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Z-CONTINUOUS MAPS IN FERMATEAN FUZZY TOPOLOGICAL SPACES AND ITS APPLICATION

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Abstract. In this paper, we undertake a detailed study of various types of functions in Fermatean fuzzy topological spaces, namely Fermatean fuzzy Z-continuous, Fermatean fuzzy Z-irresolute, strongly Fermatean fuzzy Z-continuous, and perfectly Fermatean fuzzy Z-continuous functions. We present rigorous definitions and characterizations of these functions, explore their interrelationships, and establish several fundamental properties supported by illustrative examples. Furthermore, we demonstrate the practical significance of the proposed concepts by developing a real-life decision-making application based on entropy measures defined over Fermatean fuzzy sets, thereby showcasing their potential in handling uncertainty and imprecision in complex problem-solving scenarios.

Keywords: Fermatean fuzzy Z-continuous, Fermatean fuzzy Z-irresolute, strongly Fermatean fuzzy Z-continuous, perfectly Fermatean fuzzy Z-continuous.

AMS Subject Classification: 03E72, 54A05, 54C05, 54A40.

1. Introduction

Fuzzy sets were introduced by Zadeh [30] in 1965. The fuzzy set concept corresponding to unexplained physical situations gives useful applications on many topics such as statistics, data processing and linguistics. A lot of research has been done on this subject since 1965. In 1968, Chang [6] defined the concept of fuzzy topological space and generalized some basic notions of topology such as open set, closed set, continuity and compactness to fuzzy topological spaces. The idea of intuitionistic fuzzy set was first

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published by Atanassov [1] and many works by the same author and his colleagues appeared in the literature [2, 5]. Coker [7] initiated a study of intuitionistic fuzzy topological spaces. Later Yager [28] launched a non standard fuzzy set referred to as Pythagorean fuzzy set. Olgun et al., [13] defined a Pythagorean fuzzy topological spaces. Fermatean fuzzy sets proposed by Senapati and Yager in 2020 [16], can handle uncertain information more easily in the process of decision making. They defined basic operations over the Fermatean fuzzy sets. Hariwan Z. Ibrahim defined a Fermatean fuzzy topological spaces and the continuity of a function defined among Fermatean fuzzy topological spaces. Also he proved as any Intuitionistic fuzzy subset or Pythagorean fuzzy subset of a set can be considered as Fermatean fuzzy subset, we observe that any Intuitionstic fuzzy topological space or Pythagorean fuzzy topological space is a Fermatean fuzzy topological space as well. On the other hand, it is obvious that a Fermatean fuzzy topological space need not be Intuitionistic fuzzy topological space and Pythagorean fuzzy topological space. Even an Fermatean fuzzy open set maybe neither an Intuitionistic fuzzy set nor Pythagorean fuzzy set. Saha [15] defined δ-open sets in fuzzy topological spaces, topological space by Pankajam et al. [14] and neutrosophic topological space by Vadivel et al. [22]. El-Maghrabi and Al-Juhani [9] proposed the concept of M-open sets in topological spaces in 2011 and examined some of their features. Padma et al. [17] also found M-open sets in topological spaces. Vadivel et al. [18, 19, 20] discussed some open sets in fuzzy and neutrosophic topological spaces. Kalaiyarsan et al. [11] and Vadivel et al. [21] introduced M-open sets in fuzzy and neutrosophic topological spaces. Recently, in 2025, Vadivel et. al. [23, 24, 25, 26, 8], introduced Fermatean fuzzy δ open, Fermatean fuzzy $\delta\alpha$ open and Fermatean fuzzy $\delta\mathcal{S}$ open sets in Fermatean fuzzy topological spaces.

Research Gap: To the best of our knowledge, no comprehensive investigation has been carried out on certain stronger and weaker forms of Fermatean fuzzy continuous functions, such as Fermatean fuzzy δ-continuous, Fermatean fuzzy δ-semi-continuous, Fermatean fuzzy pre-continuous, Fermatean fuzzy Z-continuous, strongly Fermatean fuzzy Z-continuous, and perfectly Fermatean fuzzy Z-continuous functions, within the framework of Fermatean fuzzy topological spaces. The absence of such studies in the existing fuzzy literature motivates the present work.

In this paper, we focus on a detailed study of Fermatean fuzzy Z-continuous (Z-Cts), Fermatean fuzzy Z-irresolute (Z-Irr), strongly Fermatean fuzzy Z-continuous, and perfectly Fermatean fuzzy Z-continuous functions in Fermatean fuzzy topological spaces. We provide precise definitions, establish essential properties, and highlight the interrelations among these function classes. Moreover, we introduce an entropy measure tailored for Fermatean fuzzy sets and demonstrate its applicability through a real-life decision-making example, thereby illustrating the practical relevance of our theoretical developments in addressing problems involving uncertainty and vagueness.

2. Preliminaries

We recall some basic notions of fuzzy sets, IFS's, pfs's and $\mathfrak{F}\mathcal{F}s$'s.

Definition 2.1. [30] Let X be a nonempty set. A fuzzy set A in X is characterized by a membership function $\mu_A : X \to [0,1]$. That is:

$$\mu_A(x) = \begin{cases} 1, & \text{if} \quad x \in X \\ 0, & \text{if} \quad x \notin X \\ (0,1) & \text{if } x \text{ is partly in } X. \end{cases}$$

Alternatively, a fuzzy set A in X is an object having the form $A = \{< x, \mu_A(x) > | x \in X\}$ or $A = \left\{ \left\langle \frac{\mu_A(x)}{x} \right\rangle | x \in X \right\}$, where the function $\mu_A(x) : X \to [0,1]$ defines the degree of membership of the element, $x \in X$.

The closer the membership value $\mu_A(x)$ to 1, the more x belongs to A, where the grades 1 and 0 represent full membership and full non-membership. Fuzzy set is a collection of objects with graded membership, that is, having degree of membership. Fuzzy set is an extension of the classical notion of set. In classical set theory, the membership of elements in a set is assessed in a binary terms according to a bivalent condition; an element either belongs or does not belong to the set. Classical bivalent sets are in fuzzy set theory called crisp sets. Fuzzy sets are generalized classical sets, since the indicator function of classical sets is special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1. Fuzzy sets theory permits the gradual assessment of the membership of element in a set; this is described with the aid of a membership function valued in the real unit interval $[0, 1]$.

Let us consider two examples:

(i) all employees of XYZ who are over $1.8m$ in height; (ii) all employees of XYZ who are tall. The first example is a classical set with a universe (all XYZ employees) and a membership rule that divides the universe into members (those over $1.8m$) and nonmembers. The second example is a fuzzy set, because some employees are definitely in the set and some are definitely not in the set, but some are borderline.

This distinction between the ins, the outs, and the borderline is made more exact by the membership function, μ. If we return to our second example and let A represent the fuzzy set of all tall employees and x represent a member of the universe X (i.e. all employees), then $\mu_A(x)$ would be $\mu_A(x) = 1$ if x is definitely tall or $\mu_A(x) = 0$ if x is definitely not tall or $0 < \mu_A(x) < 1$ for borderline cases.

Definition 2.2. [1] The intuitionistic fuzzy sets are defined on a non-empty sets X as objects having the form $I = \{\langle x, \mu_I(x), \lambda_I(x)\rangle : x \in X\}$, where $\mu_I(x) : X \to [0,1]$ and $\lambda_I(x) : X \to [0,1]$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set I, respectively, and $0 \leq \mu_I(x) + \lambda_I(x) \leq 1$, for all $x \in X$.

Definition 2.3. [1, 2, 3, 4] Let a nonempty set X be fixed. An IFS A in X is an object having the form: $A = \{< x, \mu_A(x), \lambda_A(x) > | x \in X\}$ or $A = \left\{ \left\langle \frac{\mu_A(x), \lambda_A(x)}{x} \right\rangle | x \in X \right\}$, where the functions $\mu_A(x) : X \to [0,1]$ and $\lambda_A(x) : X \to [0,1]$ define the degree of membership and the degree of non-membership, respectively, of the element $x \in X$ to A, which is a subset of X, and for every $x \in X : 0 \leq \mu_A(x) + \lambda_A(x) \leq 1$. For each A in X: $\pi_A(x) = 1 - \mu_A(x) - \lambda_A(x)$ is the intuitionistic fuzzy set index or hesitation margin of x in X. The hesitation margin $\pi_A(x)$ is the degree of nondeterminancy of $x \in X$ to the set A and $\pi_A(x) \in [0,1]$. The hesitation margin is the function that expresses lack of knowledge of whether $x \in X$ or $x \notin X$. Thus: $\mu_A(x) + \lambda_A(x) + \pi_A(x) = 1$.

Example 2.1. Let $X = \{x, y, z\}$ be a fixed universe of discourse and $A = \left\{ \left\langle \frac{0.6, 0.1}{x} \right\rangle, \left\langle \frac{0.8, 0.1}{y} \right\rangle, \left\langle \frac{0.5, 0.3}{z} \right\rangle \right\}$, be the intuitionistic fuzzy set in X. The hesitation margins of the elements x, y, z to A are as follows: $\pi_A(x) = 0.3$, $\pi_A(y) = 0.1$ and $\pi_A(z) = 0.2$.

Definition 2.4. [27, 28, 29] Let X be a universal set. Then, a Pythagorean fuzzy set A, which is a set of ordered pairs over X, is defined by the following: $A = \{< x, \mu_A(x), \lambda_A(x) | x \in X\}$ or $A = \left\{ \left\langle \frac{\mu_A(x), \lambda_A(x)}{x} \right\rangle | x \in X \right\}$, where the functions $\mu_A(x)$:

$X \to [0,1]$ and $\lambda_A(x) : X \to [0,1]$ define the degree of membership and the degree of non-membership, respectively, of the element $x \in X$ to A, which is a subset of X, and for every $x \in X$, $0 \leq (\mu_A(x))^2 + (\lambda_A(x))^2 \leq 1$. Supposing $(\mu_A(x))^2 + (\lambda_A(x))^2 \leq 1$, then there is a degree of indeterminacy of $x \in X$ to A defined by $\pi_A(x) = \sqrt{1 - [(\mu_A(x))^2 + (\lambda_A(x))^2]}$ and $\pi_A(x) \in [0,1]$. In what follows, $(\mu_A(x))^2 + (\lambda_A(x))^2 + (\pi_A(x))^2 = 1$. Otherwise, $\pi_A(x) = 0$ whenever $(\mu_A(x))^2 + (\lambda_A(x))^2 = 1$. We denote the set of all PFS's over X by $pfs(X)$.

Definition 2.5. [16] Let X be a universe of discourse. A Fermatean fuzzy set ($\mathfrak{F}\mathcal{F}s$) F in X is an object having the form $F = \{< x, \mu_F(x), \lambda_F(x) >: x \in X\}$ where $\mu_F(x) : X \to [0,1]$ and $\lambda_F(x) : X \to [0,1]$, including the condition $0 \leq (\mu_F(x))^3 + (\lambda_F(x))^3 \leq 1$, for all $x \in X$. The numbers $\mu_F(x)$ and $\lambda_F(x)$ denote, respectively, the degree of membership and the degree of non-membership of the element x in the set F. For any $\mathfrak{F}\mathcal{F}s$ F and $x \in X$, $\pi_F(x) = \sqrt[3]{1 - [(\mu_F(x))^3 - (\lambda_F(x))^3]}$ is identified as the degree of indeterminacy of x to F. In the interest of simplicity, we shall mention the symbol $F = (\mu_F, \lambda_F)$ for the $\mathfrak{F}\mathcal{F}s$ $F = \{< x, \mu_F(x), \lambda_F(x) : x \in X\}$.

