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PATH CENTER OF A FUZZY GRAPH BASED ON μ-DISTANCE

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Abstract. Graph theory has put forward a mathematical foundation for modelling and fine tuning communication and transportation networks. Centers and path centers serve as effective tools for optimizing traffic flow and efficiently allocating resources. The present article examines the concepts of eccentricity, center and path center of a fuzzy graph based on μ-distance. The major contribution of this article is an algorithm to find the path center and center of trees in fuzzy context. Many characteristics of center and path center of fuzzy graphs are explored and illustrated. Furthermore, eccentricities of adjacent nodes in a fuzzy graph and eccentricities of end nodes of effective arcs and strongly μ-related nodes are investigated.

Keywords: μ-distance; Eccentricity; Center; Path center; Tree; Fuzzy graph.

AMS Subject Classification: 05C05, 05C12, 05C72.

1. Introduction

Graph theory plays a significant role in dealing with problems related to real-world situations. In 1736, Euler solved the classic Konigsberg bridge problem by modelling it in terms of nodes and arcs, which led to the development of theory of graphs. In order to deal with the ambiguity and imprecision of some types of sets in relation with human thought, communication and knowledge, in 1965, Lotfy A. Zadeh established fuzzy set theory [14]. Based on fuzzy relation on a fuzzy set, Rosenfeld A. formulated the theory of fuzzy graphs, an extension of classical graph theory where nodes and arcs are given degrees of membership [10]. He presented a fuzzy graph as a special type of weighted graph where the weight of an arc can be at most the minimum of the weights of its end nodes. He also introduced two types of distances, later known as geodesic distance and μ-distance.

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Using the μ-distance, in 1987, Bhattacharya P. introduced and studied the concepts of eccentricity, radius, diameter and center of a fuzzy graph [1]. Later, in 2001, Sunitha M. S. further studied the concept of center of a fuzzy graph and formulated some theorems on self-centered fuzzy graphs [13]. Bhutani K. R. and Rosenfeld A. introduced strong paths and geodesic distance in a connected fuzzy graph in 2003 [2]. Nagoor Gani A. and Umamaheswari J. considered the maximal μ-length paths and defined fuzzy detour μ-distance and fuzzy detour μ-center [9]. Linda J. P. and Sunitha M. S. introduced fuzzy detour g-distance and fuzzy detour g-center in fuzzy graphs in [6]. In [8], Tom M. and Sunitha M. S. introduced ss-distance in fuzzy graphs. In [5], optimal cut nodes are introduced and studied. Recent studies in fuzzy graphs include topics such as the sombor index [11] and balanced spherical fuzzy graphs [12]. The fundamental concepts and core principles underpinning fuzzy graph theory are thoroughly explored by Sunil Mathew, Modeson J. N. and Malik D. S. in their book 'Fuzzy Graph Theory', offering an in-depth analysis of its theoretical framework [7].

In their book 'Distance in Graphs' [3], Buckley F. and Harany F. discussed several major concepts related to distances in crisp graphs. Also, they outlined various situations where one needs to choose a path such that all the other nodes in the fuzzy graph are nearest to the path. Path centers aid in recognizing essential nodes in communication or transportation networks, allowing efficient flow and reducing overall distance. In [4], Cockayne E. J., Hedetniemi S. M. and Hedetniemi S. T. presented an algorithm for determining the path center of a crisp tree. In 1869, Jordan C proved that the center of a crisp tree is either K_1 or K_2. Bhutani K. R and Rosenfeld A. generalized this result to geodesic center of a tree in [2]. Even if centers and central paths of crisp graphs have been investigated, a significant study of central paths in fuzzy graphs yet to be conducted. Such a study is clearly relevant as the majority of real-world situations are fuzzy.

This article focuses on the concepts of eccentricity, center, and path center in fuzzy graphs in terms of μ-distance. Section 2 of this article contains some preliminary data. Section 3 covers the concepts of eccentricity and centre of a fuzzy graph based on the μ-distance. It is also proven that the μ-center of a tree consists of either a single node or a pair of adjacent nodes. In section 4, an algorithm to find the center and path center of a tree is devised. Section 5 concludes the article and outlines directions for future work.

2. Preliminaries

This section provides the definitions and the notations used within this article. For detailed study [1, 7, 10] and [13] can be referred.

Definition 2.1. [10] *Let S be a set. A fuzzy subset of S is a mapping $\sigma : S \to [0,1]$ which assigns to each element $x \in S$ a degree of membership, $0 \leq \sigma(x) \leq 1$. A fuzzy relation on S is a fuzzy subset of $S \times S$; i.e., a mapping $\mu : S \times S \to [0,1]$ which assigns to each ordered pair of elements (x, y) a degree of membership, $0 \leq \mu(x, y) \leq 1$.*

A fuzzy relation μ on S is said to be a fuzzy relation on σ if $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$.

Definition 2.2. [10] *Let V be a set. A fuzzy graph (f-graph) is a pair $G(\sigma, \mu)$ where σ is a fuzzy subset of the set V and μ is a fuzzy relation on σ; i.e., V is the node set and σ is a fuzzy subset of V and μ is a fuzzy relation on V such that $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$.*

The underlying crisp graph of a fuzzy graph $G(\sigma, \mu)$ is represented as G^* with node set $V(G)$ and arc set $E(G)$. An arc with end nodes x and y is represented as (x, y). An arc (x, y) is effective if $\mu(x, y) = \sigma(x) \wedge \sigma(y)$ and a fuzzy graph is said to be a strong fuzzy graph if all of its arcs are effective. An arc (x, y) is strongly μ-related if $\mu(x, y) \geq \frac{1}{2}$.

The fuzzy graph $H(\tau, \nu)$ is called a (partial) fuzzy subgraph of $G(\sigma, \mu)$ if $\tau(x) \leq \sigma(x)$ for all $x \in V(G)$ and $\nu(x, y) \leq \mu(x, y)$ for all $x, y \in V(G)$. The fuzzy subgraph (τ, ν) spans the fuzzy graph (σ, μ) if $\tau(x) = \sigma(x)$ for all x. In this case, the two graphs have the same fuzzy node set; they differ only in the arc weights. For any fuzzy subset τ of σ, i.e., such that $\tau(x) \leq \sigma(x)$ for all x, the fuzzy subgraph of (σ, μ) induced by τ is the maximal fuzzy subgraph of (σ, μ) that has fuzzy node set τ.

