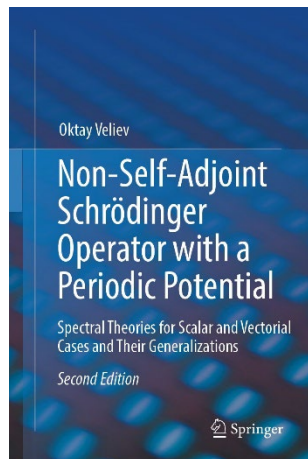


## Book Review



**Oktay Veliyev. Non-Self-Adjoint Schrödinger Operator with a Periodic Potential: Spectral Theories for Scalar and Vectorial Cases and Their Generalizations, Springer, Switzerland, 487 pp. 2025**

This monograph presents a comprehensive treatment of the spectral theory of non-self-adjoint differential operators with complex-valued periodic coefficients. A central theme of the book is the construction of spectral expansions in settings where no general spectral theorem is available. The work covers both scalar and vector Schrödinger operators, including operators with PT-symmetric periodic coefficients, and further extends the developed methods to higher-order differential operators with periodic matrix-valued coefficients.

One of the principal achievements presented in the monograph is the resolution of the longstanding problem of the spectral decomposition of differential operators with complex-valued periodic coefficients, originally posed by I. M. Gelfand in the 1950s. To address this problem, the author developed a series of new analytical methods and introduced important concepts such as “essential spectral singularities” and “singular quasimomenta.” These innovations enabled the establishment of a unified and comprehensive spectral theory for non-self-adjoint ordinary differential operators with periodic coefficients, including higher-order and vector-valued cases.

The monograph also provides a complete classification of the spectra of PT-symmetric periodic differential operators. The theory is developed systematically and rigorously, with detailed proofs and extensive analysis that illuminate the intricate structure of non-self-adjoint periodic operators.

The methods and results presented in this work open new avenues in spectral analysis and have significant potential applications in optics, quantum mechanics, wave propagation, and mathematical physics. Owing to the originality of the contributions, the depth of the analysis, and the broad scope of the material covered, the book constitutes an important reference for researchers, specialists, and graduate students working in operator theory, spectral analysis, and non-self-adjoint problems in mathematical physics.

**Prof. Elman HASANOGLU**