Definition 2.6. [16] Let $F = (\mu_F, \lambda_F)$, $F_1 = (\mu_{F_1}, \lambda_{F_1})$ and $F_2 = (\mu_{F_2}, \lambda_{F_2})$, be three Fermatean fuzzy sets ($\mathfrak{F}\mathcal{F}s$'s), then their operations are defined as follows:

(i) $F_1 \cap F_2 = (min\{\mu_{F_1}, \mu_{F_2}\}, max\{\lambda_{F_1}, \lambda_{F_2}\})$.
(ii) $F_1 \cup F_2 = (max\{\mu_{F_1}, \mu_{F_2}\}, min\{\lambda_{F_1}, \lambda_{F_2}\})$.
(iii) $F^c = (\lambda_F, \mu_F)$.

Remark 2.1. If $\mu_{F_1} = \mu_{F_2}$ and $\lambda_{F_1} = \lambda_{F_2}$, then $F_1 = F_2$

Definition 2.7. [10] Let X be a non empty set and τ be a family of Fermatean fuzzy subsets of X. If

(i) $1_F, 0_F \in \tau$
(ii) for any $F_1, F_2 \in \tau$, we have $F_1 \cap F_2 \in \tau$,
(iii) for any $\{F_i\}_{i \in I} \subset \tau$, we have $\bigcup_{i \in I} F_i \in \tau$ where I is an arbitrary index set then τ is called a Fermatean fuzzy topology on X.
The pair (X, τ) is said to be a Fermatean fuzzy topological space. Each member of τ is called an Fermatean fuzzy open set. The complement of an Fermatean fuzzy open set is called a Fermatean fuzzy closed set.

Remark 2.2. [10] As any Intuitionistic fuzzy subset or Pythagorean fuzzy subset of a set can be considered as Fermatean fuzzy subset, we observe that any Intuitionstic fuzzy topological space or Pythagorean fuzzy topological space is a Fermatean fuzzy topological space as well. On the other hand, it is obvious that a Fermatean fuzzy topological space need not be Intuitionistic fuzzy topological space and Pythagorean fuzzy topological space. Even an Fermatean fuzzy open set maybe neither an Intuitionistic fuzzy set nor Pythagorean fuzzy set.

Example 2.2. [10] Let $X = \{c_1, c_2\}$. Consider the following family Fermatean fuzzy subsets $\tau = \{1_F, 0_F, F_1, F2\}$ where $F_1 = \{\langle c_1, \mu_{F_1}(c_1) = 0.4, \lambda_{F_1}(c_1) = 0.6\rangle, \langle c_2, \mu_{F_1}(c_2) = 0.1, \lambda_{F_1}(c_2) = 0.3\rangle\}$ and $F_2 = \{\langle c_1, \mu_{F_2}(c_1) = 0.9, \lambda_{F_2}(c_1) = 0.6\rangle, \langle c_2, \mu_{F_2}(c_2) = 0.2, \lambda_{F_2}(c_2) = 0.3\rangle\}$. Observe that (X, τ) is a Fermatean fuzzy topological space but (X, τ) is neither Intuitionistic fuzzy topological space nor Pythagorean fuzzy topological space.

Definition 2.8. [10] Let (X, τ) be an $\mathfrak{F}\mathcal{F}ts$ and $A = \{< a, \mu_A(a), \lambda_A(a) > | a \in X\}$ be an $\mathfrak{F}\mathcal{F}s$ in X. Then the Fermatean fuzzy interior and the Fermatean fuzzy closure of A are denoted by $\mathfrak{F}\mathcal{F}int(A)$ and $\mathfrak{F}\mathcal{F}cl(A)$ and are defined as follows: $\mathfrak{F}\mathcal{F}int(A) =$

$\cup\{G|G$ is a $\mathfrak{F}\mathcal{F}os$ and $G \subseteq A\}$ and $\mathfrak{F}\mathcal{F}cl(A) = \cap\{K|K$ is a $\mathfrak{F}\mathcal{F}cs$ and $A \subseteq K\}$. Also, it can be established that $\mathfrak{F}\mathcal{F}cl(A)$ is an $\mathfrak{F}\mathcal{F}cs$ and $\mathfrak{F}\mathcal{F}int(A)$ is an $\mathfrak{F}\mathcal{F}os$, A is an $\mathfrak{F}\mathcal{F}cs$ if and only if $\mathfrak{F}\mathcal{F}cl(A) = A$ and A is an $\mathfrak{F}\mathcal{F}os$ if and only if $\mathfrak{F}\mathcal{F}int(A) = A$. We say that A is $\mathfrak{F}\mathcal{F}$-dense if $\mathfrak{F}\mathcal{F}cl(A) = 1_{\mathfrak{F}}$.

Lemma 2.1. [10] For any Fermatean fuzzy set A in (X, τ), we have $1_F - \mathfrak{F}\mathcal{F}int(A) = \mathfrak{F}\mathcal{F}cl(1_{\mathfrak{F}} - A)$ and $1_{\mathfrak{F}} - \mathfrak{F}\mathcal{F}cl(A) = \mathfrak{F}\mathcal{F}int(1_{\mathfrak{F}} - A)$.

Definition 2.9. [23] Let (X, τ) be an $\mathfrak{F}\mathcal{F}ts$ and A be an $\mathfrak{F}\mathcal{F}s$. Then A is said to be an Fermatean fuzzy (i) regular open set ($\mathfrak{F}\mathcal{F}ros$ in short) if $A = \mathfrak{F}\mathcal{F}int(\mathfrak{F}\mathcal{F}cl(A))$. (ii) regular closed set ($\mathfrak{F}\mathcal{F}rcs$ in short) if $A = \mathfrak{F}\mathcal{F}cl(\mathfrak{F}\mathcal{F}int(A))$. By Lemma 2.1, it follows that A is an $\mathfrak{F}\mathcal{F}ros$ iff $\bar{A}$ is an $\mathfrak{F}\mathcal{F}rcs$.

Definition 2.10. [23] Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$. Let S be a $\mathfrak{F}\mathcal{F}$ of X. Then Fermatean

(i) fuzzy δ interior of S (briefly, $\mathfrak{F}\mathcal{F}\delta int(S)$) is defined by $\mathfrak{F}\mathcal{F}\delta int(S) = \cup\{I : I \subseteq S$ & I is a $\mathfrak{F}\mathcal{F}ro$ set in $X\}$.
(ii) fuzzy δ closure of S (briefly, $\mathfrak{F}\mathcal{F}\delta cl(S)$) is defined by $\mathfrak{F}\mathcal{F}\delta cl(S) = \cap\{A : S \subseteq A$ & A is a $\mathfrak{F}\mathcal{F}rc$ set in $X\}$.

Definition 2.11. [23] Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$. Then a $\mathfrak{F}\mathcal{F}s$ S in X is said to be Fermatean

(i) fuzzy δ-open (briefly, $\mathfrak{F}\mathcal{F}\delta o$) set if $S = \mathfrak{F}\mathcal{F}\delta int(S)$.
(ii) fuzzy $\delta\alpha$-open (briefly, $\mathfrak{F}\mathcal{F}\delta\alpha o$) set if $S \subseteq \mathfrak{F}\mathcal{F}int(\mathfrak{F}\mathcal{F}cl\ (\mathfrak{F}\mathcal{F}\delta int\ (S)))$.
(iii) fuzzy δ-semi open (briefly, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$) set if $S \subseteq \mathfrak{F}\mathcal{F}cl(\mathfrak{F}\mathcal{F}\delta int(S))$.

The complement of an $\mathfrak{F}\mathcal{F}\delta o$ (resp. $\mathfrak{F}\mathcal{F}\delta\alpha o$ & $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$) set is called a Fermatean fuzzy δ (resp. Fermatean fuzzy δ-α & Fermatean fuzzy δ-semi) closed (briefly, $\mathfrak{F}\mathcal{F}\delta c$ (resp. $\mathfrak{F}\mathcal{F}\delta\alpha c$ & $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$)) in X.

Definition 2.12. [23] Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$. Let S be a $\mathfrak{F}\mathcal{F}s$ of X. Then Fermatean fuzzy

(i) δ semi interior of S (briefly, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}int(S)$) is defined by $\mathfrak{F}\mathcal{F}\delta\mathcal{S}int(S) = \cup\{I : I \subseteq S$ & I is a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ set in $X\}$.
(ii) δ semi closure of S (briefly, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(S)$) is defined by $\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(S) = \cap\{A : S \subseteq A$ & A is a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$ set in $X\}$.

Definition 2.13. [24] A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be Fermatean fuzzy

(i) pre continuous (briefly, $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$), if for each $\mathfrak{F}\mathcal{F}o$ set M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}\mathcal{P}o$ set of X_1.
(ii) Z continuous (briefly, $\mathfrak{F}\mathcal{F}ZCts$), if for each $\mathfrak{F}\mathcal{F}o$ set M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}Zo$ set of X_1.

Lemma 2.2. [24] Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a function. Then the following statements hold.

(i) If A and B are Fermatean fuzzy subsets of X_1 such that $A \subseteq B$, then $h_{\mathfrak{F}}(A) \subseteq h_{\mathfrak{F}}(B)$.
(ii) If A and B are Fermatean fuzzy subsets of X_2 such that $A \subseteq B$, then $h_{\mathfrak{F}}^{-1}(A) \subseteq h_{\mathfrak{F}}^{-1}(B)$.

Lemma 2.3. [24] Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a function. If A is a Fermatean fuzzy subset of X_1 and B is a Fermatean fuzzy subset of X_2. Then

(i) $h_{\mathfrak{F}}(h_{\mathfrak{F}}^{-1}(A)) \subseteq A$
(ii) $h_{\mathfrak{F}}(h_{\mathfrak{F}}^{-1}(A)) = A \Leftrightarrow h_{\mathfrak{F}}$ is surjective.
(iii) $h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}(A)) \supseteq A$
(iv) $h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}(A)) = A$ whenever $h_{\mathfrak{F}}$ is injective.

3. Fermatean fuzzy Z (resp. δ, $\delta\mathcal{S}$ and pre)-continuous functions

Definition 3.1. Let (X,τ) be a $\mathfrak{F}\mathcal{F}ts$. Then a $\mathfrak{F}\mathcal{F}s$ S in X is said to be a Fermatean fuzzy

(i) Z-open (briefly, $\mathfrak{F}\mathcal{F}Zo$) set if $S \subseteq \mathfrak{F}\mathcal{F}cl(\mathfrak{F}\mathcal{F}\delta int(S)) \cap \mathfrak{F}\mathcal{F}int(\mathfrak{F}\mathcal{F}cl(S))$,
(ii) Z-closed (briefly, $\mathfrak{F}\mathcal{F}Zc$) set if $\mathfrak{F}\mathcal{F}int(\mathfrak{F}\mathcal{F}\delta cl(S)) \cap \mathfrak{F}\mathcal{F}cl(\mathfrak{F}\mathcal{F}int(S)) \subseteq S$,
(iii) pre open (briefly, $\mathfrak{F}\mathcal{F}\mathcal{P}o$) set if $S \subseteq \mathfrak{F}\mathcal{F}int(\mathfrak{F}\mathcal{F}cl(S))$.