Definition 2.3. [13] *Let $G(\sigma, \mu)$ be a fuzzy graph. The degree of a node u in G is defined by*

$$d_G(u) = \sum_{u \neq v} \mu(u, v) \tag{1}$$

In this article, a node is referred to as a pendant node if its degree is 1 in G^*, and a pendant arc is the arc incident with the pendant node.

Definition 2.4. [10] *A path ρ in a fuzzy graph $G(\sigma, \mu)$ is a sequence of distinct nodes $x_0, x_1, x_2, ..., x_n$ such that $\mu(x_{i-1}, x_i) > 0, 1 \leq i \leq n$; here $n \geq 0$ is called the length of ρ. The consecutive pairs (x_{i-1}, x_i) are called the arcs of the path.*

The strength of ρ is defined as $\wedge_{i=1}^{n} \mu(x_{i-1}, x_i)$. In other words, the strength of a path is defined to be the weight of the weakest arc of the path. Two nodes that are joined by a path are said to be connected. The strength of connectedness between two nodes x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $\mu^\infty(x, y)$. A strongest path joining any two nodes x, y has strength $\mu^\infty(x, y)$ [7]. ρ is a cycle if $x_0 = x_n$ and $n \geq 3$. A Hamiltonian path is a spanning path and if a graph contains a Hamiltonian path then the graph is called traceable.

A forest is a fuzzy graph $G(\sigma, \mu)$ if the underlying crisp graph G^* is a forest. If $G(\sigma, \mu)$ is a forest and connected, then it is called a tree. For a tree, each pair of nodes is connected by a unique path.

The fuzzy graph $G(\sigma, \mu)$ is a fuzzy forest if it has a fuzzy spanning subgraph $F(\sigma, \nu)$ which is a forest, where for all arcs (x, y) not in F (i.e., such that $\nu(x, y) = 0$), we have $\mu(x, y) < \nu^\infty(x, y)$, the strength of connectedness between the nodes x and y. In other words, if $(x, y) \in G$ but $(x, y) \notin F$, there is a path in F between x and y whose strength is greater than $\mu(x, y)$. If the graph is connected, then it is called a fuzzy tree. The corresponding spanning tree F of G is unique [10] and in this article it is denoted as, $\Im_G$. It is clear that a forest is a fuzzy forest.

Definition 2.5. [10] *For any path $\rho = x_0, ..., x_n$ in $G(\sigma, \mu)$, the μ-length of ρ, $l_\mu(\rho)$ is the sum of the reciprocals of ρ's arc weights, i.e.,*

$$l_\mu(\rho) = \sum_{i=1}^{n} \frac{1}{\mu(x_{i-1}, x_i)} \tag{2}$$

If $n = 0$, then $l_\mu(\rho) = 0$ and for $n \geq 1$, $l_\mu(\rho) \geq 1$. For any two nodes x and y, their μ-distance $\delta(x, y)$ is the smallest μ-length of any path from x to y.

Remark 2.1. [10] *The μ-distance $\delta(x, y)$ is a metric.*

Based on this μ-distance Bhattacharya defined eccentricity of a node, center and radius of a fuzzy graph [1].

Definition 2.6. [1] *If $G(\sigma, \mu)$ is a fuzzy graph with node set V, then the eccentricity $e_\mu(v)$ of a node $v \in V$ is defined to be the maximum of all the μ-distances $\delta(v, w)$ for all $w \in V$.*

A node u^* such that $\delta(u, u^*) = e_\mu(u)$ is called an eccentric node of the node u.

Definition 2.7. [1] *A central node of a connected fuzzy graph is a node whose eccentricity is the minimum. The radius $r_\mu(G)$ of a connected fuzzy graph is the minimum of all eccentricities of the nodes of the fuzzy graph.*

Based on the μ-distance δ, for a fuzzy graph $G(\sigma,\mu)$, the set of central nodes is $C_\mu(G) = \{u \in V(G); e_\mu(u) = r_\mu(G)\}$. The center of a fuzzy graph is the node induced subgraph of G by the central nodes, $C_\mu(G)$ [13]. A fuzzy graph $G(\sigma,\mu)$ is self-centered if the center of G is isomorphic to G. In self-centered fuzzy graphs, each node will be a central node.

Definition 2.8. [10] *Let $G = (\sigma,\mu)$ be a fuzzy graph, x and y be any two distinct nodes, and G' be the fuzzy subgraph of G obtained by deleting the arc (x,y); i.e., $G'(\sigma,\mu')$ where $\mu'(x,y) = 0$; $\mu' = \mu$ for all other pairs. We say that (x,y) is a fuzzy bridge or a fuzzy cut arc in G if $\mu'^\infty(u,v) < \mu^\infty(u,v)$ for some u,v; in other words, if deleting the arc (x,y) reduces the strength of connectedness between some pair of nodes. A node is called a fuzzy cut node if deleting the node reduces the strength of connectedness between some other pair of nodes.*

Definition 2.9. [7] *A maximal connected fuzzy subgraph of $G(\sigma,\mu)$, induced by a subset of $V(G)$, which has no fuzzy cut nodes is called a block of G. If G has no fuzzy cut nodes, then G itself is a block.*

A block of a graph containing only one cut node of the graph is called an end block. A block may have fuzzy bridges if the fuzzy graph is not crisp.

3. Center of fuzzy graphs

The μ-distance is a metric. Using the triangular inequality of the metric, investigating the eccentricities of nodes led to the following conclusions.

Theorem 3.1. *For any two nodes u and v in a connected fuzzy graph $G(\sigma,\mu)$, $|e_\mu(u) - e_\mu(v)| \leq \delta(u,v)$.*

Proof. Let u and v be any two nodes in G. Let x be the eccentric node of u. Since μ-distance δ is a metric,

$$e_\mu(u) = \delta(u,x) \leq \delta(u,v) + \delta(v,x) \leq \delta(u,v) + e_\mu(v).$$

$$\implies e_\mu(u) - e_\mu(v) \leq \delta(u,v) \tag{3}$$

Interchanging u and v, $e_\mu(v) - e_\mu(u) \leq \delta(v,u) = \delta(u,v)$

$$\implies -\delta(u,v) \leq e_\mu(u) - e_\mu(v) \tag{4}$$

From Equation (3) and Equation (4),

$$|e_\mu(u) - e_\mu(v)| \leq \delta(u,v). \tag{5}$$

Hence the proof. □

Corollary 3.1. *For any two adjacent nodes u and v in a connected fuzzy graph $G(\sigma,\mu)$, $|e_\mu(u) - e_\mu(v)| \leq \dfrac{1}{\mu(u,v)}$.*

