 12.5p, 2\rangle$	$\langle 4 - 14p, 1\rangle$ $\langle 3, 0\rangle$	$\langle 19, 0\rangle$
P3	$\langle 8 - 14p, 2\rangle$	$\langle 9 - 11p, 2\rangle$	$\langle 4 - 8.5p, 1.5\rangle$ $\langle 14, 0\rangle$	$\langle 6 - 7.5p, 1\rangle$ $\langle 3, 0\rangle$	$\langle 17, 0\rangle$
Demand	$\langle 11, 0\rangle$	$\langle 3, 0\rangle$	$\langle 14, 0\rangle$	$\langle 16, 0\rangle$	$\langle 44, 0\rangle$

The modified versions of Vogel's Approximation method is applied for any transportation problem involving uncertain parameters. Because this method is very systematic and it takes lesser time in solving transportation problem and also less computation are involved in this method. Since the number of positive allocations is (m+n-1) =6, interval version of the MODI method is applied to check the optimality of the current solution and is found to be optimal. Now by theorem (4.1), this optimal solution will be a pareto optimal solution to the given multi objective interval transport problem.
Hence the compromised solution to the given MOITP is

$$x_{12} = \langle 3, 0\rangle = 3, x_{14} = \langle 5, 0\rangle = 5, x_{21} = \langle 11, 0\rangle = 11,$$
$$x_{24} = \langle 8, 0\rangle = 8, x_{33} = \langle 14, 0\rangle = 14, x_{34} = \langle 3, 0\rangle = 3.$$

The corresponding minimum time

$$\begin{aligned}\tilde{z}^1 &= \langle 3,0\rangle\langle 2,1\rangle + \langle 5,0\rangle\langle 6,2\rangle + \langle 11,0\rangle\langle 1.5,0.5\rangle + \langle 8,0\rangle\langle 4,1\rangle \\ &\quad + \langle 14,0\rangle\langle 4,1\rangle + \langle 3,0\rangle\langle 6,1\rangle \\ &= \langle 158.5, 2\rangle \\ &= [156.5, 160.5].\end{aligned}$$

The corresponding minimum cost

$$\begin{aligned}\tilde{z}^2 &= \langle 3,0\rangle\langle -4,2\rangle + \langle 5,0\rangle\langle -3,2\rangle + \langle 11,0\rangle\langle -5,1\rangle + \langle 8,0\rangle\langle -10,1\rangle \\ &\quad + \langle 14,0\rangle\langle -4.5,1.5\rangle + \langle 3,0\rangle\langle -1.5,0.5\rangle \\ &= \langle -229.5, 2\rangle\end{aligned}$$

Hence the corresponding maximum profit

$$\begin{aligned}&= -\text{minimum cost} \\ &= -\langle -229.5, 2\rangle = \langle 229.5, 2\rangle \\ &= [227.5, 231.5]\end{aligned}$$

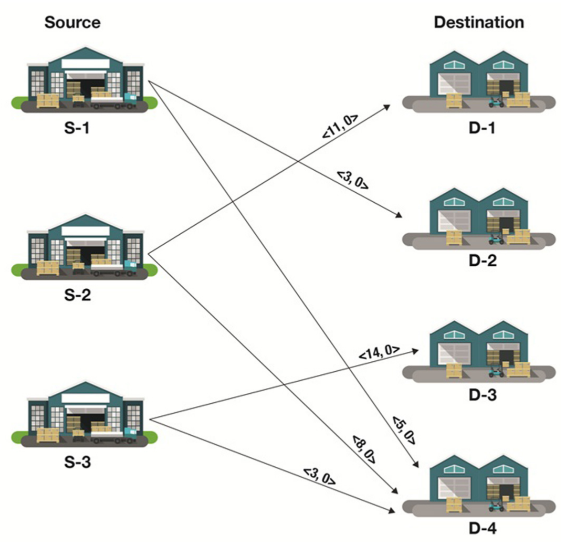

FIGURE 3. Optimal distribution route

The explanation achieved by the suggested technique yields significantly compact pareto optimum values for both objective functions, as shown in the table above.

TABLE 7. Comparison of solutions

S.No	Solution by the projected technique	Solution by Muzaffar Makhmudov with Chang Seong Ko [13]
1	$Z^1 = [156.5, 160.5]$	$Z^1 = [151, 190]$
2	$Z^2 = [227.5, 231.5]$	$Z^2 = [200, 254]$

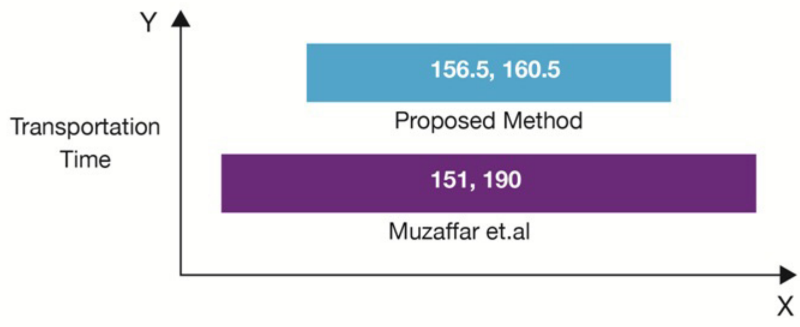

FIGURE 4. Comparision of opimum solution

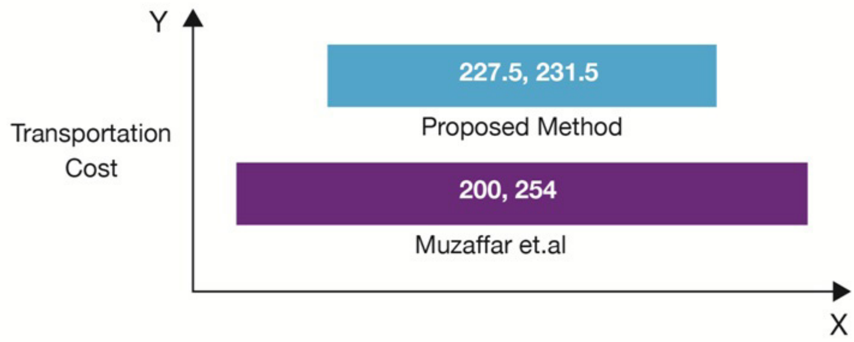

FIGURE 5. Comparision of opimum solution

It is seen from the table that the solution obtained by the proposed method gives vagueness reduced pareto optimal values for both the objective functions.

6. Conclusion

In this paper, we suggested interval variants of the ASM method approach to address multi-objective interval transportation problems without translating them to traditional multi-objective transportation problem. From a practical point of view, the ASM method process approach is very simple and easy to understand and to apply. The approach presented and discussed above gives us an initial basic feasible solution, which is closer to the optimal solution of multi-objective interval transportation problem. It also provides vagueness reduced Pareto optimal values for both the objective functions in where the source and destination parameters are chosen as interval numbers. The algorithm determines the Initial Basic Feasible(IBFS) Solution of Multi-objective interval transportation problem to minimizing time and maximizing profit. Which is very close to optimality. Ming ma's arithmetic operations in our proposed method solution obtained with less vagueness (less width). Hence our solution is better.To illustrate the proposed method, a numerical example is solved and the obtained result is compared with the results of other existing approaches. The proposed method is very easy to understand and it can be applied on real life transportation problems by the decision makers.

Acknowledgement. The author's thankful to the Editor-in-chief for the technical comments, and to the anonymous reviewers for their suggestions, which have improved the quality of the paper.

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