The complement of an $\mathfrak{F}\mathcal{F}Zo$ (resp. $\mathfrak{F}\mathcal{F}\mathcal{P}o$) set is called a Fermatean fuzzy Z (resp. Fermatean fuzzy pre) closed (briefly, $\mathfrak{F}\mathcal{F}Zc$ (resp. $\mathfrak{F}\mathcal{F}\mathcal{P}c$)) in X.

The family of all $\mathfrak{F}\mathcal{F}Zo$ (resp. $\mathfrak{F}\mathcal{F}Zc$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}\mathcal{P}c$) sets of a space (X,τ) will be as always denoted by $\mathfrak{F}\mathcal{F}ZO(X)$ (resp. $\mathfrak{F}\mathcal{F}ZC(X)$, $\mathfrak{F}\mathcal{F}\mathcal{P}O(X)$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C(X)$).

Definition 3.2. Let (X,τ) be a $\mathfrak{F}\mathcal{F}ts$. Then a $\mathfrak{F}\mathcal{F}s$ K in X, then the Fermatean

(i) fuzzy Z-interior of K is the union of all $\mathfrak{F}\mathcal{F}Zo$ sets contained in K and denoted by $\mathfrak{F}\mathcal{F}Zint(K)$.
(ii) fuzzy Z-closure of K is the intersection of all $\mathfrak{F}\mathcal{F}Zc$ sets containing K and denoted by $\mathfrak{F}\mathcal{F}Zcl(K)$.
(iii) pre interior of S (briefly, $\mathfrak{F}\mathcal{F}\mathcal{P}int(S)$) is defined by $\mathfrak{F}\mathcal{F}\mathcal{P}int(S) = \cup\{I : I \subseteq S \ \&\ I \text{ is a } \mathfrak{F}\mathcal{F}\mathcal{P}o \text{ set in } X\}$.
(iv) pre closure of S (briefly, $\mathfrak{F}\mathcal{F}\mathcal{P}cl(S)$) is defined by $\mathfrak{F}\mathcal{F}\mathcal{P}cl(S) = \cap\{A : S \subseteq A \ \&\ A \text{ is a } \mathfrak{F}\mathcal{F}\mathcal{P}c \text{ set in } X\}$.

Definition 3.3. A function $h_{\mathfrak{F}} : (X_1,\tau_1) \to (X_2,\tau_2)$ is said to be Fermatean fuzzy

(i) pre continuous (briefly, $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$), if for each $\mathfrak{F}\mathcal{F}o$ set M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}\mathcal{P}o$ set of X_1.
(ii) Z continuous (briefly, $\mathfrak{F}\mathcal{F}ZCts$), if for each $\mathfrak{F}\mathcal{F}o$ set M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}Zo$ set of X_1.

Theorem 3.1. Let $h_{\mathfrak{F}} : (X_1,\tau_1) \to (X_2,\tau_2)$ be a mapping. Then every

(i) $\mathfrak{F}\mathcal{F}\delta Cts$ is $\mathfrak{F}\mathcal{F}Cts$.
(ii) $\mathfrak{F}\mathcal{F}Cts$ is $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$.
(iii) $\mathfrak{F}\mathcal{F}\delta Cts$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$.
(iv) $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$ is $\mathfrak{F}\mathcal{F}ZCts$.
(v) $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$ is $\mathfrak{F}\mathcal{F}ZCts$.

Proof. (i) Let $h_{\mathfrak{F}} : (X_1,\tau_1) \to (X_2,\tau_2)$ be $\mathfrak{F}\mathcal{F}\delta Cts$ and L is a $\mathfrak{F}\mathcal{F}o$ in X_2. Then $h_{\mathfrak{F}}^{-1}(L)$ is $\mathfrak{F}\mathcal{F}\delta o$ in X_1. Since every $\mathfrak{F}\mathcal{F}\delta o$ is $\mathfrak{F}\mathcal{F}o$, $h_{\mathfrak{F}}^{-1}(L)$ is $\mathfrak{F}\mathcal{F}o$ in X_1. Therefore $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}Cts$.

(ii) Let $h_{\mathfrak{F}} : (X_1,\tau_1) \to (X_2,\tau_2)$ be $\mathfrak{F}\mathcal{F}Cts$ and L is a $\mathfrak{F}\mathcal{F}o$ in X_2. Then $h_{\mathfrak{F}}^{-1}(L)$ is $\mathfrak{F}\mathcal{F}o$ in X_1. Since every $\mathfrak{F}\mathcal{F}o$ is $\mathfrak{F}\mathcal{F}\mathcal{P}o$, $h_{\mathfrak{F}}^{-1}(L)$ is $\mathfrak{F}\mathcal{F}\mathcal{P}o$ in X_1. Therefore $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$.

(iii) Let $h_{\mathfrak{F}} : (X_1,\tau_1) \to (X_2,\tau_2)$ be $\mathfrak{F}\mathcal{F}\delta Cts$ and L is a $\mathfrak{F}\mathcal{F}o$ in X_2. Then $h_{\mathfrak{F}}^{-1}(L)$ is $\mathfrak{F}\mathcal{F}\delta o$ in X_1. Since every $\mathfrak{F}\mathcal{F}\delta o$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $h_{\mathfrak{F}}^{-1}(L)$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ in X_1. Therefore $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$.

(iv) Let $h_{\mathfrak{F}} : (X_1,\tau_1) \to (X_2,\tau_2)$ be $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$ and L is a $\mathfrak{F}\mathcal{F}o$ in X_2. Then $h_{\mathfrak{F}}^{-1}(L)$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ in X_1. Since every $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ is $\mathfrak{F}\mathcal{F}Zo$, $h_{\mathfrak{F}}^{-1}(L)$ is $\mathfrak{F}\mathcal{F}Zo$ in X_1. Therefore $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$.

(v) Let $h_{\mathfrak{F}} : (X_1,\tau_1) \to (X_2,\tau_2)$ be $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$ and L is a $\mathfrak{F}\mathcal{F}o$ in X_2. Then $h_{\mathfrak{F}}^{-1}(L)$ is $\mathfrak{F}\mathcal{F}\mathcal{P}o$ in X_1. Since every $\mathfrak{F}\mathcal{F}\mathcal{P}o$ is $\mathfrak{F}\mathcal{F}Zo$, $h_{\mathfrak{F}}^{-1}(L)$ is $\mathfrak{F}\mathcal{F}Zo$ in X_1. Therefore $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$. The converse of the Theorem 3.1 need not be true. □

Example 3.1. Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1 and A_2 are defined as
$\mu_{A_1}(a) = 0.4,\ \lambda_{A_1}(a) = 0.1,$
$\mu_{A_1}(b) = 0.6,\ \lambda_{A_1}(b) = 0.3;$
$\mu_{A_2}(a) = 0.9,\ \lambda_{A_2}(a) = 0.2,$
$\mu_{A_2}(b) = 0.6,\ \lambda_{A_2}(b) = 0.3;$
Let $\tau_1 = \tau_2 = \tau = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2\}$ be a $\mathfrak{F}\mathcal{F}ts$ on X and let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}Cts$ but not $\mathfrak{F}\mathcal{F}\delta Cts$. Since, A_2 is a $\mathfrak{F}\mathcal{F}o$ set in X_2 but $h_{\mathfrak{F}}^{-1}(A_2) = A_2$ is not $\mathfrak{F}\mathcal{F}\delta o$ set in X_1.

Example 3.2. Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2 and A_3 are defined as
$\mu_{A_1}(a) = 0.2,\ \lambda_{A_1}(a) = 0.8,$
$\mu_{A_1}(b) = 0.3,\ \lambda_{A_1}(b) = 0.7;$
$\mu_{A_2}(a) = 0.1,\ \lambda_{A_2}(a) = 0.9,$
$\mu_{A_2}(b) = 0.1,\ \lambda_{A_2}(b) = 0.9;$
$\mu_{A_3}(a) = 0.2,\ \lambda_{A_3}(a) = 0.8,$
$\mu_{A_3}(b) = 0.4,\ \lambda_{A_3}(b) = 0.6;$
Let $\tau_1 = \tau_2 = \tau = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2,\ A_3\}$ be a $\mathfrak{F}\mathcal{F}ts$ on X and let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$ but not $\mathfrak{F}\mathcal{F}\delta Cts$. Since, A_1 is a $\mathfrak{F}\mathcal{F}o$ set in X_2 but $h_{\mathfrak{F}}^{-1}(A_1) = A_1$ is not $\mathfrak{F}\mathcal{F}\delta o$ set in X_1.

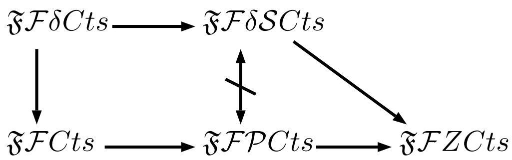
Example 3.3. Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, B_1 and B_2 are defined as
$\mu_{A_1}(a) = 0.2,\ \lambda_{A_1}(a) = 0.7,$
$\mu_{A_1}(b) = 0.1,\ \lambda_{A_1}(b) = 0.8;$
$\mu_{A_2}(a) = 0.3,\ \lambda_{A_2}(a) = 0.6,$
$\mu_{A_2}(b) = 0.4,\ \lambda_{A_2}(b) = 0.5;$
$\mu_{B_1}(a) = 0.1,\ \beta_{B_1}(a) = 0.9,$
$\mu_{B_1}(b) = 0.2,\ \beta_{B_1}(b) = 0.9;$
$\mu_{B_2}(a) = 0.2,\ \beta_{B_2}(a) = 0.3,$
$\mu_{B_2}(b) = 0.4,\ \beta_{B_2}(b) = 0.7;$ Let $\tau_1 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2\}$ and $\tau_2 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ B_1,\ B_2\}$ are $\mathfrak{F}\mathcal{F}ts$'s on X and let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$ but not $\mathfrak{F}\mathcal{F}Cts$. Since, B_2 is a $\mathfrak{F}\mathcal{F}o$ set in X_2 but $h_{\mathfrak{F}}^{-1}(B_2) = B_2$ is not $\mathfrak{F}\mathcal{F}o$ set in X_1.