Proof. Let u and v be any two adjacent nodes of a connected fuzzy graph $G(\sigma,\mu)$. Then arc (u,v) is a $u - v$ path. Length of arc $(u,v) = \dfrac{1}{\mu(u,v)}$. Since μ-distance $\delta(u,v)$ is the smallest μ-length of any path from u to v, $\delta(u,v) \leq$ length of arc (u,v). Then from Equation (5),

$$|e_\mu(u) - e_\mu(v)| \leq \frac{1}{\mu(u,v)}. \tag{6}$$

Hence the result. □

Corollary 3.2. *For any strongly μ-related adjacent nodes u and v in a connected fuzzy graph G, $|e_\mu(u) - e_\mu(v)| \leq 2$.*

Proof. Let u and v be two strongly μ-related adjacent nodes in the fuzzy graph G. Then,

$$u \text{ is strongly } \mu \text{ related to } v \implies \mu(u,v) \geq \frac{1}{2} \implies \frac{1}{\mu(u,v)} \leq 2. \tag{7}$$

Therefore, from Equation (6) and (7),

$$|e_\mu(v) - e_\mu(u)| \leq 2. \tag{8}$$

Hence the result. □

Theorem 3.2. *For any effective arc (u,v) in a connected fuzzy graph $G(\sigma,\mu)$, $\delta(u,v) = \dfrac{1}{\mu(u,v)}$.*

Proof. For any effective arc (u,v), $\mu(u,v) = \sigma(u) \wedge \sigma(v)$. Let $\rho : u = x_0, x_1, ..., x_n = v$ be any $u - v$ path in G where, $\mu(x_{i-1}, x_i) > 0$ for all $i = 1, 2, ..., n$. By Equation (2),

$$l_\mu(\rho) = \sum_{i=1}^{n} \frac{1}{\mu(x_{i-1}, x_i)} \geq \frac{1}{\mu(x_0, x_1)} + \frac{1}{\mu(x_{n-1}, x_n)}. \tag{9}$$

Now by the definition of fuzzy graphs,

$$\begin{aligned}
\mu(x_0, x_1) \leq \sigma(x_0) \wedge \sigma(x_1) &\leq \sigma(x_0) \\
&= \sigma(u) = \sigma(u) \wedge \sigma(v) && \text{if } \sigma(u) \wedge \sigma(v) = \sigma(u). \\
&= \mu(u,v) \\
\implies \frac{1}{\mu(x_0, x_1)} &\geq \frac{1}{\mu(u,v)} && \text{if } \sigma(u) \wedge \sigma(v) = \sigma(u). \\
\text{Similarly, } \frac{1}{\mu(x_{n-1}, x_n)} &\geq \frac{1}{\mu(u,v)} && \text{if } \sigma(u) \wedge \sigma(v) = \sigma(v).
\end{aligned}$$

Then from Equation (9), $l_\mu(\rho) \geq \dfrac{1}{\mu(u,v)}$ for any $u - v$ path. Therefore, $\delta(u,v) \geq \frac{1}{\mu_(u,v)}$.
Now, arc (u,v) is a $u-v$ path and length of arc $(u,v) = \dfrac{1}{\mu(u,v)}$. Therefore, $\delta(u,v) \leq \dfrac{1}{\mu_(u,v)}$.
Hence,

$$\delta(u,v) = \frac{1}{\mu(u,v)} \text{ for effective arc } (u,v). \tag{10}$$

□

Proposition 3.1. *An eccentric node u^* of a node u of a tree is always a pendant node.*

Proof. Let $T(\sigma,\mu)$ be a tree and u^* be an eccentric node of $u \in T$. Let ρ be the unique $u - u^*$ path in T. If u^* is not a pendant node of T, then there exists a node $v \notin \rho$ adjacent to u^*. Then the unique $u - v$ path in T is the union of path ρ and the arc (u^*, v), and so $\delta(u,v) = \delta(u,u^*) + \frac{1}{\mu(u^*,v)} > e_\mu(u)$, a contradiction. So the eccentric node u^* of u is always a pendant node. □

Jordan C proved that the center of a crisp tree consists of either a single node or a pair of adjacent nodes by the property that the center of tree will not change even if all the pendant nodes are removed from it [3]. Also, Bhutani and Rosenfeld pointed out that the g-center of a tree is either a single node or adjacent nodes [2]. In the case of μ-distance, first we observed that removal of pendant nodes may change the center of a tree, as obvious from the following example.

Example 3.1. *Consider the tree $T(\sigma,\mu)$ and the resultant tree T' after removing all the pendant vertices in FIGURE 1.*

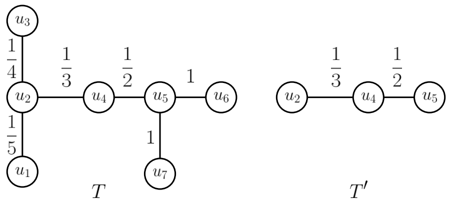

FIGURE 1. Removal of all the pendant nodes may change the center

For T, $e_\mu(u_1) = 11$, $e_\mu(u_2) = 6$, $e_\mu(u_3) = 10$, $e_\mu(u_4) = 8$, $e_\mu(u_5) = 10$, $e_\mu(u_6) = 11$ and $e_\mu(u_7) = 11$. Center $C_\mu(T) = \{u_2\}$. For T', $e_\mu(u_2) = 5$, $e_\mu(u_4) = 3$, $e_\mu(u_5) = 5$. So center $C_\mu(T') = \{u_4\}$.i.e., T and T' have different centers.

Since the removal of pendant nodes may change the center, in general, the successive removal of pendant nodes may not end up in the μ-center of a tree. But, the following theorem ensures and generalizes the result on center of trees in crisp graphs to that of fuzzy graphs.

Theorem 3.3. *The μ-center of a tree consists of either a single node or a pair of adjacent nodes.*

Proof. Consider a tree $T(\sigma,\mu)$. Assume, x, y be two distinct central nodes of T. Let x^* and y^* be the eccentric nodes of x and y respectively. Then, by the above proposition, x^* and y^* are pendant nodes. In a tree, each pair of nodes are connected by a unique path. Let P be the unique $x - x^*$ path and Q be the unique $y - y^*$ path. Then $e_\mu(x) = e_\mu(y)$ $= l_\mu(P) = l_\mu(Q) = r_\mu(T)$.

If possible, assume that P and Q have no common nodes. Since T is connected, each node of P is connected to each node of Q by a unique path. So, there exists two nodes $v \in P$ and $w \in Q$ such that R is the unique $v - w$ path in T having no common nodes with P or Q other than v and w and contain at least one arc.

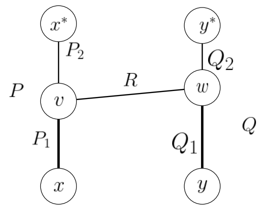

FIGURE 2. P and Q have no common nodes

Without loss of generality, assume that v divides path P into two parts P_1 and P_2 such that $l_\mu(P_1) \geq l_\mu(P_2)$. Also assume w divides path Q into two parts Q_1 and Q_2 such that $l_\mu(Q_1) \geq l_\mu(Q_2)$. See FIGURE 2.

If P_1 is the $x - v$ path, then $l_\mu(P_1 \cup R \cup Q_1) \geq \frac{l_\mu(P)}{2} + l_\mu(R) + \frac{l_\mu(Q)}{2} > l_\mu(P)$ which contradicts the eccentricity of x. Similarly, if Q_1 is the $y - w$ path, then $l_\mu(Q_1 \cup R \cup P_1) > l_\mu(Q)$ contradicts the eccentricity of y. So, neither P_1 is $x - v$ part of P nor Q_1 is $y - w$ part of Q.
Now let P_1 be the $v - x^*$ path and Q_1 is $w - y^*$ path. If $l_\mu(P_1) \geq l_\mu(Q_1)$, then

$$\begin{aligned} \delta(y, x^*) &= l_\mu(Q_2 \cup R \cup P_1) = l_\mu(Q_2) + l_\mu(R) + l_\mu(P_1) \\ &\geq l_\mu(Q_2) + l_\mu(R) + l_\mu(Q_1) > l_\mu(Q_2) + l_\mu(Q_1) = l_\mu(Q) \end{aligned}$$

$\delta(y, x^*) > l_\mu(Q)$ contradicts the eccentricity of y. Similarly, if $l_\mu(Q_1) \geq l_\mu(P_1)$ then $\delta(x, y^*) > l_\mu(P)$, contradicts the eccentricity of x. So, the existence of path R is not possible and hence, P and Q must have common nodes.

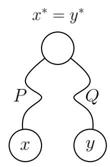

FIGURE 3. P and Q have exactly one common node, $x^* = y^*$

$x^* = y^*$ cannot be the only one node common to P and Q, as the eccentric nodes are pendant nodes. For, if $x^* = y^*$ and no other common nodes for P and Q (FIGURE 3) then $\delta(x, y) = l_\mu(P) + l_\mu(Q) > e_\mu(x) = e_\mu(y)$, which is a contradiction.

Now consider the possibility that P and Q have common nodes but not x or y. Then there are two cases, $x^* = y^*$ along with other common nodes or $x^* \neq y^*$.(FIGURE 4)

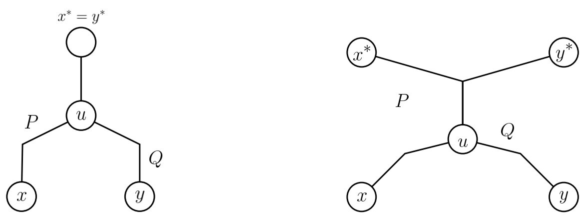

FIGURE 4. P and Q have common nodes other than x and y

Let u be the common node such that $u - x$ part of P has no common node with $u - y$ part of Q other than u. Clearly $u \neq x^*$ and $u \neq y^*$.
Claim: $e_\mu(u) < r_\mu(T)$. Let z be any node in T.
Case 1 : $u - z$ path contains a node in $u - x^*$ part of P or $u - y^*$ part of Q.: If $u - z$ part contains a node in path $u - x^*$, then, $\delta(u, z) < \delta(x, z) < e_\mu(x) = r_\mu(T)$ and if $u - z$ part contains a node in path $u - y^*$, then $\delta(u, z) < \delta(y, z) < e_\mu(y) = r_\mu(T)$.
Case 2 : $u - z$ path contains a node in $u - x$ part of P or $u - y$ part of Q.: If $u - z$ part contains a node in path $u - x$, then $\delta(u, z) < \delta(y, z) < e_\mu(y) = r_\mu(T)$ and if $u - z$ part contains a node in path $u - y$, then $\delta(u, z) < \delta(x, z) < e_\mu(x) = r_\mu(T)$.
Case 3 : $u - z$ path contains no node of P or Q.: If $u - z$ path contains no node of P or Q, then $\delta(u, z) < \delta(x, z) < \delta(x, x^*) < e_\mu(x) = r_\mu(T)$.

So for any case, $\delta(u,z) < r_\mu(T)$ for any node $z \in T$. Then, $e_\mu(u) < r_\mu(T)$, a contradiction. The only possibility remains is that, P and Q have x and y as common nodes. See FIGURE 5.

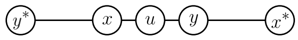

FIGURE 5. P and Q have x and y as common nodes

If $u \in P \cap Q$ is a node such that $u \neq x$ and $u \neq y$, then for any node $z \in T$, $\delta(u,z) < \delta(x,z) < \delta(x,x^*) < e_\mu(x) = r_\mu(T)$ or $\delta(u,z) < \delta(y,z) < \delta(y,y^*) < e_\mu(y) = r_\mu(T)$. i.e., $e_\mu(u) < r_\mu(T)$, a contradiction. So there exists no such node $u \in T$. Hence either $x = y$ or x is adjacent to y. If T has more than two central nodes, then all of these must be pairwise adjacent and will form a cycle which is not possible. Hence the proof. □

Proposition 3.2. *Let $T(\sigma,\mu)$ be a tree. Let p_i denote a pendant node of T and e_i be the unique arc incident with p_i. If all e_i have the same strength, then the removal of all p_i from T will not change the μ-center of T.*

Proof. For each node $u \in T$, the eccentric node u^* is a pendant node. So the removal of all p_i from T will remove all the eccentric nodes of T. If all the arcs e_i have the same strength k, the removal of all p_i from T will reduce the eccentricity of each node by $\frac{1}{k}$. So the center will remain unchanged. Hence the result. □

Remark 3.1. *As clear from the following example, the converse of the above proposition need not be true.*

Example 3.2. *Consider the tree $T(\sigma,\mu)$ in FIGURE 6 and the resultant tree T' after removing all the pendant nodes. They have the same μ-center, $\{u_3\}$.*