Example 3.4. Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, A_3, A_4, and A_5 are defined as
$\mu_{A_1}(a) = 0.2,\ \lambda_{A_1}(a) = 0.8,$
$\mu_{A_1}(b) = 0.4,\ \lambda_{A_1}(b) = 0.6;$
$\mu_{A_2}(a) = 0.1,\ \lambda_{A_2}(a) = 0.9,$
$\mu_{A_2}(b) = 0.3,\ \lambda_{A_2}(b) = 0.7;$
$\mu_{A_3}(a) = 0.9,\ \lambda_{A_3}(a) = 0.1,$
$\mu_{A_3}(b) = 0.7,\ \lambda_{A_3}(b) = 0.3;$
$\mu_{A_4}(a) = 0.2,\ \lambda_{A_4}(a) = 0.8,$
$\mu_{A_4}(b) = 0.3,\ \lambda_{A_4}(b) = 0.7.$ Let $\tau_1 = \tau_2 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2,\ A_3,\ A_4\}$ be $\mathfrak{F}\mathcal{F}ts$'s on X and let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$ but not $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$. Since, A_4 is $\mathfrak{F}\mathcal{F}o$ set in X_2 but $h_{\mathfrak{F}}^{-1}(A_4) = A_4$ is not $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ set in X_1.

Example 3.5. Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, A_3, A_4, B_1, B_2, B_3 and B_4 are defined as
$\mu_{A_1}(a) = 0.2,\ \lambda_{A_1}(a) = 0.8,$
$\mu_{A_1}(b) = 0.4,\ \lambda_{A_1}(b) = 0.6;$
$\mu_{A_2}(a) = 0.1,\ \lambda_{A_2}(a) = 0.9,$

$\mu_{A_2}(b) = 0.3,\ \lambda_{A_2}(b) = 0.7;$
$\mu_{A_3}(a) = 0.9,\ \lambda_{A_3}(a) = 0.1,$
$\mu_{A_3}(b) = 0.7,\ \lambda_{A_3}(b) = 0.3;$
$\mu_{A_4}(a) = 0.2,\ \lambda_{A_4}(a) = 0.8,$
$\mu_{A_4}(b) = 0.3,\ \lambda_{A_4}(b) = 0.7;$
$\mu_{B_1}(a) = 0.4,\ \beta_{B_1}(a) = 0.6,$
$\mu_{B_1}(b) = 0.5,\ \beta_{B_1}(b) = 0.5;$
$\mu_{B_2}(a) = 0.6,\ \beta_{B_2}(a) = 0.4,$
$\mu_{B_2}(b) = 0.6,\ \beta_{B_2}(b) = 0.4;$
$\mu_{B_3}(a) = 0.7,\ \beta_{B_3}(a) = 0.3,$
$\mu_{B_3}(b) = 0.6,\ \beta_{B_3}(b) = 0.4;$
$\mu_{B_4}(a) = 0.4,\ \beta_{B_4}(a) = 0.6,$
$\mu_{B_4}(b) = 0.4,\ \beta_{B_4}(b) = 0.6;$
Let $\tau_1 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2,\ A_3,\ A_4\}$ and $\tau_2 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ B_1,\ B_2,\ B_3,\ B_4\}$ are $\mathfrak{F}\mathcal{F}ts$'s on X and let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$ but not $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$. Since, B_4 is a $\mathfrak{F}\mathcal{F}o$ set in X_2 but $h_{\mathfrak{F}}^{-1}(B_4) = B_4$ is not $\mathfrak{F}\mathcal{F}\mathcal{P}o$ set in X_1.

Remark 3.1. The following Figure shows the relations among the different types of Fermatean fuzzy Z continuous mappings that were studied in this section.



Note: $K \to L$ denotes K implies L, but not conversely.

Theorem 3.2. A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathfrak{F}\mathcal{F}ZCts$ iff the inverse image of every $\mathfrak{F}\mathcal{F}c$ set in X_2 is $\mathfrak{F}\mathcal{F}Zc$ in X_1.

Proof. Let $h_{\mathfrak{F}}$ be $\mathfrak{F}\mathcal{F}ZCts$ & O is $\mathfrak{F}\mathcal{F}o$ in X_2. (i.e.) $X_2 - O$ is $\mathfrak{F}\mathcal{F}o$ in X_2. Since $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$, $h_{\mathfrak{F}}^{-1}(Q - O)$ is $\mathfrak{F}\mathcal{F}Zo$ in X_1. (i.e.) $X_1 - h_{\mathfrak{F}}^{-1}(O)$ is $\mathfrak{F}\mathcal{F}Zo$ in X_1. Therefore $h_{\mathfrak{F}}^{-1}(O)$ is $\mathfrak{F}\mathcal{F}Zc$ in X_1.

Conversely, let the inverse image of every $\mathfrak{F}\mathcal{F}c$ set be $\mathfrak{F}\mathcal{F}Zc$ set. Let C be $\mathfrak{F}\mathcal{F}o$ in X_2. Then $X_2 - C$ is $\mathfrak{F}\mathcal{F}c$ in X_2. $\implies$ $h_{\mathfrak{F}}^{-1}(Q - C)$ is $\mathfrak{F}\mathcal{F}Zc$ in X_1. (i.e.) $X_1 - h_{\mathfrak{F}}^{-1}(C)$ is $\mathfrak{F}\mathcal{F}Zc$ in X_1. Therefore $h_{\mathfrak{F}}^{-1}(C)$ is $\mathfrak{F}\mathcal{F}Zo$ in X_1. Thus, the inverse image of every $\mathfrak{F}\mathcal{F}o$ set in X_2 is $\mathfrak{F}\mathcal{F}Zo$ in X_1. Hence, $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$ on X_1. □

Theorem 3.3. A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathfrak{F}\mathcal{F}ZCts$ iff $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Zcl(M)) \subseteq \mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}(M))$ for every subset M of X_1.

Proof. Let $h_{\mathfrak{F}}$ be $\mathfrak{F}\mathcal{F}ZCts$ and $M \subseteq X_1$. Then $h_{\mathfrak{F}}(M) \subseteq X_2$. Since $h_{\mathfrak{F}}$ be $\mathfrak{F}\mathcal{F}ZCts$ and $\mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}(M))$ is $\mathfrak{F}\mathcal{F}c$ in X_2, $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}(M)))$ is $\mathfrak{F}\mathcal{F}Zc$ in X_1. Since $h_{\mathfrak{F}}(M) \subseteq \mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}(M))$, $h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}(M)) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}(M)))$, then $M \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}(M)))$. $\mathfrak{F}\mathcal{F}Zcl$ $(M) \subseteq \mathfrak{F}\mathcal{F}Zcl[h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl h_{\mathfrak{F}}(M))] = h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}(M)))$. Thus $\mathfrak{F}\mathcal{F}Zcl(M) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl$ $(h_{\mathfrak{F}}(M)))$. Therefore $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Zcl(M)) \subseteq \mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}(M))$ for every subset M of X_1.

Conversely, let $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Zcl(M)) \subseteq \mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}(M))$ for every subset M of X_1. If D is $\mathfrak{F}\mathcal{F}c$ in X_2 and since $h_{\mathfrak{F}}^{-1}(D) \subseteq X_1$, $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(D))) \subseteq \mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}(h_{\mathfrak{F}}^{-1}(D))) = \mathfrak{N}cl(D) = D$. That is $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(D))) \subseteq D$. Thus $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(D)) \subseteq h_{\mathfrak{F}}^{-1}(D)$. But $h_{\mathfrak{F}}^{-1}(D) \subseteq \mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(D))$. Hence, $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(D)) = h_{\mathfrak{F}}^{-1}(D)$. Therefore $h_{\mathfrak{F}}^{-1}(D)$ is $\mathfrak{F}\mathcal{F}Zc$ in X_1, for every $\mathfrak{F}\mathcal{F}c$ set D in X_2. Thus $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$. □

Remark 3.2. A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathfrak{F}\mathcal{F}ZCts$ then $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Zcl(K))$ is not necessarily equal to $\mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}(K))$ where $K \in I^{X_1}$.

Example 3.6. In Example 3.4, $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$. Then $\mathfrak{F}\mathcal{F}Zcl(A_2^c) = h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Zcl(A_2^c)) = h_{\mathfrak{F}}(A_1^c) = A_1^c$. But $\mathfrak{F}\mathcal{F}cl h_{\mathfrak{F}}(A_2^C) = \mathfrak{F}\mathcal{F}cl(A_2^c) = A_2^c$. Thus $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Zcl(A_2^c)) \neq \mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}(A_2^c))$.

Theorem 3.4. A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathfrak{F}\mathcal{F}ZCts$ iff $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(L)) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(L))$ for every subset L of X_2.

Proof. If $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$ & $L \subseteq X_2$. $\mathfrak{F}\mathcal{F}cl(L)$ is $\mathfrak{F}\mathcal{F}c$ in X_2. Thus, $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(L))$ is $\mathfrak{F}\mathcal{F}Zc$ in X_1. Therefore $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\ cl(L))) = h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(L))$. Since $L \subseteq \mathfrak{F}\mathcal{F}cl(L)$, $h_{\mathfrak{F}}^{-1}(L) \subset h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(L))$. Therefore $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(L)) \subset \mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(L))) = h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(L))$. That is $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(L)) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(L))$.

Conversely, let $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(L)) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(L))$ for every subset L of X_2. If L is $\mathfrak{F}\mathcal{F}c$ in X_2, then $\mathfrak{F}\mathcal{F}cl(L) = L$. By assumption, $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(L)) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(L)) = h_{\mathfrak{F}}^{-1}(L)$. Thus $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(L)) \subseteq h_{\mathfrak{F}}^{-1}(L)$. But $h_{\mathfrak{F}}^{-1}(L) \subseteq \mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(L))$. Therefore $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(L)) = h_{\mathfrak{F}}^{-1}(L)$. Hence $h_{\mathfrak{F}}^{-1}(L)$ is $\mathfrak{F}\mathcal{F}Zc$ in X_1, for every $\mathfrak{F}\mathcal{F}c$ set L in X_2. Therefore $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$ on X_1. □

Remark 3.3. A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathfrak{F}\mathcal{F}ZCts$ then $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(L))$ is not necessarily equal to $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(L))$ where $L \in I^{X_2}$.

Example 3.7. In Example 3.4, $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$. Then $\mathfrak{F}\mathcal{F}cl(A_2^c) = h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(A_2^c)) = h_{\mathfrak{F}}^{-1}(A_2^c) = A_2^c$. But $\mathfrak{F}\mathcal{F}Zcl h_{\mathfrak{F}}^{-1}(A_2^c) = \mathfrak{F}\mathcal{F}Zcl(A_2^c) = A_1^c$. Thus $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(A_2^c)) \neq \mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(A_2^c))$.

Theorem 3.5. A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathfrak{F}\mathcal{F}ZCts$ iff $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}int(L)) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(L))$ for every subset L of X_2.

Proof. If $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$ and $L \subseteq X_2$. $\mathfrak{F}\mathcal{F}int(L)$ is $\mathfrak{F}\mathcal{F}o$ in X_2 and hence, $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}int(L))$ is $\mathfrak{F}\mathcal{F}Zo$ in X_1. Therefore $\mathfrak{F}\mathcal{F}Zint\ (h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}int(L))) = h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}int(L))$. So, $\mathfrak{F}\mathcal{F}int(L) \subseteq L$, $\implies h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}int(L)) \subseteq h_{\mathfrak{F}}^{-1}(L)$. Then, $\mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}int\ (L))) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(L))$. That is $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}int(L)) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(L))$.