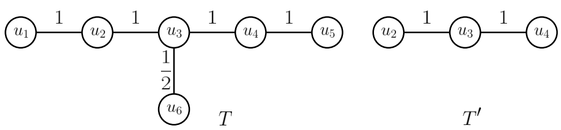

FIGURE 6. Example showing Remark 3.1

Remark 3.2. *For a fuzzy tree, the μ-center may not be K_1 or K_2. In fact, there are self-centered fuzzy trees* [13]

Remark 3.3. *For a fuzzy tree G, $\Im_G$ and G may not have the same center. Furthermore, as seen from the following example, for a node u in G, $e_\mu^{\Im_G}(u)$ may not be equal to $e_\mu^G(u)$.*

Example 3.3. *Consider the fuzzy tree G in FIGURE 7 with four nodes. $C(G) = \{u_2\}$ and $C(\Im_G) = \{u_2, u_3\}$. Also $e_\mu^G(u_2) = 4$ and $e_\mu^{\Im_G}(u_2) = 5$.*

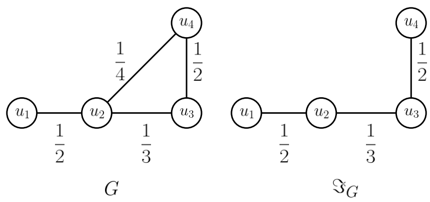

FIGURE 7. Fuzzy graph G having $e_\mu^{\Im_G}(u)$ not equal to $e_\mu^G(u)$, $u = u_2$

4. Path center of a fuzzy graph

The concept of path center can be adapted from traditional graph theory to fit the framework of fuzzy graphs. In this context, we define the eccentricity of a subgraph within a fuzzy graph. Also we define the eccentric node and proximal node of a subgraph. This definition allows us to establish a central path, which represents the path that maintains a minimum distance from all other nodes not in the path.

Definition 4.1 (***Eccentricity of subgraph***). *Let $G(\sigma,\mu)$ be a fuzzy graph and let W be a subgraph of G. For any node u in G, the μ-distance $\delta(u,W)$ from u to W is the minimum μ-distance from u to a node in W; i.e., $\delta(u,W) = \min_{x\in W}\{\delta(u,x)\}$. The eccentricity of W is $e_\mu(W) = \max_{u\in G}\{\delta(u,W)\}$.*

An eccentric node of W is a node u in G such that $\delta(u,W) = e_\mu(W)$. A proximal node of W is a node w in W such that $\delta(u,w) = e_\mu(W)$ where u is an eccentric node of W.

A path P in a fuzzy graph G is a subgraph $P(\tau,\nu)$ of the fuzzy graph $G(\sigma,\mu)$, where $\tau(x) = \sigma(x)$ for all nodes $x \in P^*$ and $\nu(x,y) = \mu(x,y)$ for all arcs $(x,y) \in P^*$. Considering eccentricity of subgraphs in paths, the central paths and path centers of a graph are defined as follows:

Definition 4.2 (***Central Path and Path center***). *A path $P(\tau,\nu)$ in a fuzzy graph $G(\sigma,\mu)$ is a central path of G if P has minimum eccentricity. A central path having minimum μ-length is called a path center of G. Path center of a graph G can be represented as P_c or P_c^G.*

Example 4.1. *Consider the following tree T as in FIGURE 8 having 9 nodes $u_i, i = 1,2,...,9$.*

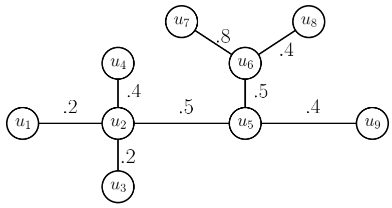

FIGURE 8. $T(\sigma,\mu)$

Consider all the paths P_i in T in FIGURE 8. Of these paths, $P_1 = \{u_2,u_5,u_6\}$ and $P_2 = \{u_2,u_5\}$ have the minimum eccentricity,

$5 = e_\mu(P_1) = e_\mu(P_2)$. So P_1 and P_2 are the central paths.
But, $l_\mu(W_1) = 4$ and $l_\mu(W_2) = 2$. So, the path center is P_2.

A fuzzy graph is traceable if and only if it has a spanning path P. So the eccentricity of P, $e_\mu(P) = 0$. Hence the eccentricity of path center of a traceable fuzzy graph will be always 'zero'. i.e., outside the path center there is no nodes for the graph. So, the path center is a spanning path. This will give a characterization of path center of a fuzzy graph as given in the following proposition.

Proposition 4.1. *A fuzzy graph G is traceable if and only if $n(P_c) = n(G)$, where $n(P_c)$ and $n(G)$ represents the number of nodes in path center P_c and G respectively.*

Remark 4.1. *Path center of a fuzzy graph may not be unique. See Example 4.2.*

Example 4.2. *Consider the fuzzy graph G in FIGURE 9 with 11 nodes*

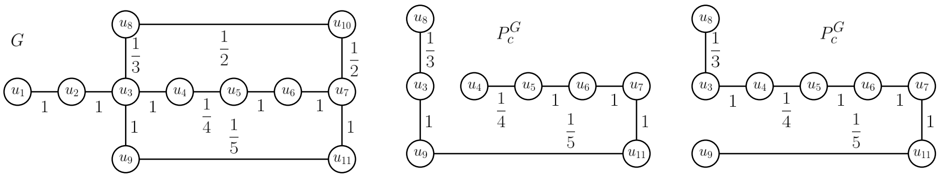

FIGURE 9. G has two different path centers

Theorem 4.1. *For a fuzzy graph $G(\sigma,\mu)$ with n nodes, $1 \le n(P_c) \le n-k+2$ where n is the number of nodes of G and k denotes the number of end blocks of G^*.*

Proof. $1 \le n(P_c)$ is trivial. Since P_c is a path, no nodes in it can repeat. Once P_c enters into an end block of G^* through the unique cut node u in the block, P_c can't leave the block without revisiting u. So if the path center P_c visits a non cut node of a block, P_c has to end in the block itself. Also, P_c can start from an end block of G^*. i.e., P_c can at most go through two end blocks of G^*. Since, each end block of G^* contains at least one node other than the cut node, P_c can't contain at least $k-2$ nodes of G. So $n(P_c) \le n-k+2$. □

Remark 4.2. *For any crisp path graph, P_n with number of nodes, $n>2$, $k=2$ and path center P_c is the path P_n itself. Clearly, $n(P_c)=n-k+2$.*

Example 4.3. *Consider the graph G in FIGURE 10, $n=6, k=3$. Number of nodes of path center is $5=n-k+2$.*

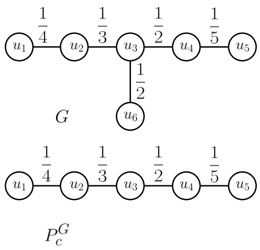

FIGURE 10. Example for graphs having $n(P_c)=n-k+2$

Remark 4.3. *A fuzzy graph can have more than one spanning tree. The path center of a spanning tree and that of the fuzzy graph may not be same.*

The following theorem establishes the existence of a spanning tree with the same path center as the fuzzy graph.

Theorem 4.2. *For any fuzzy graph $G(\sigma,\mu)$, there is a spanning tree such that path center of the spanning tree is a path center of G.*

Proof. Let P_c^G be the path center of the fuzzy graph G.
Claim : There is a spanning tree T of G such that $P_c^T = P_c^G$.
If $n(P_c^G)=n(G)$, then $T=P_c^G$ will do.
Now, assume $n(P_c^G) < n(G)$. Let $\{u_{f_i}\}$ be the collection of eccentric nodes of P_c^G and $\{u_i\}$ be the collection of proximal nodes of P_c^G. Let P_{f_i} be a $u_i - u_{f_i}$ path such that

$\delta(u_i, u_{f_i}) = e_\mu(P_c^G)$. Now consider $T_1 = P_c^G \cup (\cup_i P_{f_i})$. By the method of construction, path center of T_1 will be P_c^G. If $n(T_1) = n(G)$, then $T = T_1$.
If $n(T_1) \neq n(G)$, as above, we can construct a new subtree of G, by adding the nodes in $V(G)\backslash V(T_1)$ which are farthest from P_c such that the new tree has path center P_c^G. If the new subtree is not a spanning tree of G, we can repeat the process of constructing new subtrees of G until we get a spanning tree T of G, and the resultant tree will have the same path center as G. □

Example 4.4. *Consider the fuzzy graph G in FIGURE 11 with 15 nodes. There is a spanning tree of G having path center same as G.*

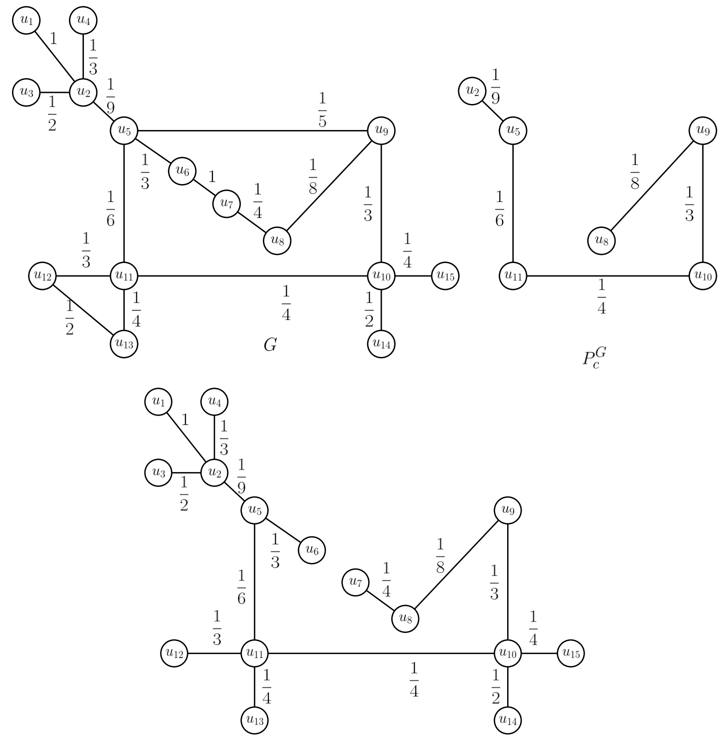

FIGURE 11. Spanning tree having path center same as G

It is clear from the following example that for a fuzzy tree G, the corresponding spanning tree $\Im_G$ and G may not have same path center.

Example 4.5. *Consider the fuzzy tree G and corresponding spanning tree $\Im_G$ in FIGURE 12, having 5 nodes. They have different path centers.*

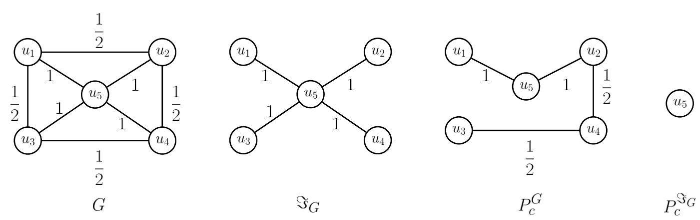

FIGURE 12. G and $\Im_G$ have different path centers

Theorem 4.3. *For a fuzzy graph G with strength of all arcs equals 1, (i.e. a crisp graph) and having at least one pendant node, the following statements are equal.*

(1) *G is not traceable;*
(2) *every spanning tree of G have at least three pendant nodes;*
(3) *path center of G, P_c contains no pendant node of G;*
(4) $n(P_c) \leq n(G) - 3$.

Proof. Assume (1). If there exists a spanning tree T with only two pendant nodes, then T will become a path and hence G is traceable. This contradiction leads to (2).

Now assume (2). By Theorem. 4.2 there exist a spanning tree T of G having path center P_c, same as that of G. Since P_c is a path, it has at most two pendant nodes. So there exists a pendant node u of T, such that $u \notin P_c$. Then $\delta(u, P_c) \geq 1$. If v is a pendant node of T such that $v \in P_c$, then $\delta(v, P_c) = 0$ and $\delta(v, P_c \backslash \{v\}) = 1$. i.e., $\delta(u, P_c) \geq 1 = \delta(v, P_c \backslash \{v\})$ which is a contradiction to the definition of path center and so there exists no pendant node v of T such that $v \in P_c$. If G contains a pendant node, then it should be a pendant node of every spanning tree of G and so not a part of P_c. So (2) $\implies$ (3). Now, Assume (3). Clearly $n(P_c) < n(G)$ and so G is not traceable. So (3) $\implies$ (1). Assume (2). Then the subtree T which has path center P_c, also have at least three pendant nodes say, u, v and w. Then by (3), $u, v, w \notin P_c$. i.e., $n(P_c) \leq n(G) - 3$. So (2) $\implies$ (4). Assume (4). Then $n(P_c) < n(G)$ and so G is not traceable. i.e., (4) $\implies$ (1). □

The following example shows that path center of a fuzzy graph may contain pendant nodes.