Conversely, let $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}int(L)) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(L))$ for every subset L of X_2. If L is $\mathfrak{F}\mathcal{F}o$ in X_2, then $\mathfrak{F}\mathcal{F}int(L) = L$. Based on our assumption, $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}int(L)) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(L))$. Thus $h_{\mathfrak{F}}^{-1}(L) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(L))$. But $\mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(L)) \subseteq h_{\mathfrak{F}}^{-1}(L)$. Therefore $\mathfrak{F}\mathcal{F}Zint\ (h_{\mathfrak{F}}^{-1}(L)) = h_{\mathfrak{F}}^{-1}(L)$. That is, $h_{\mathfrak{F}}^{-1}(L)$ is $\mathfrak{F}\mathcal{F}Zo$ in X_1, for every $\mathfrak{F}\mathcal{F}o$ set L in X_2. Therefore $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$ on X_1. □

Remark 3.4. A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathfrak{F}\mathcal{F}ZCts$ then $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}int(L))$ is not necessarily equal to $\mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(L))$ where $L \in I^{X_2}$.

Example 3.8. In Example 3.4, $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$. Then $\mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(A_1)) = \mathfrak{F}\mathcal{F}Zint(A_1) = A_1^c$. But $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}int(A_1)) = h_{\mathfrak{F}}^{-1}(A_1) = A_1$. Thus $\mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(A_1)) \neq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}int(A_1))$.

Theorem 3.6. Let (X_1, τ_1) and (X_2, τ_2) be two $\mathfrak{F}\mathcal{F}ts$'s and $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a function. Then, $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}ZCts$ function iff $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(B)) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zcl(B))$ for all fuzzy set B in X_2.

Proof. Let B be any Fermatean fuzzy set in X_2 and $h_{\mathfrak{F}}$ be a $\mathfrak{F}\mathcal{F}ZCts$ function. From Theorem 3.4 (i), $h_{\mathfrak{F}}^{-1}(B) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zcl(B))$. Then, $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(B)) \subseteq \mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zcl(B)))$. Since $\mathfrak{F}\mathcal{F}Zcl(B)$ is $\mathfrak{F}\mathcal{F}Zc$ set in X_2, by Theorem 3.4, $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zcl(B))$ is $\mathfrak{F}\mathcal{F}Zc$ set in X_1. Thus, $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(B)) \subseteq \mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zcl(B))) = h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zcl(B))$.

Conversely, $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(B)) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zcl(B))$ for all fuzzy set B in X_2. Let F be a $\mathfrak{F}\mathcal{F}c$ set in X_2. Since every $\mathfrak{F}\mathcal{F}c$ set is $\mathfrak{F}\mathcal{F}Zc$ set, $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}})) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zcl(F)) = h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}})$. From Theorem 3.4, $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}ZCts$ function.

Other cases are similar. □

Theorem 3.7. Let (X_1, τ_1) and (X_2, τ_2) be two $\mathfrak{F}\mathcal{F}ts$'s, $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a bijective function. Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$ iff $\mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(A)) \subseteq h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Zint(A))$ for all Fermatean fuzzy set A in X_1.

Proof. Let A be any Fermatean fuzzy set in X_1 and $h_{\mathfrak{F}}$ be a bijective and $\mathfrak{F}\mathcal{F}ZCts$ function. Let $h_{\mathfrak{F}}(A) = B$. From Theorem 3.5, $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zint(B)) \subseteq h_{\mathfrak{F}}^{-1}(B)$. Since $h_{\mathfrak{F}}$ is an injective function, $h_{\mathfrak{F}}^{-1}(B) = A$, so that $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zint(B)) \subseteq A$. Therefore, $\mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zint(B))) \subseteq \mathfrak{F}\mathcal{F}Zint(A)$. Since $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$, $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zint(B))$ $\mathfrak{F}\mathcal{F}Zo$ set in X_1 and $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zint(B)) \subseteq \mathfrak{F}\mathcal{F}Zint(A), h_{\mathfrak{F}}(h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zint(B))) \subseteq h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Zint(A))$. Hence, $\mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(A)) \subseteq h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Zint(A))$.

Conversely, $\mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(A)) \subseteq h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Zint(A))$ for all Fermatean fuzzy set A in X_1. Let V be a $\mathfrak{F}\mathcal{F}o$ set in X_2. Then V is $\mathfrak{F}\mathcal{F}Zo$ in X_2. Since $h_{\mathfrak{F}}$ is surjective and Theorem 3.5, $V = \mathfrak{F}\mathcal{F}Zint(V) = \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(h_{\mathfrak{F}}^{-1}(V))) \subseteq h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(V))$. It follows that, $h_{\mathfrak{F}}^{-1}(V) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(V))$. Therefore $h_{\mathfrak{F}}^{-1}(V)$ is $\mathfrak{F}\mathcal{F}Zo$ set in X_1. Hence by Definition 3.3, $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}ZCts$ function. □

Definition 3.4. For any two Fermatean fuzzy subsets A and B, we shall write AqB to mean that A is Fermatean fuzzy quasi-coincident with B if there exists $x \in X$ such that $A(x) + B(x) > 1$. That is $\mu_A(x) + \mu_B(x) : x \in X > 1$.

If A is not Fermatean fuzzy quasi-coincident with B, then we write $A \not{q} B$.

Definition 3.5. Let A and B be any two Fermatean fuzzy subsets of a $\mathfrak{F}\mathcal{F}ts$'s. Then A is Fermatean fuzzy q-neighborhood with B if there exists a $\mathfrak{F}\mathcal{F}o$ set O with $AqO \subseteq B$.

Definition 3.6. A Fermatean fuzzy set A in a $\mathfrak{F}\mathcal{F}ts$ (X, τ) is called a Fermatean fuzzy Z q-neighborhood (briefly, $\mathfrak{F}\mathcal{F}Zq$-*nbhd*) of a Fermatean fuzzy point x_r if there exists a $\mathfrak{F}\mathcal{F}Zo$ set V in (X, τ) such that $x_r qV \subseteq A$.

Proposition 3.1. Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}ts$. Then the following assertions are equivalent.

(i) $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$.
(ii) For each Fermatean fuzzy point $x_r \in X_1$ and every $\mathfrak{F}\mathcal{F}Zq$-*nbhd* A of $h_{\mathfrak{F}}(x_r)$, there exists a $\mathfrak{F}\mathcal{F}Zo$ set B in X_1 such that $x_r \in B \subseteq h_{\mathfrak{F}}^{-1}(A)$.
(iii) For each Fermatean fuzzy point $x_r \in X_1$ and every $\mathfrak{F}\mathcal{F}Zq$-*nbhd* A of $h_{\mathfrak{F}}(x_r)$, there exists a $\mathfrak{F}\mathcal{F}Zo$ set B in X_1 such that $x_r \in B$ and $h_{\mathfrak{F}}(B) \subseteq A$.

Proof. (i) ⇒ (ii) Let x_r be a Fermatean fuzzy point in X_1 and let A be a $\mathfrak{F}\mathcal{F}Zq$-*nbhd* of $h_{\mathfrak{F}}(x_r)$. Then there exists a $\mathfrak{F}\mathcal{F}Zo$ set B in X_2 such that $h_{\mathfrak{F}}(x_r) \in B \subseteq A$. Since $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$, we know that $h_{\mathfrak{F}}^{-1}(B)$ is a $\mathfrak{F}\mathcal{F}Zo$ set in X_1 and $x_r \in h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}(x_r)) \subseteq h_{\mathfrak{F}}^{-1}(B) \subseteq h_{\mathfrak{F}}^{-1}(A)$. Consequently (ii) is valid.

(ii) ⇒ (iii) Let x_r be a Fermatean fuzzy point in X_1 and let A be a $\mathfrak{F}\mathcal{F}Zq$-$nbhd$ of $h_{\mathfrak{F}}(x_r)$. The condition (ii) implies that there exists a $\mathfrak{F}\mathcal{F}Zo$ set B in X_1 such that $x_r \in B \subseteq h_{\mathfrak{F}}^{-1}(A)$ so that $x_r \in B$ and $h_{\mathfrak{F}}(B) \subseteq h_{\mathfrak{F}}(h_{\mathfrak{F}}^{-1}(A)) \subseteq A$. Hence (iii) is true.

(iii) ⇒ (i) Let B be a $\mathfrak{F}\mathcal{F}o$ set in X_2 and let $x_r \in h_{\mathfrak{F}}^{-1}(B)$. Since B is $\mathfrak{F}\mathcal{F}o$ set, $h_{\mathfrak{F}}(x_r) \in B$, and so B is a $\mathfrak{F}\mathcal{F}Zq$-$nbhd$ of $h_{\mathfrak{F}}(x_r)$. It follows from (iii) that there exists a $\mathfrak{F}\mathcal{F}Zo$ set A in X_1 such that $x_r \in A$ and $h_{\mathfrak{F}}(A) \subseteq B$ so that $x_r \in A \subseteq h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}(A)) \subseteq h_{\mathfrak{F}}^{-1}(B)$. Applying Definition 3.6 induces that $h_{\mathfrak{F}}^{-1}(B)$ is a $\mathfrak{F}\mathcal{F}Zo$ set in X_1. Therefore, $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}ZCts$ function. □

Remark 3.5. The composition of two $\mathfrak{F}\mathcal{F}ZCts$ functions need not be $\mathfrak{F}\mathcal{F}ZCts$ as seen from the following example.

Example 3.9. Let $X_1 = X_2 = X_3 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, B_1, B_2 and C_1 are defined as
$\mu_{A_1}(a) = 0.2,\ \lambda_{A_1}(a) = 0.7,$
$\mu_{A_1}(b) = 0.1,\ \lambda_{A_1}(b) = 0.8;$
$\mu_{A_2}(a) = 0.3,\ \lambda_{A_2}(a) = 0.6,$
$\mu_{A_2}(b) = 0.4,\ \lambda_{A_2}(b) = 0.5;$
$\mu_{B_1}(a) = 0.1,\ \beta_{B_1}(a) = 0.9,$
$\mu_{B_1}(b) = 0.2,\ \beta_{B_1}(b) = 0.9;$
$\mu_{B_2}(a) = 0.2,\ \beta_{B_2}(a) = 0.3,$
$\mu_{B_2}(b) = 0.4,\ \beta_{B_2}(b) = 0.7;$
$\mu_{C_1}(a) = 0.3,\ \beta_{C_1}(a) = 0.2,$
$\mu_{C_1}(b) = 0.7,\ \beta_{C_1}(b) = 0.4;$
Let $\tau_1 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2\}$, $\tau_2 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ B_1,\ B_2\}$ and $\tau_3 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ C_1\}$ are $\mathfrak{F}\mathcal{F}ts$'s on X and

let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ and $g_{\mathfrak{F}} : (X_2, \tau_2) \to (X_3, \tau_3)$ be an identity function, Then $h_{\mathfrak{F}}$ and $g_{\mathfrak{F}}$ are $\mathfrak{F}\mathcal{F}ZCts$ but $(g_{\mathfrak{F}} \circ h_{\mathfrak{F}})$ is not $\mathfrak{F}\mathcal{F}ZCts$. Since, C_1 is a $\mathfrak{F}\mathcal{F}o$ set in X_3 but $(g_{\mathfrak{F}} \circ h_{\mathfrak{F}})^{-1}(C_1) = C_1$ is not $\mathfrak{F}\mathcal{F}Zo$ set in X_1.