Example 4.6. *Consider the graph G in FIGURE 13*

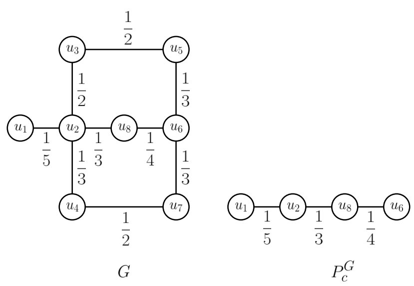

FIGURE 13. P_c^G contains pendant node u_1 of G

Theorem 4.4. *The path center of a tree is unique.*

Proof. Let $T(\sigma, \mu)$ be a tree. If possible, let P_c and P'_c be two different path centers of T. Then $e_\mu(P_c) = e_\mu(P'_c)$. Let w be the eccentric node of P_c and w' be the eccentric node of P'_c. Let $x \in P_c$ be the node such that $\delta(x, w) = e_\mu(P_c)$ and $y \in P_C$ be the node such that $\delta(y, w') = e_\mu(P'_c)$. Then, there are two cases.

Case 1 : P_c and P'_c have no common nodes - There are two nodes $u \in P_c$ and $v \in P'_c$ such that the path $u - v$ have no internal nodes common with P_c or P'_c. Then the eccentric node w of P_c should lie in P'_c. For, if $w \notin P'_c$ then $\delta(y, w) = \delta(y, v) + \delta(v, u) + \delta(u, x) + \delta(x, w) > \delta(x, w) = e_\mu(P_c) = e_\mu(P'_c)$, a contradiction. So, $w \in P'_c$. Similarly, $w' \in P_c$.

Now consider the $u - v$ path P. For any node $z \in T$, either $\delta(u, z) < \delta(y, z) \leq e_\mu(P'_c)$ or $\delta(v, z) < \delta(x, z) \leq e_\mu(P_c)$. i.e., $e_\mu(P) < e_\mu(P_c) = e_\mu(P'_c)$, a contradiction. So P_c and P'_c must have common nodes.

Case 2 : P_c and P'_c have common nodes - Assume P_c and P'_c have at least one node not common with each other and let the $u-v$ path ρ be the part common to both P_c and P'_c. Then clearly $l_\mu(\rho) < l_\mu(P_c) = l_\mu(P'_c)$ and for any node $z \in T$, $\delta(z,\rho) \leq e_\mu(T)$, a contradiction to the definition of path center. So the $u-v$ path ρ must be of length that of P_c and P'_C. i.e., $P_c = P'_c$. □

Proposition 4.2. *Let $T(\sigma,\mu)$ be a tree which is not a path. Then, removal of all the pendant nodes of T incident with the pendant arcs having maximum strength do not change the path center of T.*

Proof. Let $u_i, i = 1,2,...,n$ be the n nodes of T and $e_{k,j}$ denotes the pendant arc of T incident with a pendant node u_k, which is adjacent to node u_j. Let M be the maximum of arc strengths of $e_{k,j}$. Then $\frac{1}{M}$ is the minimum among μ-lengths of pendant arcs. Then removal of all pendant nodes incident with pendant arcs $e_{k,j}$ having μ-length $\frac{1}{M}$ will give a subtree T' of T with minimum eccentricity and having proximal nodes u_j. Then the path center of T' will be same as that of T as the path center P_c of T is the path in T that has minimum eccentricity and minimum length. □

Remark 4.4. *Consider the subtree T' of T used in Proposition 4.2. For those pendant nodes of T', which are proximal nodes, consider the sum of eccentricity of T' and μ-length of the pendant arc incident with the pendant node and consider their minimum, say $\frac{1}{M'}$. For those pendant nodes which are not the proximal nodes of T', consider the minimum length $\frac{1}{M''}$ of pendant arcs as same as we calculated in the above proposition. Then, the removal of all those pendant nodes having* $\min\{\frac{1}{M'}, \frac{1}{M''}\}$ *will result into a new subtree T'' of T having eccentricity* $\min\{\frac{1}{M'}, \frac{1}{M''}\}$. *It is clear that $e_\mu(T') < e_\mu(T'')$ and there exists no other subtree T_1 such that $e_\mu(T') < e_\mu(T_1) < e_\mu(T'')$, as to reach the removed pendant nodes of T in the above proposition, one have to necessarily consider the μ-length of pendant arcs that are incident with the removed nodes.*

4.1. **Algorithm to find path center of a tree fuzzy graph.** From Theorem 4.4, Prosition 4.2 and Remark 4.4 we can devise the following algorithm to find the path center of a tree fuzzy graph. Consider the tree fuzzy graph T with n nodes $u_1, u_2, \ldots, u_n$. Let $\mu_{i,j}$ be the strength of the arc (u_i, u_j).

- Step 1 : Set a weight $\eta_i = 0$ for $i = 1,2,\ldots,n$;
- Step 2 : For each pendant node u_i, find $x_i = \dfrac{1}{\mu_{i,j}} + \eta_i$ where $\mu_{i,j} > 0$;
- Step 3 : Find $\min_i\{x_i\}$ and let it be η;
- Step 4 : Find all pendant nodes u_k of the tree T such that $x_k = \eta$ and let $\eta_j = \eta$ for $\mu_{k,j} > 0$;
- Step 5 : Remove all pendent nodes x_k from T. Let the resultant tree be $T^{(1)}$. If $T^{(1)}$ is a path then $T^{(1)}$ be the path center, else let $T = T^{(1)}$ and go to step 2 and repeat the steps.

Remark 4.5. *If we did not stop the iteration even if we got a path $T^{(1)}$ in step 5, continuing the iteration will lead to K_1 or K_2 which is the center of T. Clearly, the path $T^{(1)}$ will contain this K_1 or K_2. Thus path center of a tree fuzzy graph contains the center of the graph.*

Example 4.7. *Consider the tree T in FIGURE 14 having* 17 *nodes $u_i; i = 1,2,\ldots,17$. and arc strength $\mu_{i,j}$ for the arc (u_i, u_j).*

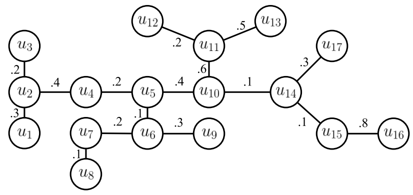

FIGURE 14. Tree T in Example 4.7

To find the path center of the given tree T:

- *Iteration 1 : Set a weight $\eta_i = 0$ for every node u_i, $i = 1, 2, ..., 17$. Determine $x_i = \frac{1}{\mu_{i,j}} + \eta_i$ for every pendant node u_i where u_i and u_j are adjacent. Then, $x_1 = 3.3$, $x_3 = 5$, $x_8 = 10$, $x_9 = 3.3$, $x_{12} = 5$, $x_{13} = 2$, $x_{17} = 3.3$, $x_{16} = 1.25$. $\min_i\{x_i\} = 1.25 = x_{16}$. $T^{(1)} = \{T \setminus \{u_{16}\}\}$ is not a path (FIGURE 15). So let $\eta_{15} = x_{16} = 1.25$. Let $T = T^{(1)}$.*

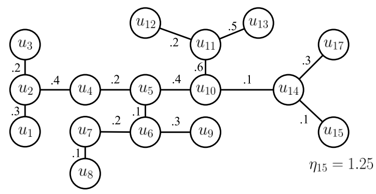

FIGURE 15. Tree obtained from Iteration 1

- *Iteration 2 : On new T, $\eta_{15} = 1.25$, $\eta_i = 0$ for all $i \neq 15$. $x_3 = 5$, $x_1 = 3.3$, $x_8 = 10$, $x_{12} = 5$, $x_{13} = 2$, $x_{17} = 3.3$, $x_{15} = 11.25$, $x_9 = 3.3$. $\min_i\{x_i\} = 2 = x_{13}$. $T^{(1)} = T \backslash \{u_{13}\}$ is not a path. So let $\eta_{11} = x_{13} = 2$ and $T = T^{(1)}$.*
- *Iteration 3 : Now, $\eta_{15} = 1.25$, $\eta_{11} = 2$, $\eta_i = 0$ for all other nodes. Then, $x_1 = 3.3$, $x_3 = 5$, $x_8 = 10$, $x_9 = 3.3$, $x_{12} = 5$, $x_{17} = 3.3$, $x_{15} = 11.25$. $\min_i\{x_i\} = 3.3 = x_1 = x_9 = x_{17}$. $T^{(1)} = T \backslash \{u_{15},\ u_9,\ u_1\}$ is not a path. So let $\eta_2 = \eta_6 = \eta_{14} = 3.3$.*
- *Iteration 4 : $\eta_2 = \eta_6 = \eta_{14} = 3.3$, $\eta_{15} = 1.25$, $\eta_{11} == 2$, $\eta_i = 0$ for all other nodes. Then, $x_3 = 5$, $x_8 = 10$, $x_{12} = 5$, $x_{15} = 11.25$. $\min_i\{x_i\} = 5 = x_3 = x_{12}$. $T^{(1)} = T \backslash \{u_3,\ u_{12}\}$ is not a path. So let $\eta_2 = \eta_{11} = 5$.*
- *Iteration 5 : $\eta_2 = \eta_{11} = 5$, $\eta_6 = \eta_{14} = 3.3$, $\eta_{15} = 1.25$, $\eta_i = 0$ for all other nodes. Then, $x_2 = 7.5$, $x_8 = 10$, $x_{11} = 6.6$, $x_{15} = 11.25$. $\min_i\{x_i\} = 6.6 = x_{11}$. $T^{(1)} = T \backslash \{u_{11}\}$ is not a path. So let $\eta_{10} = 6.6$.*
- *Iteration 6 : $\eta_{10} = 6.6$, $\eta_2 = 5$, $\eta_6 = \eta_{14} = 3.3$, $\eta_{15} = 1.25$, $\eta_i = 0$ for all other nodes. Then, $x_2 = 7.5$, $x_8 = 10$, $x_{15} = 11.25$. $\min_i\{x_i\} = 7.5 = x_2$. $T^{(1)} = T \backslash \{u_2\}$ is not a path. So let $\eta_4 = 7.5$.*
- *Iteration 7 : $\eta_{10} = 6.6$, $\eta_4 = 7.5$, $\eta_6 = \eta_{14} = 3.3$, $\eta_{15} = 1.25$, $\eta_i = 0$ for all other nodes. Then, $x_4 = 12.5$, $x_8 = 10$, $x_{15} = 11.25$. $\min_i\{x_i\} = 10 = x_8$. $T^{(1)} = T \backslash \{u_8\}$ is not a path. So let $\eta_7 = 10$.*
- *Iteration 8 : $\eta_7 = 10$, $\eta_{10} = 6.6$, $\eta_4 = 7.5$, $\eta_6 = \eta_{14} = 3.3$, $\eta_{15} = 1.25$, $\eta_i = 0$ for all other nodes. Then, $x_4 = 12.5$, $x_7 = 15$, $x_{15} = 11.25$. $\min_i\{x_i\} = 11.25 = x_{15}$. $T^{(1)} = T \backslash \{u_{15}\}$ is not a path.*

- *Iteration 9 :* $\eta_7 = 10$, $\eta_{10} = 6.6$, $\eta_4 = 7.5$, $\eta_6 = 3.3$, $\eta_{14} = 11.25$, $\eta_i = 0$ *for all other nodes. Then* $x_4 = 12.5$, $x_7 = 15$, $x_{14} = 21.25$. $\min_i\{x_i\} = 12.5 = x_4$. $T^{(1)} = T\setminus\{u_4\}$ *is a path. So the path center is the* $u_7 - u_{14}$ *path (FIGURE 16).*

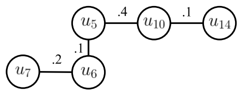

FIGURE 16. Path center of given tree T

Remark 4.6. *Using the algorithm for crisp trees stated in* [4]*, the path center of* G^* *of the above example is* $\{u_5, u_{10}\}$ *which is different from path center of tree* G*. But, by the above algorithm we can obtain path center of* G *as well as* G^**.*

5. Conclusions

The fuzzy graph framework, including centers and path centres, facilitates the study of complex network systems, revealing insights into their structure, connectivity, and behaviour. In this article, we have proposed an algorithm to determine the center and path center of a tree. Also, we have proved that the difference between eccentricities of any two strongly μ-related adjacent nodes u and v in a connected fuzzy graph is at most two. Furthermore, the μ-center of a tree in fuzzy context consists of a single node or a pair of adjacent nodes whereas the result does not hold for fuzzy trees. A bound to the number of nodes of path center of a fuzzy graph in terms of end blocks is derived. Further studies on the path center of different classes of fuzzy graphs beyond trees can be pursued in the future work.

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