Theorem 3.8. Let $h_1 : (X_1, \tau_1) \to (X_2, \tau_2)$ and $h_2 \colon (X_2, \tau_2) \to (X_3, \tau_3)$ be any two functions. If h_1 is a $\mathfrak{F}\mathcal{F}ZCts$ and h_2 is $\mathfrak{F}\mathcal{F}Cts$ functions, then $h_2 \circ h_1$ is $\mathfrak{F}\mathcal{F}ZCts$.

Proof. Let C be any $\mathfrak{F}\mathcal{F}c$ set in S. As h_2 is $\mathfrak{F}\mathcal{F}Cts$, $h_2^{-1}(C)$ is $\mathfrak{F}\mathcal{F}c$ in X_2. Since h_1 is $\mathfrak{F}\mathcal{F}ZCts$, implies $h_1^{-1}(h_2^{-1}(C)) = (h_2 \circ h_1)^{-1}(C)$ is $\mathfrak{F}\mathcal{F}Zc$ in X_1. Therefore $h_2 \circ h_1$ is $\mathfrak{F}\mathcal{F}ZCts$. □

4. Fermatean fuzzy Z-irresolute Functions

Definition 4.1. A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is called Fermatean fuzzy

(i) irresolute [24] (briefly, $\mathfrak{F}\mathcal{F}ZIrr$) function, if for each $\mathfrak{F}\mathcal{F}\mathcal{S}o$ subset M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}\mathcal{S}o$ subset of X_1.
(ii) pre irresolute (briefly, $\mathfrak{F}\mathcal{F}\mathcal{P}Irr$) function, if for each $\mathfrak{F}\mathcal{F}\mathcal{P}o$ subset M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}\mathcal{P}o$ subset of X_1.
(iii) δ semi irresolute [24] (briefly, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Irr$) function, if for each $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ subset M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ subset of X_1.
(iv) Z irresolute (briefly, $\mathfrak{F}\mathcal{F}ZIrr$) function, if for each $\mathfrak{F}\mathcal{F}Zo$ subset M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}Zo$ subset of X_1.

Theorem 4.1. A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is called

(i) $\mathfrak{F}\mathcal{F}Irr$, then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}Cts$.

(ii) $\mathfrak{F}\mathcal{F}\mathcal{P}Irr$, then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$.
(iii) $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Irr$, then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$.
(iv) $\mathfrak{F}\mathcal{F}ZIrr$, then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$.

Proof. (iv) Let C be $\mathfrak{F}\mathcal{F}o$ set in X_2, then C is $\mathfrak{F}\mathcal{F}Zo$ set in X_2, since every $\mathfrak{F}\mathcal{F}o$ set is $\mathfrak{F}\mathcal{F}Zo$. By hypothesis, $h_{\mathfrak{F}}^{-1}(C)$ is $\mathfrak{F}\mathcal{F}Zo$. Therefore $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$. Proof of the other cases are similar. □

The converse of the Theorem 4.1 need not be true.

Example 4.1. Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, B_1 and B_2 are defined as
$\mu_{A_1}(a) = 0.2,\ \lambda_{A_1}(a) = 0.7,$
$\mu_{A_1}(b) = 0.1,\ \lambda_{A_1}(b) = 0.8;$
$\mu_{A_2}(a) = 0.3,\ \lambda_{A_2}(a) = 0.6,$
$\mu_{A_2}(b) = 0.4,\ \lambda_{A_2}(b) = 0.5;$
$\mu_{B_1}(a) = 0.1,\ \beta_{B_1}(a) = 0.9,$
$\mu_{B_1}(b) = 0.2,\ \beta_{B_1}(b) = 0.9;$
$\mu_{B_2}(a) = 0.2,\ \beta_{B_2}(a) = 0.3,$
$\mu_{B_2}(b) = 0.4,\ \beta_{B_2}(b) = 0.7;$
Let $\tau_1 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2\}$ and $\tau_2 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ B_1,\ B_2\}$ are $\mathfrak{F}\mathcal{F}ts$'s on X and let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\mathcal{S}Cts$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Cts$) but not $\mathfrak{F}\mathcal{F}Irr$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Irr$). Since, A_2^c is a $\mathfrak{F}\mathcal{F}\mathcal{S}o$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{P}o$) set in X_2 but $h_{\mathfrak{F}}^{-1}(A_2^c) = A_2^c$ is not $\mathfrak{F}\mathcal{F}\mathcal{S}o$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{P}o$) set in X_1.

Example 4.2. Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, A_3, A_4, B_1, B_2, B_3 and B_4 are defined as
$\mu_{A_1}(a) = 0.4,\ \lambda_{A_1}(a) = 0.6,$
$\mu_{A_1}(b) = 0.5,\ \lambda_{A_1}(b) = 0.5;$
$\mu_{A_2}(a) = 0.6,\ \lambda_{A_2}(a) = 0.4,$
$\mu_{A_2}(b) = 0.6,\ \lambda_{A_2}(b) = 0.4;$
$\mu_{A_3}(a) = 0.7,\ \lambda_{A_3}(a) = 0.3,$
$\mu_{A_3}(b) = 0.6,\ \lambda_{A_3}(b) = 0.4;$
$\mu_{A_4}(a) = 0.4,\ \lambda_{A_4}(a) = 0.6,$
$\mu_{A_4}(b) = 0.4,\ \lambda_{A_4}(b) = 0.6;$
$\mu_{B_1}(a) = 0.2,\ \beta_{B_1}(a) = 0.8,$
$\mu_{B_1}(b) = 0.4,\ \beta_{B_1}(b) = 0.6;$
$\mu_{B_2}(a) = 0.1,\ \beta_{B_2}(a) = 0.9,$
$\mu_{B_2}(b) = 0.3,\ \beta_{B_2}(b) = 0.7;$
$\mu_{B_3}(a) = 0.9,\ \beta_{B_3}(a) = 0.1,$
$\mu_{B_3}(b) = 0.7,\ \beta_{B_3}(b) = 0.3;$
$\mu_{B_4}(a) = 0.2,\ \beta_{B_4}(a) = 0.8,$
$\mu_{B_4}(b) = 0.3,\ \beta_{B_4}(b) = 0.7;$
Let $\tau_1 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2,\ A_3,\ A_4\}$ and $\tau_2 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ B_1,\ B_2,\ B_3,\ B_4\}$ are $\mathfrak{F}\mathcal{F}ts$'s on X and let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$ but not $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Irr$. Since, A_4 is a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ set in X_2 but $h_{\mathfrak{F}}^{-1}(A_4) = A_4$ is not $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ set in X_1.

Example 4.3. Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, A_3, A_4, B_1, $B_2$and B_3 are defined as
$\mu_{A_1}(a) = 0.2,\ \lambda_{A_1}(a) = 0.8,$
$\mu_{A_1}(b) = 0.4,\ \lambda_{A_1}(b) = 0.6;$
$\mu_{A_2}(a) = 0.1,\ \lambda_{A_2}(a) = 0.9,$

$\mu_{A_2}(b) = 0.3,\ \lambda_{A_2}(b) = 0.7;$
$\mu_{A_3}(a) = 0.9,\ \lambda_{A_3}(a) = 0.1,$
$\mu_{A_3}(b) = 0.7,\ \lambda_{A_3}(b) = 0.3;$
$\mu_{A_4}(a) = 0.2,\ \lambda_{A_4}(a) = 0.8,$
$\mu_{A_4}(b) = 0.3,\ \lambda_{A_4}(b) = 0.7;$
$\mu_{B_1}(a) = 0.9,\ \beta_{B_1}(a) = 0.1,$
$\mu_{B_1}(b) = 0.7,\ \beta_{B_1}(b) = 0.3;$
$\mu_{B_2}(a) = 0.1,\ \beta_{B_2}(a) = 0.9,$
$\mu_{B_2}(b) = 0.3,\ \beta_{B_2}(b) = 0.7;$
$\mu_{B_3}(a) = 0.2,\ \beta_{B_3}(a) = 0.8,$
$\mu_{B_3}(b) = 0.4,\ \beta_{B_3}(b) = 0.6.$
Let $\tau_1 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2,\ A_3,\ A_4\}$ and $\tau_2 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ B_1,\ B_2,\ B_3\}$ are $\mathfrak{F}\mathcal{F}ts$'s on X and let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$ (resp. $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$) but not $\mathfrak{F}\mathcal{F}\mathcal{P}Irr$ (resp. $\mathfrak{F}\mathcal{F}ZIrr$). Since, (i) A_4^c is a $\mathfrak{F}\mathcal{F}\mathcal{P}o$ set in X_2 but $h_{\mathfrak{F}}^{-1}(A_4^c) = A_4^c$ is not $\mathfrak{F}\mathcal{F}\mathcal{P}o$ set in X_1, (ii) A_1^c is a $\mathfrak{F}\mathcal{F}Zo$ set in X_2 but $h_{\mathfrak{F}}^{-1}(A_1^c) = A_1^c$ is not $\mathfrak{F}\mathcal{F}Zo$ set in X_1.

Theorem 4.2. A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathfrak{F}\mathcal{F}ZIrr$ iff for every $\mathfrak{F}\mathcal{F}Zc$ set K in X_2, $h_{\mathfrak{F}}^{-1}(K)$ is $\mathfrak{F}\mathcal{F}Zc$ in X_1.

Proof. Follows from the fact that the complement of $\mathfrak{F}\mathcal{F}Zo$ set is $\mathfrak{F}\mathcal{F}Zc$ and vice versa. □

Theorem 4.3. Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a function. Then the following are equivalent:

(i) $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZIrr$.
(ii) $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(B)) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zcl(B))$ for every Fermatean fuzzy set B of X_2.
(iii) $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Zcl(A)) \subseteq \mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}(A))$ for every Fermatean fuzzy set A of X_1.
(iv) $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zint(B)) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(B))$ for every Fermatean fuzzy set B of X_2.

Proof. (i) $\Rightarrow$ (ii): Let B be any Fermatean fuzzy set in X_2. Then by Definition 3.2, $\mathfrak{F}\mathcal{F}Zcl(B)$ is $\mathfrak{F}\mathcal{F}Zc$ in X_2. Since $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZIrr$, $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zcl(B))$ is $\mathfrak{F}\mathcal{F}Zc$ in X_1. Then $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zcl(B))) = h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zcl(B))$. By Theorems 3.4 and 3.5, $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(B)) \subseteq \mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zcl(B))) = h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zcl(B))$. This proves (ii).

(ii) $\Rightarrow$ (iii): Let A be any Fermatean fuzzy set in X_1. Then $h_{\mathfrak{F}}(A) \subseteq X_2$. By (ii), $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}(A))) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}(A)))$. But $\mathfrak{F}\mathcal{F}Zcl(A) \subseteq \mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}(A)))$, $\mathfrak{F}\mathcal{F}Zcl(A) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}(A)))$. That implies, $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Zcl(A)) \subseteq \mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}(A))$.

(iii) $\Rightarrow$ (i): Let $h_{\mathfrak{F}}$ be any $\mathfrak{F}\mathcal{F}Zc$ set in X_2. Then $h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}) = h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zcl(F))$. By (iii), $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}))) \subseteq \mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}(h_{\mathfrak{F}}^{-1}(F))) \subseteq \mathfrak{F}\mathcal{F}Zcl(F) = F$. That implies, $(\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}))) \subseteq h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}})$. But $h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}) \subseteq \mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}})), \mathfrak{F}\mathcal{F}Z\ cl(h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}})) = h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}})$ and so $h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}})$ is $\mathfrak{F}\mathcal{F}Zc$ set X_1. Therefore $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZIrr$.

(i) $\Rightarrow$ (iv): Let B any Fermatean fuzzy set in X_2. By Definition 3.2, $\mathfrak{F}\mathcal{F}Zint(B)$ is $\mathfrak{F}\mathcal{F}Zo$ in X_2. Since $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZIrr$, $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zint(B))$ is $\mathfrak{F}\mathcal{F}Zo$ in X_1. Then $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zint(B)) = \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zint(B))) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(B))$.

(iv) $\Rightarrow$ (i): Let V be any $\mathfrak{F}\mathcal{F}Zc$ in X_2. Then by (iv), $h_{\mathfrak{F}}^{-1}(V) = h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Zint(V)) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(V))$. But, $\mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(V)) \subseteq h_{\mathfrak{F}}^{-1}(V), \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}^{-1}(V)) = h_{\mathfrak{F}}^{-1}(V)$ and by Theorem 3.2 (ii), $h_{\mathfrak{F}}^{-1}(V)$ is $\mathfrak{F}\mathcal{F}Zo$. Thus $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZIrr$. □

Theorem 4.4. If $h_1 : (X_1, \tau_1) \to (X_2, \tau_2)$ and $h_2 : (X_2, \tau_2) \to (X_3, \tau_3)$ are both $\mathfrak{F}\mathcal{F}ZIrr$, then $h_2 \circ h_1 : (X_1, \tau_1) \to (X_3, \tau_3)$ is $\mathfrak{F}\mathcal{F}ZIrr$.

Proof. Let K be $\mathfrak{F}\mathcal{F}Zo$ in S. Then $h_2^{-1}(K)$ is $\mathfrak{F}\mathcal{F}Zo$ in X_2, since h_2 is $\mathfrak{F}\mathcal{F}ZIrr$ and $h_1^{-1}(h_2^{-1}(K)) = (h_2 \circ h_1)^{-1}(K)$ is $\mathfrak{F}\mathcal{F}Zo$ in P, since h_1 is $\mathfrak{F}\mathcal{F}ZIrr$. Hence $h_2 \circ h_1$ is $\mathfrak{F}\mathcal{F}ZIrr$. □

Theorem 4.5. (i) If $h_1 : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathfrak{F}\mathcal{F}ZIrr$ and $h_2 : (X_2, \tau_2) \to (X_3, \tau_3)$ is $\mathfrak{F}\mathcal{F}ZCts$, then $h_2 \circ h_1 : (X_1, \tau_1) \to (X_3, \tau_3)$ is $\mathfrak{F}\mathcal{F}ZCts$.

(ii) If $h_1 : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathfrak{F}\mathcal{F}ZCts$ and $h_2 : (X_2, \tau_2) \to (X_3, \tau_3)$ is $\mathfrak{F}\mathcal{F}Cts$, then $h_2 \circ h_1 : (X_1, \tau_1) \to (X_3, \tau_3)$ is $\mathfrak{F}\mathcal{F}ZCts$.

5. Strongly Fermatean fuzzy Z continuous and Perfectly Fermatean fuzzy Z continuous

In this section, we introduce the concept of strongly Fermatean fuzzy (resp. Z) continuous and perfectly Fermatean fuzzy (resp. Z) continuous functions in $\mathfrak{F}\mathcal{F}ts$ and we discuss the relation with the above-mentioned functions.

Definition 5.1. Let (X_1, τ_1) and (X_2, τ_2) be two $\mathfrak{F}\mathcal{F}ts$'s. A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is called strongly Fermatean fuzzy

(i) continuous (briefly, $St\mathfrak{F}\mathcal{F}Cts$) function if the inverse image of every subset in X_2 is Fermatean fuzzy clopen (i.e both $\mathfrak{F}\mathcal{F}o$ and $\mathfrak{F}\mathcal{F}c$) (briefly, $\mathfrak{F}\mathcal{F}clo$) in X_1.

(ii) Z continuous (briefly, $St\mathfrak{F}\mathcal{F}ZCts$) function if the inverse image of every $\mathfrak{F}\mathcal{F}Zo$ set in X_2 is $\mathfrak{F}\mathcal{F}o$ in X_1.

Definition 5.2. Let (X_1, τ_1) and (X_2, τ_2) be two $\mathfrak{F}\mathcal{F}ts$'s. A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is called a perfectly Fermatean fuzzy

(i) continuous (briefly, $Pe\mathfrak{F}\mathcal{F}Cts$) function if the inverse image of every $\mathfrak{F}\mathcal{F}o$ set in X_2 is $\mathfrak{F}\mathcal{F}clo$ in X_1.

(ii) Z continuous (briefly, $Pe\mathfrak{F}\mathcal{F}ZCts$) function if the inverse image of every $\mathfrak{F}\mathcal{F}Zo$ set in X_2 is $\mathfrak{F}\mathcal{F}clo$ in X_1.

Theorem 5.1. Let (X_1, τ_1) and (X_2, τ_2) be two $\mathfrak{F}\mathcal{F}ts$'s and $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a function. Then

(i) If $h_{\mathfrak{F}}$ is $Pe\mathfrak{F}\mathcal{F}ZCts$, then $h_{\mathfrak{F}}$ is $Pe\mathfrak{F}\mathcal{F}Cts$.

(ii) If $h_{\mathfrak{F}}$ is $St\mathfrak{F}\mathcal{F}ZCts$, then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}Cts$.

Proof. (i) Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be $Pe\mathfrak{F}\mathcal{F}ZCts$. Let V be a $\mathfrak{F}\mathcal{F}o$ set in X_2. Since $h_{\mathfrak{F}}$ is $Pe\mathfrak{F}\mathcal{F}ZCts$, $h_{\mathfrak{F}}^{-1}(V)$ is $\mathfrak{F}\mathcal{F}clo$ in X_1. Therefore $h_{\mathfrak{F}}$ is $Pe\mathfrak{F}\mathcal{F}Cts$.

(ii) Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be $St\mathfrak{F}\mathcal{F}ZCts$. Let G be a $\mathfrak{F}\mathcal{F}o$ set in X_2. Since $h_{\mathfrak{F}}$ is $St\mathfrak{F}\mathcal{F}ZCts$, $h_{\mathfrak{F}}^{-1}(G)$ is $\mathfrak{F}\mathcal{F}o$ in X_1. Therefore $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}Cts$. □

Theorem 5.2. Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be $St\mathfrak{F}\mathcal{F}ZCts$ and A be $\mathfrak{F}\mathcal{F}o$ in X_1. Then the restriction, $h_{\mathfrak{F}_A} : A \to X_2$ is $St\mathfrak{F}\mathcal{F}ZCts$.

Proof. Let V be any $\mathfrak{F}\mathcal{F}Zo$ set in X_2. Since $h_{\mathfrak{F}}$ is $St\mathfrak{F}\mathcal{F}ZCts$, $h_{\mathfrak{F}}^{-1}(V)$ is $\mathfrak{F}\mathcal{F}o$ in X_1. But $h_{\mathfrak{F}_A}^{-1}(V) = A \wedge h_{\mathfrak{F}}^{-1}(V)$. Since A and $h_{\mathfrak{F}}^{-1}(V)$ are $\mathfrak{F}\mathcal{F}o$, $h_{\mathfrak{F}_A}^{-1}(V)$ is $\mathfrak{F}\mathcal{F}o$ in A. Hence $h_{\mathfrak{F}_A}$ is $St\mathfrak{F}\mathcal{F}ZCts$. □

Theorem 5.3. Every $Pe\mathfrak{F}\mathcal{F}ZCts$ is $St\mathfrak{F}\mathcal{F}ZCts$.

Proof. Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be $Pe\mathfrak{F}\mathcal{F}ZCts$ and V be $\mathfrak{F}\mathcal{F}Zo$ in X_2. Since $h_{\mathfrak{F}}$ is $Pe\mathfrak{F}\mathcal{F}ZCts$, $h_{\mathfrak{F}}^{-1}(V)$ is $\mathfrak{F}\mathcal{F}clo$ in X_1. That is, $h_{\mathfrak{F}}^{-1}(V)$ is both $\mathfrak{F}\mathcal{F}o$ and $\mathfrak{F}\mathcal{F}c$ in X_1. Hence $h_{\mathfrak{F}}$ is $St\mathfrak{F}\mathcal{F}ZCts$. □

Theorem 5.4. If $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ and $g_{\mathfrak{F}} : (X_2, \tau_2) \to (X_3, \tau_3))$ are $St\mathfrak{F}\mathcal{F}ZCts$, then their composition $g_{\mathfrak{F}} \circ h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_3, \tau_3))$ is also $St\mathfrak{F}\mathcal{F}\ ZCts$.

Proof. Let V be a $\mathfrak{F}\mathcal{F}Zo$ set in X_3. Since $g_{\mathfrak{F}}$ is a $St\mathfrak{F}\mathcal{F}ZCts$ function, $g_{\mathfrak{F}}^{-1}(V)$ is $\mathfrak{F}\mathcal{F}o$ in X_2. Since $h_{\mathfrak{F}}$ is a $St\mathfrak{F}\mathcal{F}ZCts$ function, $h_{\mathfrak{F}}^{-1}(g_{\mathfrak{F}}^{-1}(V)) = (g_{\mathfrak{F}} \circ h_{\mathfrak{F}})^{-1}(V)$ is $\mathfrak{F}\mathcal{F}o$ in X_1. Therefore $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is $St\mathfrak{F}\mathcal{F}ZCts$. □

Theorem 5.5. If $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ and $g_{\mathfrak{F}} : (X_2, \tau_2) \to (X_3, \tau_3))$ are $Pe\mathfrak{F}\mathcal{F}ZCts$, then their composition $g_{\mathfrak{F}} \circ h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_3, \tau_3))$ is also $Pe\mathfrak{F}\mathcal{F}ZCts$.

Proof. Let V be a $\mathfrak{F}\mathcal{F}Zo$ set in X_3. Since $g_{\mathfrak{F}}$ is a $Pe\mathfrak{F}\mathcal{F}ZCts$ function, $g_{\mathfrak{F}}^{-1}(V)$ is $\mathfrak{F}\mathcal{F}clo$ in X_2. That is $g_{\mathfrak{F}}^{-1}(V)$ is both $\mathfrak{F}\mathcal{F}o$ and $\mathfrak{F}\mathcal{F}c$. Since $h_{\mathfrak{F}}$ is a $Pe\mathfrak{F}\mathcal{F}ZCts$ function, $h_{\mathfrak{F}}^{-1}(g_{\mathfrak{F}}^{-1}(V)) = (g_{\mathfrak{F}} \circ h_{\mathfrak{F}})^{-1}(V)$ is $\mathfrak{F}\mathcal{F}clo$ in X_1. Therefore $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is $Pe\mathfrak{F}\mathcal{F}ZCts$. □

Theorem 5.6. Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ and $g_{\mathfrak{F}} : (X_2, \tau_2) \to (X_3, \tau_3))$ be functions. Then,

(i) If $g_{\mathfrak{F}}$ is $St\mathfrak{F}\mathcal{F}ZCts$ and $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$, then $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZIrr$.
(ii) If $g_{\mathfrak{F}}$ is $Pe\mathfrak{F}\mathcal{F}ZCts$ and $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}Cts$, then $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is $St\mathfrak{F}\mathcal{F}ZCts$.
(iii) If $g_{\mathfrak{F}}$ is $St\mathfrak{F}\mathcal{F}ZCts$ and $h_{\mathfrak{F}}$ is $Pe\mathfrak{F}\mathcal{F}ZCts$, then $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is $Pe\mathfrak{F}\mathcal{F}ZCts$.
(iv) If $g_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$ and $h_{\mathfrak{F}}$ is $St\mathfrak{F}\mathcal{F}ZCts$, then $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}Cts$.

Proof. (i) Let V be a $\mathfrak{F}\mathcal{F}Zo$ set in X_3. Since $g_{\mathfrak{F}}$ is a $St\mathfrak{F}\mathcal{F}ZCts$ function, $g_{\mathfrak{F}}^{-1}(V)$ is $\mathfrak{F}\mathcal{F}o$ in X_2. Since $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}ZCts$ function, $h_{\mathfrak{F}}^{-1}(g_{\mathfrak{F}}^{-1}(V)) = (g_{\mathfrak{F}} \circ h_{\mathfrak{F}})^{-1}(V)$ is $\mathfrak{F}\mathcal{F}Zo$ in X_1. Hence $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZIrr$.

(ii) Let V be a $\mathfrak{F}\mathcal{F}Zo$ set in X_3. Since $g_{\mathfrak{F}}$ is a $Pe\mathfrak{F}\mathcal{F}ZCts$ function, $g_{\mathfrak{F}}^{-1}(V)$ is $\mathfrak{F}\mathcal{F}clo$ in X_2. That is, $g_{\mathfrak{F}}^{-1}(V)$ is both $\mathfrak{F}\mathcal{F}o$ and $\mathfrak{F}\mathcal{F}c$. Since $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}Cts$, $h_{\mathfrak{F}}^{-1}(g_{\mathfrak{F}}^{-1}(V)) = (g_{\mathfrak{F}} \circ h_{\mathfrak{F}})^{-1}(V)$ is $\mathfrak{F}\mathcal{F}o$ in X_1. Therefore $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is $St\mathfrak{F}\mathcal{F}ZCts$.

(iii) Let V be a $\mathfrak{F}\mathcal{F}Zo$ set in X_3. Since $g_{\mathfrak{F}}$ is a $St\mathfrak{F}\mathcal{F}ZCts$ function, $g_{\mathfrak{F}}^{-1}(V)$ is $\mathfrak{F}\mathcal{F}o$ in X_2. Since $h_{\mathfrak{F}}$ is a $Pe\mathfrak{F}\mathcal{F}ZCts$ function, $h_{\mathfrak{F}}^{-1}(g_{\mathfrak{F}}^{-1}(V)) = (g_{\mathfrak{F}} \circ h_{\mathfrak{F}})^{-1}(V)$ is $\mathfrak{F}\mathcal{F}clo$ in X_1. Hence $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is $Pe\mathfrak{F}\mathcal{F}ZCts$.

(iv) Let V be a $\mathfrak{F}\mathcal{F}o$ set in X_3. Since $g_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}ZCts$ function, $g_{\mathfrak{F}}^{-1}(V)$ is $\mathfrak{F}\mathcal{F}Zo$ in X_2. Since $h_{\mathfrak{F}}$ is a $St\mathfrak{F}\mathcal{F}ZCts$ function, $h_{\mathfrak{F}}^{-1}(g_{\mathfrak{F}}^{-1}(V)) = (g_{\mathfrak{F}} \circ h_{\mathfrak{F}})^{-1}(V)$ is $\mathfrak{F}\mathcal{F}o$ in X_1. Therefore $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}Cts$. □

6. Application

Entropy as a measure of fuzziness was first proposed by Zadeh [31]. Later many mathematicians defined several entropy measures. In this section, we focus on defining an entropy measure for $\mathfrak{F}\mathcal{F}s$ that connects the degree of membership and non-membership. As an example, we have applied the proposed entropy measure in decision making.

Definition 6.1. Let $A = \{< x, \mu_A(x), \lambda_A(x) | x \in X\}$ be a $\mathfrak{F}\mathcal{F}s$ in X. The new entropy measure for A denoted by $\varepsilon_{\mathfrak{F}\mathcal{F}s}(A)$, is a function, $\varepsilon_{\mathfrak{F}\mathcal{F}s} : \tau_{\mathfrak{F}\mathcal{F}s}(X) \to [0, 1]$ and is defined as $\varepsilon_{\mathfrak{F}\mathcal{F}s}(A) = 1 - \frac{1}{n}\sum_{i=1}^{n}(\mu_A - \lambda_A)^2$; for every'$x_i \in A$, where $\tau_{\mathfrak{F}\mathcal{F}s}(X)$ denote the family of all $\mathfrak{F}\mathcal{F}s$'s on X.

Example 6.1. A certain company intends to procure a set of computers from a pool of five alternative model options, denoted as Mi $(i = 1, 2, 3, 4, 5)$. The company has identified four crucial attributes for selection, namely, Processor, Ram of the computer, Memory capacity and Power of battery Aj $(j = 1, 2, 3, 4)$. These attributes are allocated weights, denoted in the Table 1, Using Fermatean fuzzy entropy measure. We calculate for the application of the MADM model to identify the most suitable solution.

Table 1. Weight of the attributes

	Attribute 1 (A_1)	Attribute 2 (A_2)	Attribute 3 (A_3)	Attribute 4 (A_4)
Model 1 (M_1)	$< M_1, A_1; 0.42, 0.72 >$	$< M_1, A_2; 0.60, 0.59 >$	$< M_1, A_3; 0.60, 0.20 >$	$< M_1, A_4; 0.70, 0.20 >$
Model 2 (M_2)	$< M_2, A_1; 0.72, 0.37 >$	$< M_2, A_2; 0.19, 0.70 >$	$< M_2, A_3; 0.32, 0.57 >$	$< M_2, A_4; 0.40, 0.28 >$
Model 3 (M_3)	$< M_3, A_1; 0.42, 0.27 >$	$< M_3, A_2; 0.20, 0.40 >$	$< M_3, A_3; 0.90, 0.20 >$	$< M_1, A_4; 0.20, 0.52 >$
Model 4 (M_4)	$< M_4, A_1; 0.12, 0.65 >$	$< M_4, A_2; 0.25, 0.35 >$	$< M_4, A_3; 0.12, 0.55 >$	$< M_4, A_4; 0.52, 0.12 >$
Model 5 (M_5)	$< M_5, A_1; 0.10, 0.60 >$	$< M_5, A_2; 0.30, 0.12 >$	$< M_5, A_3; 0.10, 0.20 >$	$< M_5, A_4; 0.50, 0.09 >$

Clearly, all values in the Table 1 are $\mathfrak{F}\mathcal{F}s$'s. Now we calculate the $\varepsilon_{\mathfrak{F}\mathcal{F}s}$ of each Computer for the corresponding attributes.

Table 2. Entropy measure of each Computer for the corresponding attributes.

	Attribute 1 (A_1)	Attribute 2 (A_2)	Attribute 3 (A_3)	Attribute 4 (A_4)
Model 1 (M_1)	0.91	1	0.84	**0.75**
Model 2 (M_2)	0.88	**0.74**	0.94	0.99
Model 3 (M_3)	0.98	0.96	**0.51**	0.90
Model 4 (M_4)	**0.72**	0.99	0.82	0.84
Model 5 (M_5)	**0.75**	0.97	0.99	0.83

From Table 2, it is clear that $\varepsilon_{\mathfrak{F}\mathcal{F}s}(M1, A4) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(M1, A3) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(M1, A1) < \varepsilon_{\mathfrak{F}\mathcal{F}s}$ $(M1, A2)$

Similarly $\varepsilon_{\mathfrak{F}\mathcal{F}s}(M2, A2) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(M2, A1) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(M2, A3) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(M2, A4)$

$\varepsilon_{\mathfrak{F}\mathcal{F}s}(M3, A3) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(M3, A4) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(M3, A2) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(M3, A1)$

$\varepsilon_{\mathfrak{F}\mathcal{F}s}(M4, A1) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(M4, A3) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(M4, A4) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(M4, A2)$

$\varepsilon_{\mathfrak{F}\mathcal{F}s}(M5, A1) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(M5, A4) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(M5, A2) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(M5, A3)$

It is clear that Model 1 is suitable for A4, Model 2 is suitable for A2, Model 3 is suitable for A3, Model 4 and 5 is suitable for A1.

7. Conclusions

In this paper, we have presented a comprehensive study of Fermatean fuzzy Z-continuous, Fermatean fuzzy Z-irresolute, strongly Fermatean fuzzy Z-continuous, and perfectly Fermatean fuzzy Z-continuous functions within the framework of Fermatean fuzzy topological spaces. The theoretical exploration includes precise definitions, fundamental properties, and relevant examples that illustrate the behavior and significance of these functions. Looking ahead, this line of research can be extended to the investigation of Fermatean fuzzy Z-open maps, Fermatean fuzzy Z-closed maps, and Fermatean fuzzy Z-homeomorphism functions, which may further enrich the structural understanding of Fermatean fuzzy topological mappings. In addition to the theoretical contributions, we have introduced a novel entropy measure for Fermatean fuzzy sets and demonstrated its practical utility through a real-life application. This proposed entropy measure is consistent with analogous measures developed for other set-theoretic frameworks, such as classical fuzzy sets and Pythagorean fuzzy sets. Consequently, it serves as a robust tool for quantifying uncertainty, making it applicable to a wide range of real-world problems where imprecision and vagueness play a crucial role. In future work, these investigations can be extended to include Fermatean fuzzy open maps, Fermatean fuzzy closed maps, Fermatean fuzzy homeomorphisms, and Fermatean fuzzy contra maps, thereby broadening the scope of functional analysis within Fermatean fuzzy topological spaces. Furthermore, in this study, we have applied the proposed entropy measure to solve a Multi-Criteria Decision-Making (MCDM) problem arising from a real-world scenario, thereby demonstrating its practical applicability and effectiveness in handling uncertainty. As a potential direction for future research, the MCDM framework can be further developed and adapted to the structure of $\mathfrak{F}\mathcal{F}ts$, enabling more refined and topology-oriented decision-making models.

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